

# IMPROVING THE RATE OF CONVERGENCE OF ‘HIGH ORDER FINITE ELEMENTS’ ON POLYHEDRA I: A PRIORI ESTIMATES

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ABSTRACT. Let  $\mathcal{T}_k$  be a sequence of triangulations of a polyhedron  $\Omega \subset \mathbb{R}^n$  and let  $S_k$  be the associated finite element space of continuous, piecewise polynomials of degree  $m$ . Let  $u_k \in S_k$  be the finite element approximation of the solution  $u$  of a second order, strongly elliptic system  $Pu = f$  with zero Dirichlet boundary conditions. We show that a weak approximation property of the sequence  $S_k$  ensures optimal rates of convergence for the sequence  $u_k$ . The method relies on certain a priori estimates in weighted Sobolev spaces for the system  $Pu = 0$  that we establish. The weight is the distance to the set of singular boundary points. We obtain similar results for the Poisson problem with mixed Dirichlet–Neumann boundary conditions on a polygon.

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