PHYS208 Fundamentals of Physics II

Useful Constants:

$\kappa = 8.99 \times 10^{9}\, Nm^2/C^2$

$\varepsilon_0 = 8.85 \times 10^{-12}\, F/m$

$e = 1.60 \times 10^{-19}\, C$

Force between two point charges

$$F = k\frac{|q_1||q_2|}{r^2}$$

Electric field for a point charge

$$E_1 = k\frac{|q_1|}{r^2}$$

Force on a charge in an electric field

$$\vec{F}_{21} = q_2\vec{E}_1$$

Gauss's law

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enl}$$

note that this is a surface integral

Definition of electric potential

$$V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{S} \qquad \begin{cases} V_\infty = 0 \\ V_G = 0 \end{cases}$$

or $\quad V_i = \int_i^\infty \vec{E} \bullet d\vec{S}$

note that this a line integral

Electric potential for a point charge

$$V = k\frac{q}{r}$$

Capacitance

$$C = \frac{q}{V}$$

Integrals Table

$$\int \frac{dx}{(a+bx)} = \frac{1}{b}\ln(a+bx)$$

$$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{dx}{\sqrt{(a^2+x^2)}} = \ln\left(x+\sqrt{(a^2+x^2)}\right)$$

$$\int \frac{dx}{(a^2+x^2)} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{(a^2+x^2)}}$$

Exam 1, Phys208 **September, 2017**

Name: ____________________________

Discussion section number____________________

1. **(15 Pts)** Use Gauss's law to calculate the electric field of the following charged objects. Use any variable or symbol stated below and constant: ε_0.).

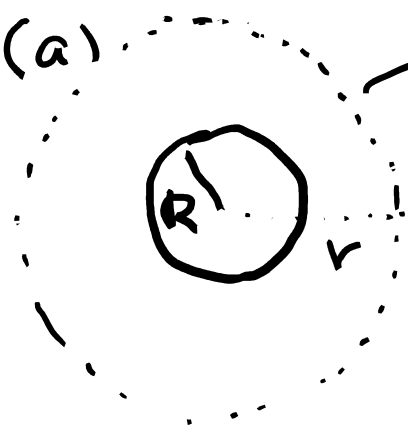
Show details to earn credits. Just write down the answer will earn no credits. You can use these results in other problems if electric field is needed in your calculation

(a). Outside an insulating sphere with a radius *R* and total positive charge Q uniformly distributed over the sphere.

(b). Outside a very long wire with a linear charge density λ.

(c). Outside a very large sheet with a surface charge density σ.

(a)



Gaussian Surface

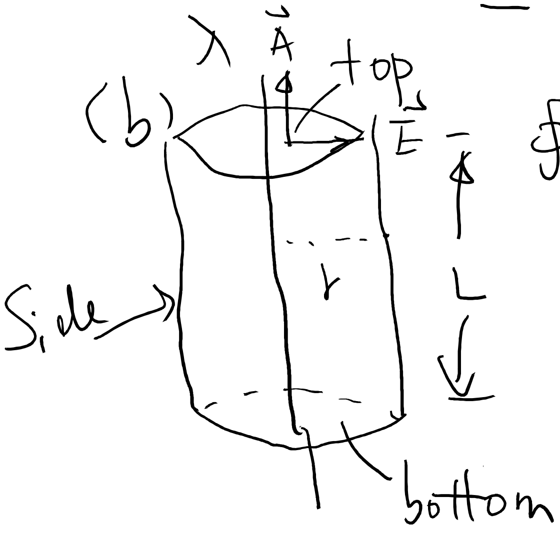
$$\oint \vec{E}\cdot d\vec{A} = q_{en}/\varepsilon_0$$

$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = E\cdot 4\pi r^2$$

$$E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

(b)

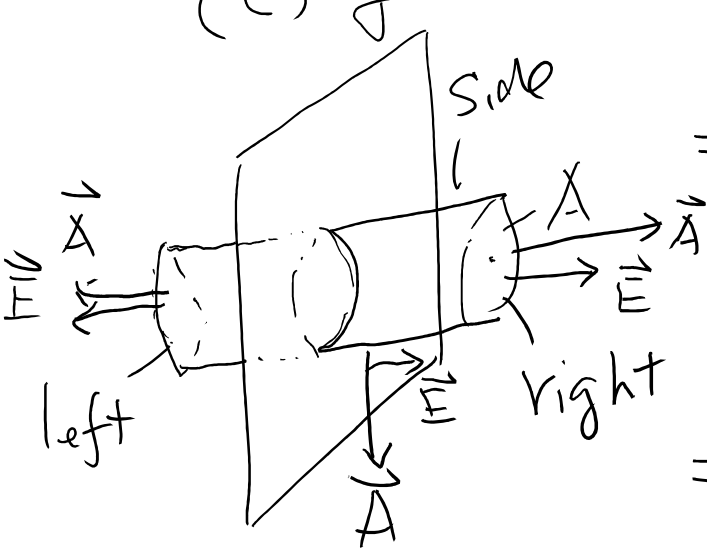


$$\oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0$$

$$\int_{Top} \cancel{\vec{E}\cdot d\vec{A}}^{\,0} + \int_{bottom} \cancel{\vec{E}\cdot d\vec{A}}^{\,0} + \int_{side} \vec{E}\cdot d\vec{A}$$

$$= \int_{side} E\,dA = E\cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

(C)



$$\oint \vec{E}\cdot d\vec{A}$$

$$= \int_L \vec{E}\cdot d\vec{A} + \int_R \vec{E}\cdot d\vec{A} + \underbrace{\cancel{\int_{side} \vec{E}\cdot d\vec{A}}}_{\to 0}$$

$$= 2\int_R E\, dA$$

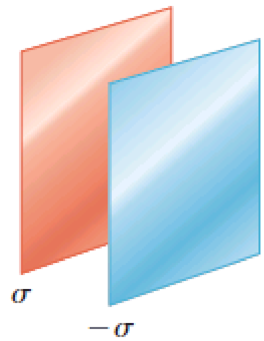
$$= 2E\int_R dA$$

$$= 2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

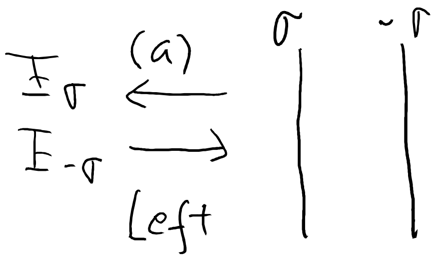
$$E = \frac{\sigma}{2\varepsilon_0} \quad *$$

2. (15 pts) Two infinite, nonconducting sheets of charge are parallel to each other and separated d as shown in the figure below. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at the following points. (Use any variable or symbol stated above along with the following as necessary: ε_0.)

(a) to the left of the two sheets
(b) in between the two sheets
(c) calculate the potential difference between two sheets
(d) Find the electric fields in all three regions if both sheets have *positive* uniform surface charge densities of value σ.
(e) Under the condition (e), calculate the potential difference between two sheets.



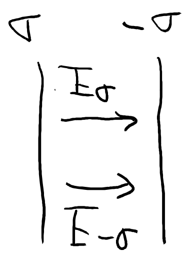
(a)



Left :

$$E = E_\sigma - E_{-\sigma} = 0$$

(b)

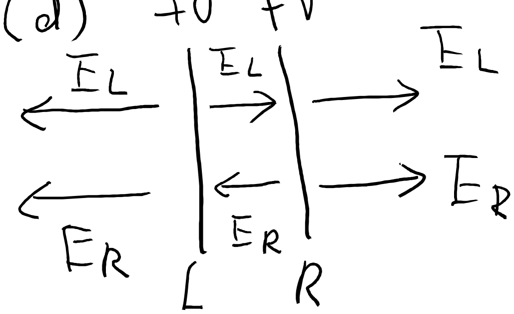


$$E = E_\sigma + E_\sigma = 2 \times \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

(c) $$V_{\sigma,-\sigma} = E \cdot d = \frac{\sigma d}{\varepsilon_0}$$

$+\sigma$ sheet has higher potential

(d)



Left : $E = \frac{\sigma}{\varepsilon_0}$

middle : $E = 0$

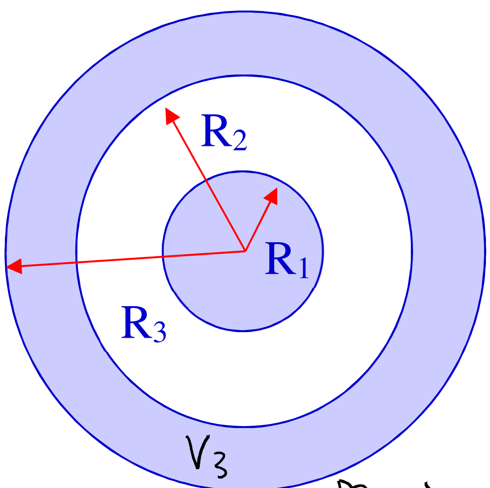
Right : $E = \frac{\sigma}{\varepsilon_0}$

(e) $$V_{\sigma,+\sigma} = E_m \cdot d = 0\ (V)$$

3 (40pts) A conducting sphere of radius R_1 is placed at the center of a conducting spherical shell of inner and outer radii of R_2 and R_3. The solid sphere and spherical shell carry charges of $+q_1$ and $+q_2$, respectively.

(1) (2.5 pts) Calculate E(r) at $r>R_3$.
(2) (2.5 pts) Calcularte E(r) at $R_3 > r>R_2$
(3) (5 pts) Calculate the potential of the spherical shell.
(4) (5pts) Calculate the potential at $R_3 > r>R_2$
(5) (5 pts) Calculate the potential difference between two charged spheres.
(6) (5 pts) what are charges on the inner and outer surfaces of the spherical shell.

Now, the outer conducting spherical shell is grounded.
(7) (2.5 pts) Calculate the potential of the spherical shell.
(8) (2.5 pts) Calcularte E(r) at $R_3 > r>R_2$
(9) (5 pts) Calculate the potential difference between two charged cylinders.
(10) (5 pts) what are charges on the inner and outer surfaces of the spherical shell.



(1) From P. 1

$$E(r) = \frac{q_1+q_2}{4\pi\varepsilon_0 r^2} \qquad r > R_3$$

(2) Since it is a conducting Shell

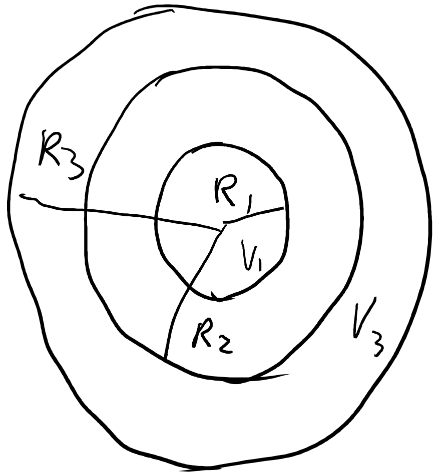
$$E(r) = 0$$

(3)
$$V_3 = \int_{R_3}^{\infty} \vec{E}(r)\cdot d\vec{r} = \int_{R_3}^{\infty} \frac{q_1+q_2}{4\pi\varepsilon_0 r^2}\,dr$$
$$= -\frac{q_1+q_2}{4\pi\varepsilon_0}\,\frac{1}{r}\Big|_{R_3}^{\infty} = \frac{q_1+q_2}{4\pi\varepsilon_0 R_3}$$

(4) Spherical shell is an equal potential She :

$$V_3(r) = V_3 = \frac{q_1+q_2}{4\pi\varepsilon_0 R_3}$$

(3)



Both spheres are conducting so they are both equal potential objects

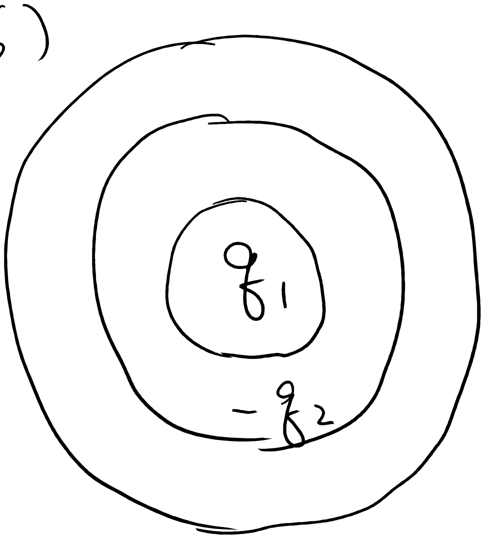
$$\Delta V = V_1 - V_3 = -\int_3^1 \vec{E} \cdot d\vec{s}$$

$$= -\int_{R_2}^{R_1} E(R_2 > r > R_1)\, dr$$

$$= -\int_{R_2}^{R_1} \frac{q_1}{4\pi\varepsilon_0 r^2}\, dr$$

$$= \frac{q_1}{4\pi\varepsilon_0} \left.\frac{1}{r}\right|_{R_2}^{R_1} = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(6)



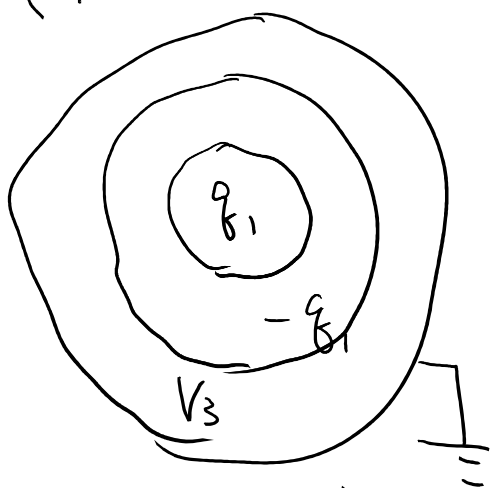
Inner surface: $-q_1$

outer surface:

$$q_{outer} = \underbrace{q_2}_{\text{total charge}} - \underbrace{(-q_1)}_{\text{inner surface}}$$

$$= q_2 + q_1$$

(7)



Now the charge distribution on the shell:

inner surface : $-q_1$

outer Surface : 0

because of grounding

$$V_3 = \int_{R_3}^{\infty} \vec{E}(r > R_3) \cdot d\vec{r}$$

$$E(r > R_3) = \frac{q_1 - q_1}{4\pi\varepsilon_0 r^2} = 0$$

$$V_3 = 0$$

(8) It is still a conducting shell

$$E(R_3 > r > R_2) \equiv 0$$

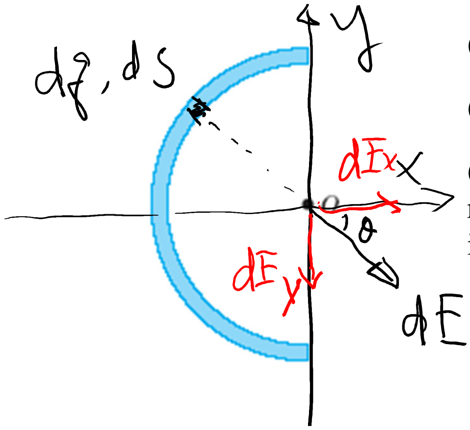
(9) Same as (5)

(10) inner surface : $-q_1$

outer surface : 0

#

4. 30pts) A semicircle wire with a radius of *R* carries a total charge +Q that is uniformly distributed over the wire.



(a) (10 pts) calculate the electric field at the center *O*.

(b) (10 pts) calculate the potential at the center.

(c) (30 pts) if a positive point charge +q with mass *m* is release from still at point *O*, it will be pushed away to infinity. Calculate the final velocity at the infinity.

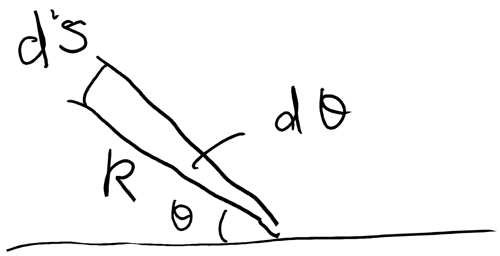
(a) $$d\vec{E} = \frac{K_e\, dq}{R^2}$$

due to the symmetry

$$E_y = 0$$

$$dE_x = dE\cos\theta = \frac{K_e\, dq\cos\theta}{R^2}$$

$$E_x = \int \frac{K_e \lambda\, ds\cos\theta}{R^2} = \frac{K_e\lambda}{R^2}\int \cos\theta\, ds$$



$$ds = R\,d\theta$$

$$E_x = \frac{K_e\lambda}{R^{\cancel{2}}}\int \cancel{R}\cos\theta\, d\theta$$

$$= \frac{K_e\lambda}{R}\int_{-\frac{\pi}{2}}^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{2K_e\lambda}{R}$$

(b) $$dV_0 = \frac{K_e\, dq}{R}$$

$$V_0 = \int_q dV_0 = \frac{K_e}{R} \int_q dq = \frac{K_e Q}{R}$$

(c) Use energy conservation

$$\frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}^{\to 0} = q\,(V_0 - \infty)$$

$$v_f = \sqrt{\frac{2qV_0}{m}}$$ ✓