

# Contributions to the Superduck Experiment, 1986

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U.S. Army Corps of Engineers  
Contract No. DACW 3986M3310



Research Report No. CE-88-66

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**DEPARTMENT OF CIVIL ENGINEERING**

Ocean Engineering Group

**UNIVERSITY OF DELAWARE**

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## Abstract

The University of Delaware's Ocean Engineering Group has carried out a project to participate in the Superduck Experiment, 1986 and to investigate some aspects of nearshore circulation and the influence of piling on the incident wave field.

A nonlinear nearshore circulation model of shoreline at the Field Research Facility is developed based on finite difference methods. Time-averaged depth-integrated flow equations are approximated by the method of Ebersole and Dalrymple (1979) and a simple refraction model is incorporated.

Reflection of normally incident waves on a single line of piling for long waves is seen to be small, based on a matched asymptotic analysis.

## 1 Introduction

This report reviews results obtained by the project—An Experimental Study of Nearshore Hydrodynamics, which consisted of three endeavors—1. preliminary numerical modelling of the nearshore region of the Field Research Facility (FRF), 2. determination of the influence of pier-like structures on the wave field and 3. participation in the Superduck experiment to obtain data on rip currents.

## 2 Nearshore Current Modelling

A finite difference method is presented to predict nearshore circulation for a given train of monochromatic waves. Several studies have been conducted at the University of Delaware to calculate nearshore wave-induced currents and mean water levels. This report utilizes the previous nonlinear model in Ebersole and Dalrymple (1979) for calculating the mean steady drift, but presents a simpler method to provide wave directions and heights. A brief description of the time-averaged depth integrated mass and momentum equations will be made in this report.

When a train of waves propagates over a zone of slowly varying depth and/or current, a gradual change of phase velocity results, called refraction. Wave direction  $\theta$  and height  $H$  are obtained on each grid, using the irrationality of the wave number  $k$  and conservation of wave action, defined as  $E/\sigma$ ,

where  $E$  is wave energy proportional to  $H^2$  and  $\sigma$ , the intrinsic frequency. Mean horizontal velocity  $(u, v)$  and mean elevation  $\bar{e}\bar{a}$  are calculated by time-averaged equations. Some difficulties lie on the fact that not only momentum equations are nonlinear but also all equations are coupled. Iteration method is used to solve nonlinear coupled equations until the difference of consecutive horizontal velocities are insignificant.

The present model does not consider the effect of diffraction and nonlinearity on the change of phase velocity. In a region of slowly varying depth and/or current, however, the refraction has been found the dominant factor for most of engineering purposes and a numerous methods of the refraction have been developed. The present simple refraction model, in slightly different form, is published in Dalrymple, 1988.

## 2.1 Refraction model

The wave number and frequency  $\sigma$  of slowly varying water waves are governed by the dispersion relationship.

$$\sigma^2 = gk \tanh kh \quad (1)$$

where  $g$  denotes gravitational acceleration and  $h$  the water depth. The absolute frequency  $\omega$ , defined as  $2\pi/T$ , is related to the intrinsic frequency as follows:

$$\sigma = \omega - k \cos \theta u - k \sin \theta v \quad (2)$$

The wave number vector  $\mathbf{k}$  is represented by a gradient of the phase function and by vector identities the irrationality condition of wave number is obtained.

$$\frac{\partial k \sin \theta}{\partial x} - \frac{\partial k \cos \theta}{\partial y} = 0 \quad (3)$$

It is more convenient to define  $A = k \sin \theta$  and  $B = k \cos \theta$ . Hence equation (3) can be rewritten as

$$\frac{\partial A}{\partial x} - \frac{\partial B}{\partial y} = 0 \quad (4)$$

Once the wave direction is found, the wave height is obtained from the conservation of wave action, which has been found to be invariant for slowly varying water waves. In the absence of dissipation, it becomes

$$\frac{\partial}{\partial x} \left( \frac{E(u + C_g \cos \theta)}{\sigma} \right) + \frac{\partial}{\partial y} \left( \frac{E(v + C_g \sin \theta)}{\sigma} \right) = 0 \quad (5)$$

where  $C_g$  is the group velocity. Defining  $A^*$  as  $E(v + C_g \sin \theta)/\sigma$  and  $B^*$  as  $E(u + C_g \cos \theta)/\sigma$ , equation (5) becomes

$$\frac{\partial A^*}{\partial x} + \frac{\partial B^*}{\partial y} = 0 \quad (6)$$

It should be noted that equation (4) differs from equation (6) by only a sign and therefore the same numerical scheme can be used to discretize both equations.

Equations (4) and (6) are first order hyperbolic equations and said to be in conservation form. The Lax-Wendroff method has been widely used in modelling for these types of equations. In the finite difference method, the domain of interest is partitioned by straight lines into a finite number of meshes, and the points of intersection of the mesh are called nodes  $(x_m, y_m)$ .

$$x_m = x_1 + (m - 1)\Delta x, \quad m = 1, 2, \dots \quad (7)$$

$$y_n = y_1 + (n - 1)\Delta y, \quad n = 1, 2, \dots \quad (8)$$

Let's consider the Taylor expansion of  $A$  at  $(x_{m+1}, y_n)$  in terms of quantities at the previous level in  $x$ .

$$A_{m+1,n} = A_{m,n} + \Delta x \left( \frac{\partial A}{\partial x} \right)_{m,n} + \frac{\Delta x^2}{2} \left( \frac{\partial^2 A}{\partial x^2} \right)_{m,n} + \dots \quad (9)$$

Keeping terms up to the second order and using (4), we have

$$A_{m+1,n} = A_{m,n} + \Delta x \left( \frac{\partial B}{\partial y} \right)_{m,n} + \frac{\Delta x^2}{2} \frac{\partial}{\partial y} \left( \frac{\partial B}{\partial A} \frac{\partial B}{\partial y} \right)_{m,n} + \dots \quad (10)$$

Here

$$\frac{\partial}{\partial y} \left( \frac{\partial B}{\partial y} \frac{\partial A}{\partial y} \right)_{m,n} = \frac{1}{\Delta y} \left[ \left( \frac{\partial B}{\partial A} \right)_{m,n+\Delta y/2} \frac{(B_{n+1} - B_n)}{\Delta y} - \left( \frac{\partial B}{\partial A} \right)_{m,n-\Delta y/2} \frac{(B_n - B_{n-1})}{\Delta y} \right] \quad (11)$$

An average operator is utilized in  $(\partial B / \partial A)$  to give the following difference equation.

$$\begin{aligned} A_{m+1,n} = & A_{m,n} + \frac{\gamma}{2} (B_{m,n+1} - B_{m,n-1}) \\ & + \frac{\gamma^2}{4} \left[ A_{m,n+1} \left( \frac{B_{m,n}}{B_{m,n+1}} - 1 \right) + A_{m,n} \left( 2 - \frac{B_{m,n+1}}{B_{m,n}} - \frac{B_{m,n}}{B_{m,n-1}} \right) + A_{m,n-1} \left( \frac{B_{m,n}}{B_{m,n-1}} - 1 \right) \right] \end{aligned} \quad (12)$$

where  $\gamma = \Delta x / \Delta y$  and  $\partial B / \partial A = -A/B$  are used. In the equation,  $A_{m+1,n}$  is exclusively expressed by the quantities of the previous level and the truncation error of the method is of order  $O(\Delta x^2 + \Delta y^2)$ . Figure 1 shows the schematic representation of computational molecule.

Similarly, wave action equation can be approximated by

$$\begin{aligned} A_{m+1,n}^* = & A_{m,n}^* - \frac{\gamma}{2} (B_{m,n+1}^* - B_{m,n-1}^*) \\ & + \frac{\gamma^2}{4} \left\{ \left[ \left( \frac{v + C_g \sin \theta}{u + C_g \cos \theta} \right)_{m,n+1} + \left( \frac{v + C_g \sin \theta}{u + C_g \cos \theta} \right)_{m,n} \right] (B_{m,n+1}^* - B_{m,n}^*) \right. \\ & \left. - \left[ \left( \frac{v + C_g \sin \theta}{u + C_g \cos \theta} \right)_{m,n} + \left( \frac{v + C_g \sin \theta}{u + C_g \cos \theta} \right)_{m,n-1} \right] (B_{m,n}^* - B_{m,n-1}^*) \right\} \end{aligned} \quad (13)$$

In the case that periodic conditions can be applied, the periodicity requirement for  $A$  can be written by

$$A_{m,n+1-i} = A_{m,i} \text{ for } i = 1, 2, \dots, N \quad (14)$$

The same equation also holds for  $B$ ,  $A^*$  and  $B^*$ . In this report, only periodic boundary conditions are considered.

## 2.2 Time-averaged depth-integrated equations

Since vertical structure of flow is not so important in the nearshore circulation model, by integrating mass and momentum equations over depth the problem is reduced to the more tractable two-dimensional one in space. Slow variation or mean motion induced by waves is obtained by time averaging over a wave period. The detail derivation can be found in the previous reports (Birkemeier and Dalrymple, 1976; Ebersole and Dalrymple, 1979; Kirby and Dalrymple, 1981).

Mean water level,  $\bar{\eta}$  and horizontal velocities ( $u, v$ ) are governed by conservation of mass and momentum equations. The mass equation is, in its final form

$$\frac{\partial \bar{\eta}}{\partial t} + \frac{\partial ud}{\partial x} + \frac{\partial vd}{\partial y} = 0 \quad (15)$$

where  $d$  is the total depth, i.e., sum of depth  $h$  and mean water level,  $\bar{\eta}$ . The horizontal momentum equations can be written as

$$\frac{\partial ud}{\partial t} + \frac{\partial u^2 d}{\partial x} + \frac{\partial uv d}{\partial y} = -gd \frac{\partial \bar{\eta}}{\partial x} - \frac{d}{\rho} \frac{\partial \bar{\tau}_x}{\partial y} - \frac{1}{\rho} \left\{ \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right\} + \frac{1}{\rho} \{ \bar{\tau}_{sx} - \bar{\tau}_{bx} \} \quad (16)$$

$$\frac{\partial vd}{\partial t} + \frac{\partial uv d}{\partial x} + \frac{\partial v^2 d}{\partial y} = -gd \frac{\partial \bar{\eta}}{\partial y} - \frac{d}{\rho} \frac{\partial \bar{\tau}_y}{\partial x} - \frac{1}{\rho} \left\{ \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \right\} + \frac{1}{\rho} \{ \bar{\tau}_{sy} - \bar{\tau}_{by} \} \quad (17)$$

Radiation stresses have been given by Longuet-Higgins and Stewart (1964) as

$$S_{xx} = E \left[ \left( 2n - \frac{1}{2} \right) \cos 2\theta + \left( n - \frac{1}{2} \right) \sin 2\theta \right] \quad (18)$$

$$S_{yy} = E \left[ \left( 2n - \frac{1}{2} \right) \sin 2\theta + \left( n - \frac{1}{2} \right) \cos 2\theta \right] \quad (19)$$

$$S_{xy} = S_{yx} = \frac{E}{2} n \sin(2\theta) \quad (20)$$

$$(21)$$

where

$$E = \frac{1}{8} \rho g H^2 \quad (22)$$

$$n = \frac{C_g}{C} = \frac{1}{2} \left[ 1 + \frac{2kh}{\sinh 2kh} \right] \quad (23)$$

The surface wind stress terms developed by Van Dorn (1953) are written as

$$\bar{\tau}_{sx} = \rho K |W| W_x \quad (24)$$

$$\bar{\tau}_{sy} = \rho K |W| W_y \quad (25)$$

where  $W$  is the wind speed, and  $W_x, W_y$  are wind velocity components in the  $x$  and  $y$  direction. The wind stress coefficient  $K$  is empirically determined and given in the previous reports. Bottom shear stress representations by Liu and Dalrymple (1978) are utilized also.

$$\bar{\tau}_{bx} = \frac{\rho f}{16\pi} \int_0^{2\pi} (u + u_m \cos \theta \cos \sigma t) |u_b| d(\sigma t) \quad (26)$$

$$\bar{\tau}_{by} = \frac{\rho f}{16\pi} \int_0^{2\pi} (v + u_m \sin \theta \cos \sigma t) |u_b| d(\sigma t) \quad (27)$$

$$(28)$$

where  $u_m$  is the maximum wave orbital velocity given by

$$u_m = \frac{\sigma H}{2 \sinh kh}, \quad (29)$$

$u_b$  is the velocity vector at the bottom and  $f$  is the Darcy-Weisbach friction factor. Longuet-Higgins (1970) included the effect of lateral mixing to prevent discontinuity in velocity distribution at the breaker line.

$$\tau_t = -\rho \left( \epsilon_y \frac{\partial u}{\partial y} + \epsilon_x \frac{\partial v}{\partial x} \right) \quad (30)$$

where  $\epsilon_x$  and  $\epsilon_y$  are semi-empirical coefficients. In order to handle wave propagation after wave breaking, wave breaking criteria is used by Noda et al. (1974).

$$\left( \frac{H}{L} \right) = 0.12 \tanh \left( \frac{d}{L} \right)$$

### 2.3 Numerical examples

The present circulation model was tested on the plane beach and found in good agreement with previous results. The big advantage of present model lies on the refraction method which is faster and more stable.

## 3 Wave Reflection from Piling

As waves pass under a pier, the regular and symmetric spacing of the piling result in a reflection of some of the incident wave energy. This problem was examined for the case of long waves approaching a single line of piles with normal incidence. The work has been reported by Martin and Dalrymple (1988) and the highlights will be presented here. The assumptions are that the wavelength of the incident wave is long with respect to the pile spacing and pile diameter. For this case, Martin and Dalrymple show that the reflection  $R$  and transmission  $T$  coefficients can be written as

$$R = \frac{ik(m - \ell)}{(1 - ik\ell)(1 - ikm)} \quad (32)$$

$$T = \frac{1 + k^2\ell m}{(1 - ik\ell)(1 - ikm)} \quad (33)$$

where again  $k$  is the wavenumber,  $m$  is related to the cross-section of the cylinder,  $m = -S/4s$ , where  $S$  is the cross-sectional area of the cylinder,  $2s$  is the center-to-center spacing of the cylinders and  $\ell$  is the *blockage coefficient*, which is a measure of the obstruction of the flow by the presence of the cylinder.

For small cylinders,  $a/s \ll 1$ ,  $m = -\pi a^2/(4s^2)$  and  $\ell = 2m$ . This yields

$$R \equiv -\frac{3ik\pi a^2}{4s}; \text{ and } T = 1 + \frac{i\pi k a^2}{4s} \quad (34)$$

In general these relationships, which are linear in  $ka$  and  $a/s$ , yield very small reflection coefficients for the range of validity. For example, for  $ka = 0.10$ , the reflection coefficients are 0.002, 0.012, 0.024 for the  $a/s$  values: 0.01, 0.05, 0.10.

For larger cylinders, the method of Richmond is used to determine more appropriate values of  $\ell$ . Values are tabulated in Martin and Dalrymple.

Martin and Dalrymple's formulae, (33), also applies to the case of elliptic cylinders, with

$$m = -\frac{\pi ab}{4s}, \quad \ell = \frac{\pi a(a+b)}{4s}, \quad (35)$$

where  $a$  is the semi-major axis in the direction along the piles array and  $b$  is the semi-major axis in the wave direction. For the case that the elliptic cylinder is flattened in the wave direction, so that the cylinders become flat plates normal to the wave direction ( $b = 0$ ),  $m = 0$  and  $\ell$  can be found from Lamb's formula (Lamb, 1946, §306),

$$\ell = \frac{2h}{\pi} \log \left( \sec \left( \frac{\pi a}{2s} \right) \right) \quad (36)$$

This expression is simplified for very long waves to  $\ell = \pi a^2 / (4h)$ . This formula provides a rough estimate for the amount of energy transmitted past an offshore breakwater system, although in general it is likely that the small  $ka$  assumption will be violated for most practical cases.

Further work involving two rows of piling will be reported on at the 21<sup>st</sup> International Coastal Engineering Conference, Dalrymple and Martin, 1988. Oblique incidence can be incorporated through the work of Twersky (1962).

## 4 Field Experiment

During September, 6 - 13, 1986, the U. S. Army Engineering Waterways Experiment Station, Coastal Engineering Research Center, Field Research Facility, located at Duck, N. C., hosted a major nearshore processes experiment known as Superduck. The University of Delaware (Robert A. Dalrymple, Seung Nam Seo and Donald Young) joined the Superduck experiment and helped to collect a wide variety of data such as rip current, wave, sediment transport, and nearshore geomorphology measurements. On the last three experimental days, a strong rip current developed at the experimental site at the north end of the property line. The current field appeared to be strongly affected by the presence of longshore bars and a rip channel. On September 12, five electromagnetic current meters were fixed in the rip current and the feeder current system to characterize the two-dimensional nature of the flow field. Fluorescein dye studies showed that the meters were well located in the rip current, which hugged the north side of the rip channel, presumably forced there by the momentum of the northward flowing longshore current. These data, unfortunately were part of the data lost by the CERC computer system after the experiment and were unavailable for use in calibrating the numerical model results.

Figure 2 shows the simulated wave field based on the data of July 23, 1986. The flow field is shown in Figure 3. Rip current is clearly shown along the scour trench near the pier.

## 5 Conclusions

A simple refraction model is presented and incorporated into the previous flow model developed at University of Delaware. The present model is found to be faster than the previous ones and utilizes Lax-Wendroff method for modelling the equations in conservation form.

Reflection of normally incident water waves from a single line of piling is very small for the case of wide spacing,  $ks < 1$ , and small diameter piles,  $ka < 1$ . There may under some circumstances be a small amount of reflection from pier structures, which would influence the nearshore circulation.

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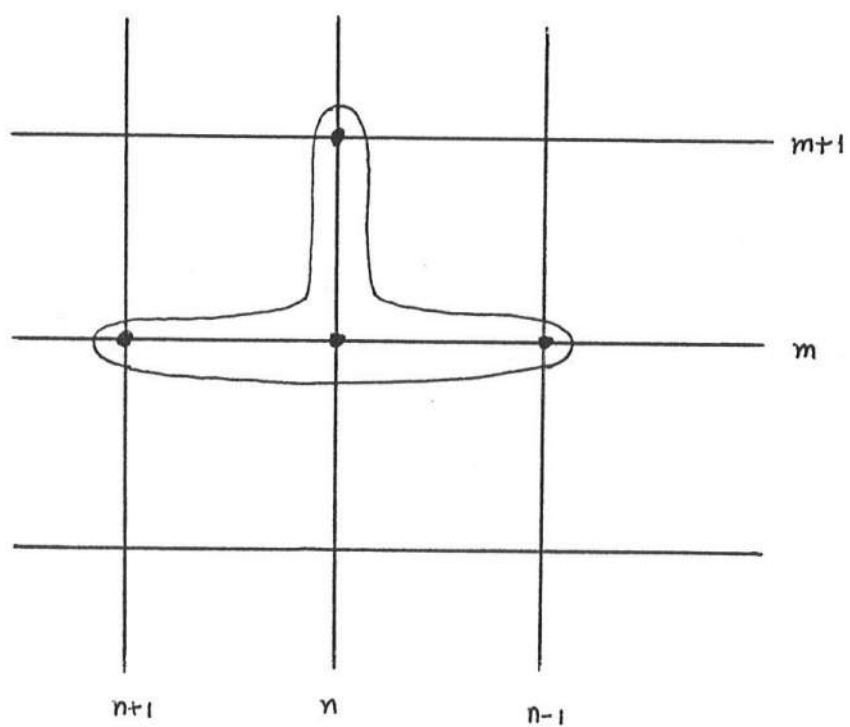
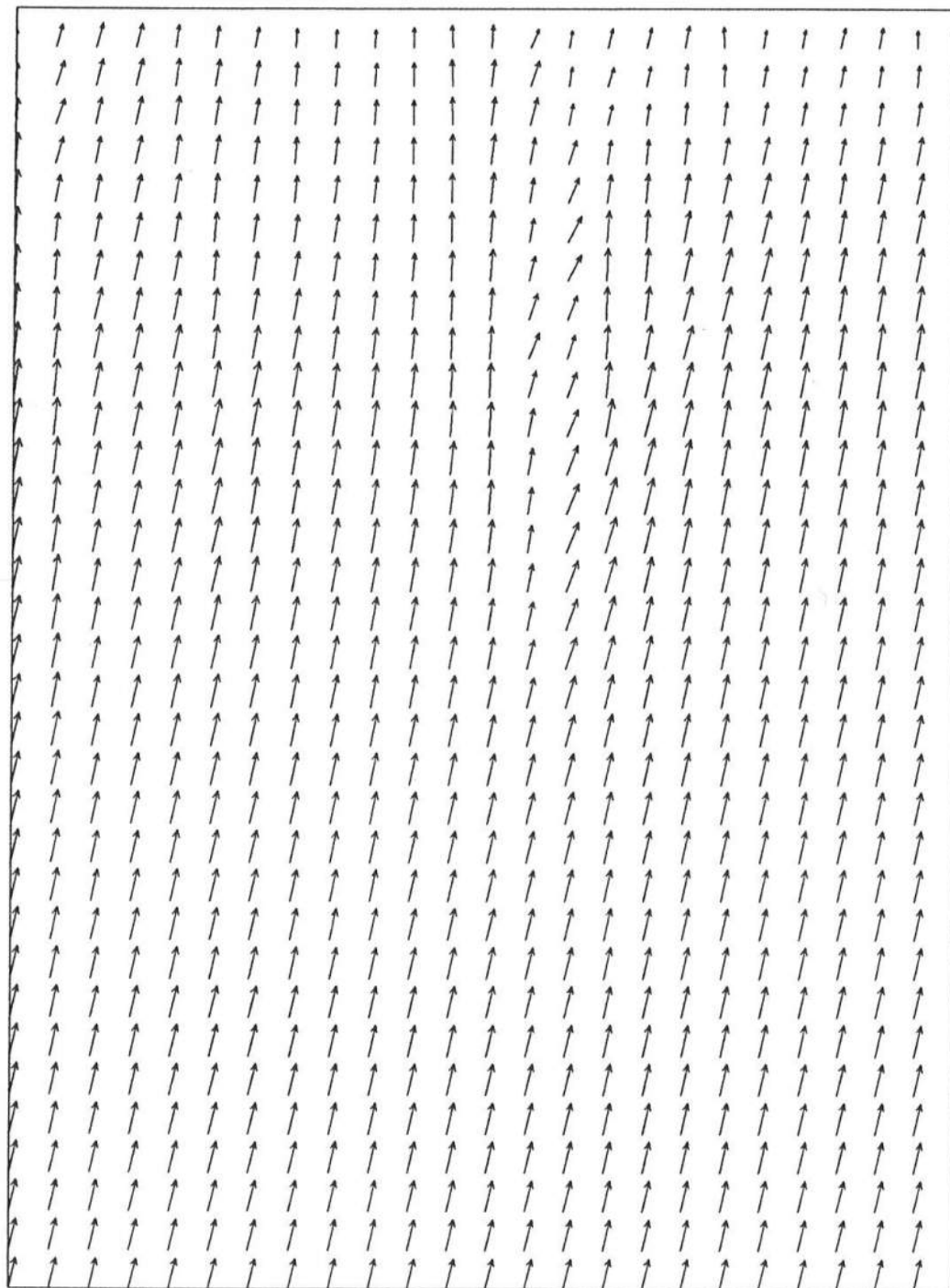


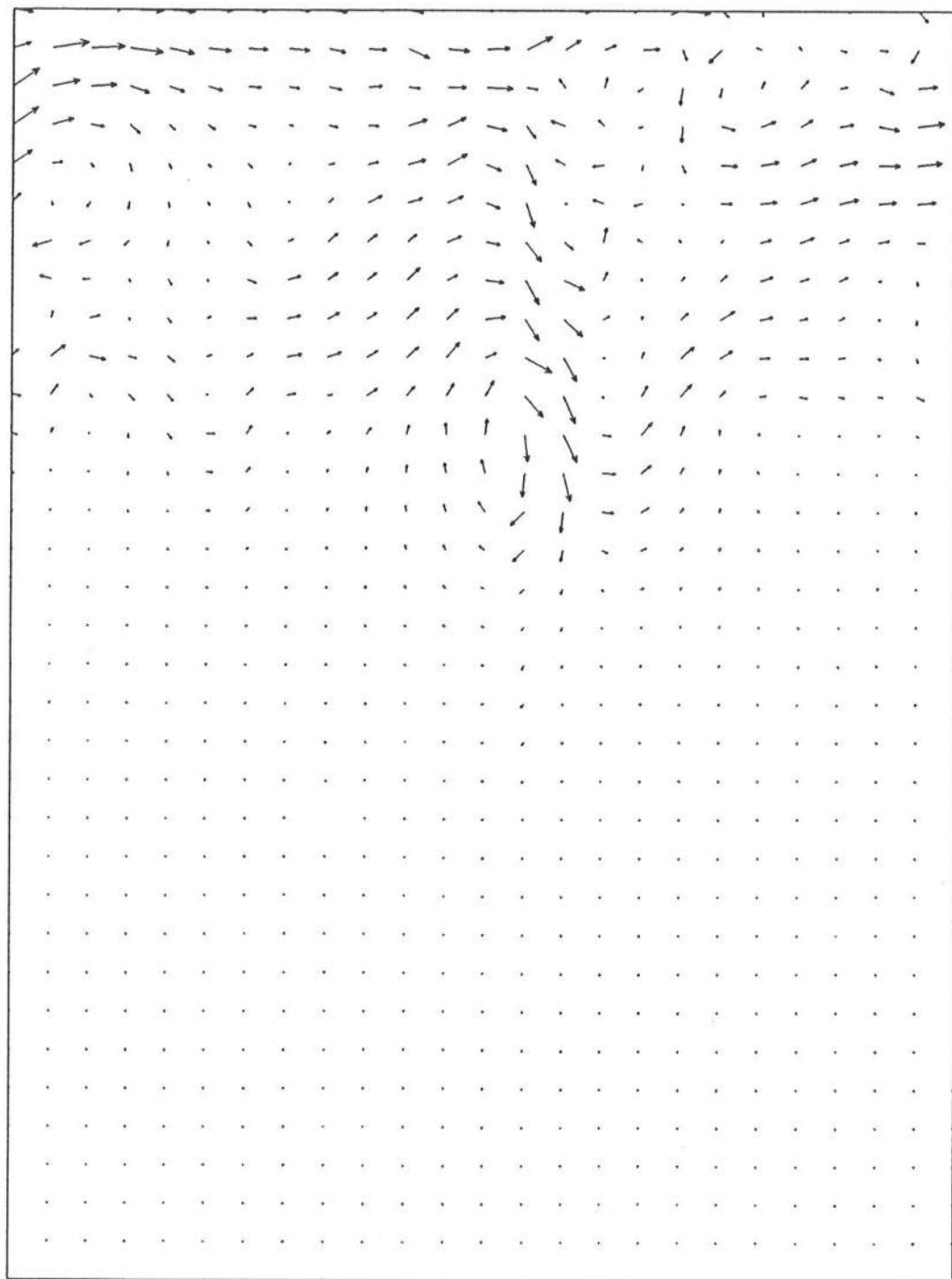
Figure 1 .

# WAVE FIELD OF DUCK CIRCULATION MODEL

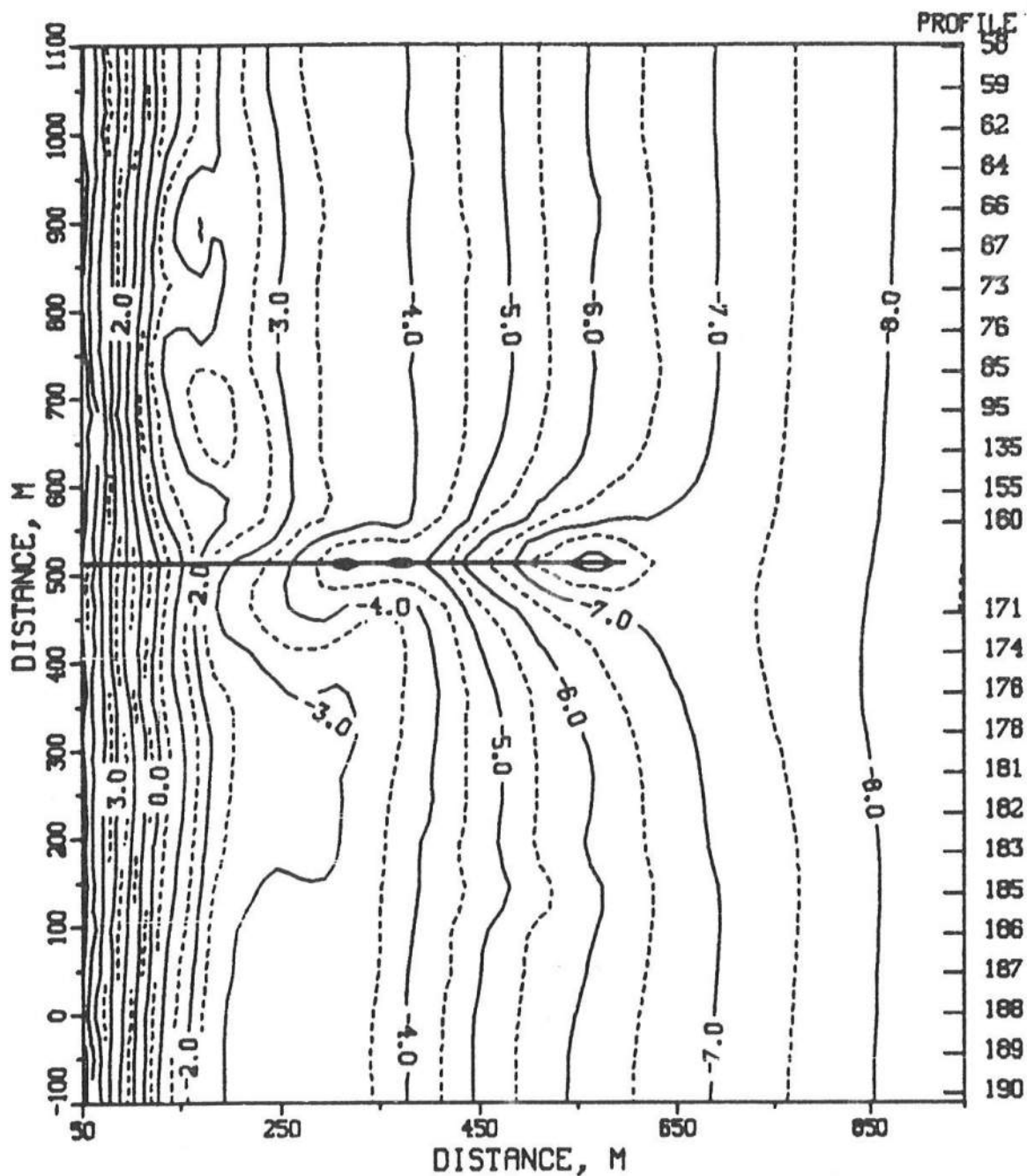


0.309E+01  
→  
MAXIMUM VECTOR

# FLOW FIELD OF DUCK CIRCULATION MODEL



0.145E+01  
MAXIMUM VECTOR



FRF BATHYMETRY 23 JUL 86  
CONTOURS IN METERS