IMPROVING THE RATE OF CONVERGENCE OF 'HIGH ORDER FINITE ELEMENTS' ON POLYGONS AND DOMAINS WITH CUSPS

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ABSTRACT. Let u and $u_V \in V$ be the solution and, respectively, the discrete solution of the non-homogeneous Dirichlet problem $\Delta u = f$ on $\mathbb{P}$, $u|_{\partial \mathbb{P}} = 0$. For any $m \in \mathbb{N}$ and any bounded polygonal domain $\mathbb{P}$, we provide a construction of a new sequence of finite dimensional subspaces V_n such that $\|u - u_{V_n}\|_{H^1} \leq C \dim(V_n)^{-m/2} \|f\|_{H^{m-1}}$, where $f \in H^{m-1}(\mathbb{P})$ is arbitrary and C is a constant that depends only on $\mathbb{P}$ and not on n (we do *not* assume $u \in H^{m+1}(\mathbb{P})$). The existence of such a sequence of subspaces was first proved in a ground–breaking paper by Babuška [1]. Our method is different; it is based on the homogeneity properties of Sobolev spaces with weights and the well–posedness of non-homogeneous Dirichlet problem in suitable Sobolev spaces with weights, for which we provide a new proof, and which is a substitute of the usual "shift theorems" for boundary value problems in domains with smooth boundary. Our results extend right away to domains whose boundaries have conical points. We also indicate some of the changes necessary to deal with domains with cusps. Our numerical computation are in agreement with our theoretical results.

REFERENCES

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