# Topics Course for Undergraduates. UD. Spring 2022.

Course Title: Mathematical Stories: Problems, History, Solutions, Theories.

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Course Summary.

1. Most of mathematics that I know began with interesting and hard problems. Those were hard to solve using method available at the time, and people who solve them often had to come up with new ideas. Later some of these ideas were developed into theories. These theories were presented in books, and taught to students. Sometimes, methods that were successful in solutions of some problems found applications in the areas seemingly far removed from those, and new more powerful theories were formed. That is how Mathematics develops.

2. Sometimes a solution of a famous problems is presented as an applications of the theory that the problem generated. In these cases, one gets an impression that it is not suitable for presentation in undergraduate courses because the students may have no sufficient prerequisites to understand the solutions. Often it is not the case. Sometimes the original solutions were hard to understand or were long. Later, those solutions were understood much better and could be presented in a much faster and self-contained way.

In this course, we will consider several illustration of the phenomena described in 1. and 2.

Syllabus / topics outline.

This is a list of possible topics. Probably only 6 or 7 of the topics will be used (total about 36-38 class hours). The decision which topics to include will be influenced by the interests of the students enrolled in the course.

- Rational, irrational, algebraic and transcendental numbers. Continuous fractions. Theorems by Lagrange, Dirichlet, Liouville and Cantor. Irrationality of numbers $e$ and $\pi$.
- Equidecomposability in the plane. Wallace - Bolyai - Gerwein Theorem: any two polygons in a plane with equal areas are equidecomposable, i.e., one can be cut by straight lines in such a way that another can be assembled out of the obtained parts. Non-equidecomposability in 3-space. Hilbert's Third Problem and its solution.
- Borsuk's Conjecture. Its history, proofs of several cases, and its refutation by Kalai and Kahn using Extremal combinatorics.
- Solutions of polynomial equations in radicals. Cardano and Ferrari formuli, and how one can use them. Abel's theorem (without Galois theory): Insolvability of the quantic .
- Affine and projective geometry. Applications to Euclidean geometry. Theorems of Desargues and Pappus and their relation to associative and commutative laws of number systems. Quaternions. Finite planes and combinatorics. Non-Desarques planes.

- Cycloid. Brachistochrone and Tautochrone problems and their solutions.
- The Isoperimetric problem: Let C be a simple closed curve in the plane with length $L$ and bounding a region of area $A$. Then $L^2 \geq 4\pi A$, with equality if and only if $C$ is a circle.
- Fermat - Torricelli point in a triangle and beyond. Minimum spanning tree of a graph and connector problem in the plane: Given $N$ points on the plane. We are allowed to add k more points, and then put $N + k - 1$ segments with both endpoints among the $N + k$ points such that obtained connected graph (it must be a tree) is of the minimum length. (About 4 hours.)
- Fermat's challenges. Theorem of Euler, bounds on the function $\pi(x)$. Number theory and Cryptography: Public-key cryptography, RSA, digital signatures, Quadratic residues modulo prime $p$ and cryptography.

Assessment: Four-five open book homework assignments. I will comment on students solutions, grade them and return.

Two close book exams in class, each exam covering about half of the course. Each exam will be made of theoretical questions covered in the course (list of those will be provided in advance) and problems. Problems will be chosen from those discussed on lectures and given as homework.

Weights: Homework 40%, Each of the two exams: – 30%.

Textbook and other sources: The material is scattered over many books, MAA journals, internet. A list will be send to the class before the semester begins, but it can be done earlier by request. All materials will be available to students in the form of electronic files or web links.

Prerequisites: Math 210, Calculus I, II, III ( III is preferable but not mandatory), Linear algebra (preferable, but not mandatory).