What is a good teaching? Who is a better teacher?

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Here I wish to share several thoughts on these questions. Usually they are discussed at lunches and dinners, and having drinks ready seems to help.

Is such a discussion really needed? I am not sure. I think some people have their answers already. Many others may think that the questions have no answers, and I agree with them. Nevertheless, discussing questions that we have no answers to is still worthwhile, I think.

Mathematicians know well that not every mathematical question has an answer. Not just that we do not know it, but in a well defined sense, giving an answer is impossible. Like the "Word problem for groups", or the "Mortal matrix problem", or Hilbert's 10th problem, or Conway's Game of Life. Outside of mathematics there are many questions like this. They are grammatically perfect, but have no answers. For example, *What is love? What is happiness*? Most people agree that it is impossible to define or to measure love or happiness, or to argue about religious or political choices. People agree that not everything in life can be compared or measured. *Is X funnier than Y? Or better looking? Does X play Hamlet better than Y*?

At the same time, when it comes to measuring the quality of teaching, we rarely raise any objection to the whole idea. Why? **I think the problem of defining good teaching and comparing different teachers in any serious way has no solution.** An immediate objection to this statement is that it defies common sense: Did not everyone of us have good or bad teachers? But when it comes to common sense we all have to be careful: according to common sense the Earth is flat, and the Sun rotates around it. About having good or bad teachers: Of course, we did! Only they were good or bad for **me**, or they were good or bad for **you**.

There exists a belief among some people, that good teaching can be defined, measured, and teachers can and must be compared. That it is possible to say where the median score is, and who is 2 points below or above the median, or even 0.2 of a point. Through

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At this time, it is impossible for me to distinguish clearly my own thoughts related to these questions from the ones I heard from others during many years, or read somewhere. I wish to apologize in advance to everyone who I forget to mention.

many conversations I had with my colleagues from this department or from other schools, including people who moved to administration, I concluded that most of them agree that it is **not possible** to solve this problem in any satisfactory manner. Then why we go along with the current practice and do it? Often, after this question, the conversations died. I believe that people did have an answer to the last question, but did not want to discuss it.

I will not discuss it here either. Instead I will try to substantiate the claim that the problem of defining good teaching and comparing different teachers has no solution. Then I will suggest what can be done about it.

We are mathematicians. Do we not have to define first what good teaching is before we measure or compare it? I do not know such a definition. I know that others attempted to give it, and I will be grateful to someone who can show me one which is hard to criticize.

Here we go. Let us simplify the question and consider just teaching a mathematics course.

1. Teaching any course, one has to decide, first of all, about the **goals**.

Yes, the goals. And often this is the hardest decision for me. Even in courses that I have taught many times.

(a) The first goal is, of course, to "cover the syllabus".

What does this mean?

- Maybe every topic from the syllabus should be touched, and x problems related to topic N should be solved. What is $x = x(N)$? How many of those should be trivial, easy, medium or hard?
- What is the level of difficulty of problems that students should be able to solve on homework or on exams?
- Should students be able to say or write the definitions of the notions they deal with or not? Should they be able to memorize main formuli, or state the most important theorems from the course?

- Should my (or every) course help students to develop their problem solving abilities, or should this be delegated to only special courses on Problem Solving?

- Should students be presented with some history of the course content and ideas? If Yes, what should be the total number of hours per semester spent on this? Spending even 5 minutes each time will result in hours total. Should students know the names of the main players, should they be able to place them in the right century and/or say in 2 – 3 sentences about what these people did? Or should this be completely delegated to a History of Mathematics course?

- Should my course reinforce students previous knowledge of mathematics, deepen it, and how much time should be spent on that?

- Should the course treat mathematics as a part of human culture and an intellectual endeavor, or just a collection of techniques to solve certain classes of problems? Here are just a few quotations by the best of us:

  The real end of science is the honour of the human mind. – Carl Jacobi.

  Mathematics is the most beautiful and most powerful creation of the human spirit. — Stefan Banach.

  Mathematics are the result of mysterious powers which no one understands, and in which the unconscious recognition of beauty must play an important part. Out of an infinity of designs a mathematician chooses one pattern for beauty's sake and pulls it down to earth. — Marston Morse.

  If one thinks that the age of romanticism in mathematics is over, here is a more recent mentioning of *'beauty'*:

  It is our responsibility to succeed at teaching mathematics ... so that our discipline realizes its full potential as a subject of beauty... – Manifesto: A declaration of values, MAA Instructional Practices Guide.

  So, should one of my goals be to emphasize the beauty of mathematics in my course, or *"There is no lack of better things for us [me] to do"*?[2]

- Should a goal be that in the future some students will decide to acquire and read a book related to mathematics just for pleasure, or for intellectual growth? Or one

[2] As Huygens commented on Fermat's number theoretic challenges.

day they decide, like James R. Newman, who was a lawyer and then a government official, to publish something like four-volume *"The World of Mathematics"*? Or they will vote to give money to mathematical community, or will donate their own money?

A conventional answer to all these questions is that a good teacher must address all of them. Those of us who tried it, soon realized that it was clearly impossible to make a satisfactory progress on all of them. Therefore one has to choose.

I could not find teaching goals on our department web page. The department does not set nor discusses them.

How, without first agreeing on goals, can we decide whose course or whose teaching is better? Should our choice of a subset of these goals be reflected in evaluation of our teaching?

(b) Here are other questions that may not be directly related to the syllabus, but are of great importance when we prepare for a course.

- Who is "an average" student, what does she/he know at the beginning of the course, and how much time should she/he spend on homework?
- Should students submit solutions of homework problems and how many submissions per semester should we ask?

  Should they be solving only problems they are asked to submit, or also many other problems? How many others?
- Should the collected solutions be graded and returned to each student with comments?
- Should the solutions of submitted homework problems be distributed when the graded homework is returned?
- Should we curve the final grades for the course at the end, or it is fine to have 65% A's or to have 65% F's?
- Should the students be held responsible for the quality of the presented solutions? Should they be given 2 points out of 10 for presenting a meaningless "solution"

(long or short), or 0 points, or should they be penalized with receiving -2 points for it?

I have heard answers to these questions from many experienced teachers, and **often they were opposite**.

Let us continue.

- Should teachers care about the ethical development of their students?
  Help them to develop their characters? Social graces? Honesty? Responsibility, and other important traits that make for an upstanding citizen?

  Should we care about this at all? If the answer is No, skip this part. If it is Yes,

  - Should we require attendance, or give extra points for it? Or should it be their choice and we treat them as adults?
  - Should we offer extra points to students for completing the instructor's evaluation at the end of the course (seems that three reminders often are not enough)?
  - Should we give extra points to students for attempting some harder problems, those that will not appear on tests and not affect their grade? Or should we just say "enjoy", and that we will be happy to discuss the problems with them any time later?
  - Should we require that students submit all work in time, or any later submission is ok too?
  - Should they be typing their solutions?
  - Should they be asked whether they have read the chapter of the text or their class notes before they come to an office hour with questions? Of course, when it happens the first time, we just try to find the answers in those sources together with them (being careful not making them feel bad). But what do we say when this happens for the second or for the fifth time? Keep doing the same? Or, maybe, they have a reading disability without knowing about it? What if it takes them five minutes to find/recall the question they have?

– How much extra help should we offer in addition to office hours, and should it depend on the student?

Well, it is time to discuss how we measure and compare teaching.

2. Students evaluations and Faculty evaluation by the Chair.

First, a few questions about the evaluations students are asked to write.

- What if **at the beginning of the course**, when instructor passes a questionnaire with the question *"Would you take this course if it was not required?"*, and 28% of students answer No?
- How should the chair view students evaluations if only $n\%$ of the class submitted them? $n = 30, 50$, or $70, \ldots$.
- **If we really care** about what students think of our teaching, why does the university allow them to see their grades **only after** they complete their evaluations? Are there legal problems with this? If No, why is this not done? If Yes, why is the faculty required to solicit the evaluations?
- What if there are obvious contradictions in the numbers? For example, 85% of those who submitted evaluations claim that the instructor was ALWAYS available during the assigned office hours, while 15% say NEVER? Should those students be trusted with their answers to other questions?
- What if 80% of the class say that the instructor was ALWAYS prepared for the class, but 20% disagree? How do students judge this? Do they expect Power Point presentations? Videos? Slides? Teacher's typed notes? Do they disapprove the instructor's decision to write on the board and without looking in her/his notes? Do they disapprove of the instructor's decision to react to a question asked by a student during the lecture (that the instructor finds very useful), and spend 15 minutes answering and discussing it? What if such an interruption happens, and the instructor asks students to read a page or two from the text or her/his notes on the missing part of the lecture, and ask questions before the next meeting if they have any?

- What if students write that the course was so hard that 2/3 of the class dropped it during the first two weeks of the course, when really 1/8 of the class did it? Should those students be trusted with their answers to other questions?

- What if some students write that the course was very hard, but only 15% of the class ever attended office hours (usually the same people, and often not those who complained)? Or, despite the encouragement from the teacher, never asked mathematical questions over email? If a question is asked in email, is it too much for a teacher to ask students instead of just writing *"I cannot do #10 from the Exercise set for Section 2.4"*, to type a question in its entirety (can sometimes help them to answer the question immediately), or indicate what their difficulty is (e.g., *"What does this word mean?", "... I do not know how to begin", "... I came to this part, but do not know how to continue", ...*)?

- Is it possible to be an excellent teacher to every student in the class? I was fortunate to have several great teachers (**great for ME**!), but some of my classmates did not like them at all. And these teachers taught in very different ways, and would definitely receive **opposite** scores on many question in our students' evaluations.

(b) What other tools can a Chair use besides students' evaluations?

- Other teaching related activities, beside the students evaluations, should be given **substantial weight** when the Chair evaluates Faculty. Like undergraduate and graduate research, publishing on teaching or teaching practices, giving reading courses (in addition to regular ones), publishing expository articles in MAA journals, sharing teaching materials with the world on personal web pages, etc.

- What about organizing teaching related seminars? There are 6 regular research seminars in the department, but only 1 regular teaching seminar. At the same time, **every** faculty member teaches. Is this not strange? What does it say about the commitment of Faculty to teaching? The statement about high quality teaching is at the top of the UD Mission statement, and it is the first question that we answer when preparing our web form for the annual evaluation by the Chair.

- The chair's impression of the classes she/he visited? Yes.

- The letters from other faculty visiting the classes? Yes. What if we submit not 1 or 2 such letters, but 5 or 6? Suppose all are strongly positive? Will it positively affect Chair's teaching evaluation of the faculty?

  But these visits allow people to only see a minuscule part of **teaching**, and the reports reflect the **visitors'** ideas about good teaching. I am not mentioning here The Hawthorne Effect, also called the Observer Effect, which is where people in studies change their behavior because they are watched. In addition, I think that in order to make any reasonable judgement one has to attend **many** classes, see **many** homework and feedback students receive from grading, see **every** exam and its relation to lectures or homework, attend office hours or read all emails related to teaching that the teacher and the students exchanged during the semester... .

Too much to ask? I agree. **No Chair can do it**, and **no faculty member** has enough time to help the Chair with this task.

I do have **my answers** to many questions above, and other teachers have **their answers**.

What can be done then about evaluating teaching?

I suggest:

- When some teachers have **unusually high or unusually low students evaluations**, or **unusually high or unusually low final grades in their courses**, or give some other reasons for the Chair to be concerned, the Chair should, perhaps together with a committee of faculty, look very carefully into **all** those cases. The committee may have about 1/6 of the total number of faculty, representing people with various views on teaching, of different ranks, and different tenure status. Serving on this committee can be one or two year task, and the committee should be selected by the faculty.

  The excellent teachers should be applauded, and should be encouraged to share their teaching philosophies, methods, and course materials with others. Personal web pages are great for this.

  And bad teachers should be dealt with in some ways.

In all other cases teachers should be given **THE SAME** grade.

- No formula is good or is needed. Paraphrasing a famous statement by Hermann Weyl about the tax law: *an archeologist who, five thousand years from now, shall unearth some of our methods for evaluation of teaching, will probably date them centuries earlier, certainly much before Galileo and Viète.*

  It is possible to trick formuli, and it is not hard to trick students into believing that we teach them well.

  At the same time, it is very hard to teach well, or even to learn the subjects we teach really well.

Most of us are here because we want to teach mathematics. To share what we love, to keep the flame of mathematics alive, to pass it on to future generations...

The following thought is close to my heart:

The essence of mathematics lies in its freedom. -— Georg Cantor.

I think, the essence of teaching also lies in its freedom, and that teaching the best way we can is a

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for every one of us. Faculty needs help and encouragement, not a competition and ranking when it comes to teaching.

I do not suggest to abolish evaluations by students. I like to see them myself, and sometimes, they are illuminating. I do not suggest that they are not seen by the Chair or other administrators. I suggest that they **are not used in any substantial way** to assess the quality of someone's teaching.

During a presentation of my talk on this subject, I conducted an anonymous poll, asking 31 attendees to answer the following question:

Suppose you know that the students' evaluations of your courses will not be used by the Chair or administration in assessing your teaching. Will it affect your teaching (a) in a positive way, (b) in no way, (c) in a negative way?

The results were: 35% chose (a), 65% chose (b), no one chose (c).

I plan to continue my discussion of teaching in the subsequent versions of this document. At this time I plan to write sections titled

- Knowledge of Course Material,
- What does “Understanding of Mathematics" mean?,
- My Teachers,
- Successes and Failures,
- My Favorite Books on Teaching Mathematics,
- Comments on the book by Parker J. Palmer *“The Courage to Teach"*.

When a new version is ready, it will replace the old one on my “teaching page"

`http://www.math.udel.edu/~lazebnik/Info/teaching.html`

Stay tuned.