

Schur Complements on Hilbert Spaces and Saddle Point Systems

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Abstract

For any continuous bilinear form defined on a pair of Hilbert spaces satisfying the compatibility Ladyshenskaya-Babuška-Brezzi condition, symmetric Schur complement operators can be defined on each of the two Hilbert spaces. In this paper, we find bounds for the spectrum of the Schur operators only in terms of the compatibility and continuity constants. In the light of the new spectral results for the Schur complements, we review the classical Babuška-Brezzi theory, find sharp stability estimates, and improve a convergence result for the inexact Uzawa algorithm. We prove that for any symmetric saddle point problem, the inexact Uzawa algorithm converges provided that the inexact process for inverting the residual at each step has the relative error smaller than $\frac{1}{3}$. As a consequence, we provide a new type of algorithm for discretizing saddle point problems, which combines the inexact Uzawa iterations with standard a posteriori error analysis and does not require the discrete stability conditions.

Key words: inexact Uzawa algorithms, saddle point system, multilevel methods, adaptive methods

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