Chapter 5 Study Questions

1. Describe the motion of a non-Brownian spheroidal particle suspended in a fluid in simple shear flow.

2. What is the effect of the aspect ratio of prolate spheroidal particles on their random maximum packing?

3. Brownian motion of nonspherical particles is more complex than for spheres. What other dimensionless group is required to describe shear flow and how is it defined?

4. The concentration dependence for the viscosity of nonspherical particles can become very nonlinear at very low volume fractions. Explain.

5. Describe the evolution of the viscosity with shear rate in steady state shear flow for dilute suspensions of Brownian spheroids (*alternative question: the same for the frequency dependence in oscillatory flow).*

Chapter 5 Answers

1. In simple shear flow a spheroid rotates at a constant angle around the vorticity axis; making a kayaking motion. The projection on the vorticity plane (plane of the velocity and shear rate axes) makes a tumbling motion, rotating at a rate that depends on the angular position. Long slender particles spent more time in orientations close to the velocity axis than in the perpendicual orientation. The total time for a full rotation is inversely proportional to the shear rate, as with spheres, but now the time depends on the aspect ratio, actually the proportionality factor $\left(p_a + 1/p_a\right)$.

2. The random maximum packing increases for slight aspect ratios up to $p_a \simeq 1.5$, reaching a value of ~ 0.71. At higher values of p_a the RMP drops systematically, for very long slender particles RMP $\propto 1/p_a$.

3. Brownian forces on nonspherical particles not only cause translational motion but also rotational motion, described by a rotational diffusivity $\mathcal{D}_r$. The balance between this rotational motion and the advection of shear is, as in the translational case, expressed by a ratio of the time constants of these motions, i.e., a rotational Péclet number $Pe_r = \dot{\gamma}/\mathcal{D}_r$. The characteristic time in rotation is equal to the rotational diffusivity. Different expressions exist for $\mathcal{D}_r$, depending on the shape and size of the particles.

4. Nonlinearity appears when particles influence each other during flow. Because of the particle rotation non-interaction requires that the volumes swept out by the particle do not overlap. For long slender particle the difference can be substantial, as the volume that is swept out is determined by the longest dimension L, and hence is related to L^3.

5. At low shear rates, more specifically at low Pe_r, Brownian motion dominates advection and therefore the orientation distribution is random, which maximizes the zero shear viscosity as compared to values at finite Pe_r. This depends on the aspect ratio through the factor $p_a^2/\ln p_a$. With increasing Pe_r the long particles spend more time in the flow direction and hence, become aligned on average in the flow direction. This reduces the hydrodynamic forces on the particle and hence, lowers the particle contribution to the viscosity. The global behaviour is therefore that of a shear thinning fluid. The Brownian forces tend to randomize the orientation, thus causing elastic effects, including a normal force difference. In contrast to the case of spherical particles, anisotropic particles in dilute suspensions can generate elasticity solely via particle rotation (distortion of the equilibrium orientation probability function).