# IMPROVING THE RATE OF CONVERGENCE OF 'HIGH ORDER FINITE ELEMENTS' ON POLYHEDRA II: MESH REFINEMENTS AND INTERPOLATION

CONSTANTIN BACUTA, VICTOR NISTOR, AND LUDMIL T. ZIKATANOV

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Abstract. We construct a sequence of meshes $\mathcal{T}'_k$ that provides *quasi-optimal rates of convergence* for the solution of the Poisson equation on a bounded polyhedral domain with right hand side in $H^{m-1}$, $m \geq 2$. More precisely, let $\Omega \subset \mathbb{R}^3$ be a bounded polyhedral domain and let $u \in H^1(\Omega)$ be the solution of the Poisson problem $-\Delta u = f \in H^{m-1}(\Omega)$, $m \geq 2$, $u = 0$ on $\partial\Omega$. Also, let $S_k$ be the Finite Element space of continuous, piecewise polynomials of degree $m \geq 2$ on $\mathcal{T}'_k$ and let $u_k \in S_k$ be the finite element approximation of $u$, then $\|u - u_k\|_{H^1(\Omega)} \leq C \dim(S_k)^{-m/3} \|f\|_{H^{m-1}(\Omega)}$, with $C$ independent of $k$ and $f$. Our method relies on the a priori estimate $\|u\|_{\mathcal{D}} \leq C\|f\|_{H^{m-1}(\Omega)}$ in certain anisotropic weighted Sobolev spaces $\mathcal{D} = \mathcal{D}^{m+1}_{a+1}(\Omega)$, with $a > 0$ small, determined only by $\Omega$. The weight is the distance to the set of singular boundary points (*i.e.,* edges). The main feature of our mesh refinement is that a segment $AB$ in $\mathcal{T}'_k$ will be divided into two segments $AC$ and $CB$ in $\mathcal{T}'_{k+1}$ as follows: $|AC| = |CB|$ if $A$ and $B$ are equally singular and $|AC| = \kappa|AB|$ if $A$ is more singular than $B$. We can choose $\kappa \leq 2^{-m/a}$. This allows us to use a uniform refinement of the tetrahedra that are away from the edges to construct $\mathcal{T}'_k$.

## Contents

*E-mail address*: bacuta@math.udel.edu

University of Delaware, Mathematical Sciences, Newark, DE 19716

*E-mail address*: nistor@math.psu.edu

*E-mail address*: ltz@math.psu.edu

Pennsylvania State University, Math. Dept., University Park, PA 16802

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