Electrical Engineering for Wind Engineers (ELEG467/667-010, embedded in ELEG437/637)-

This short course will provide non-Electrical Engineers with sufficient instruction to understand the conversion of rotary motion into electrical current and voltage, i.e., the operation of an electrical generator. As it is intended for novices, basic principles of charges and forces will be given, electric circuit elements and analysis, relevant electromagnetic theory, electric power and electric power transmission, and DC and AC electrical generators. A background including differential equations is assumed.

I Charge, Current, Voltage, Electric and Magnetic Fields, and Electromagnetic Forces

Electrical Engineering is all about the controlling the flow of charge, or current, through a circuit, or on the electric power side, producing a current by converting from another energy source. This energy source is usually something that causes the turning of a shaft, that in turn causes the movement of wires and magnets that creates current. The primary energy source could then be wind or water flow that causes the shaft to turn, or the burning of a fuel that then is used to cause mechanical movement through a heat to work engine.

Electrostatics

Here in this lecture, we will learn about charge, current and other basic electrical quantities. While "charge" is a term used sometimes in everyday language, it can be a bit mysterious if one thinks about it too deeply. We should say at the outset that this deep thinking is usually the reserve of physicists, and this is an engineering course. As engineers, we are not required to have the deep understanding of the nature of matter, but rather need to be able to apply certain principles of physics to make something work. That being said, "charge" is a quantity of elementary particles, particularly protons and electrons, just as "mass" is a quantity of them. And, just as gravitational attraction exists between two masses, another type of force, much stronger, exists between two charges, with the difference that the charge can be attractive (between an electron of negative charge and a proton of positive charge) or repulsive (between like charges).

An electron has a charge of $1.6\text{x}10^{-19}$ coulombs. How much charge a coulomb is will be defined below; but here let's begin with an understanding that all electrical units are part of the MKS (meter-kilogram-second) or metric system. Thus, a coulomb is a unit of measurement for charge, just as a meter is a unit of measurement for distance.

To understand how much charge a coulomb is, we can examine one of the first electrical experiments done, by Sir Coulomb. He placed charge on two balls attached to a spring, and noted that the more (like) charge he placed on them, the more the balls pushed apart against the force of the spring. Sir Coulomb noted that the force on the balls went as $1/r^2$, the distance between them; later when forces were quantified, with a force of 1 Newton being the force necessary to accelerate a 1 kilogram weight at 1 (meter/second)/second, it was defined that one coulomb is the amount of charge on balls 1 meter apart that result in a force of $1/(4\pi\varepsilon)$, where ε is something called the permittivity. The permittivity of air is $9\text{x}10^{-12}$ farads/meter. Note that later we will learn how a farad, the unit of capacitance, is related to force and charge via force = coulomb^2/farad-meter. For now, let's just plug in and see that the force exerted upon two 1 coulomb charges 1 meter apart is 1/(4x3.14x9E-12) = 8,800,000,000 newtons! This is the same amount of force as the weight of a 900,000,000 kg mass! So, a coulomb is a lot of charge, more than we normally ever experience in our lives.

Mathematically, then, the force between charges is given by

$F = \frac{q_1 q_2}{4\pi r^2}$ (1.1).

So far, this is pretty straightforward: an observation is made, the forces between charges, and an equation is written to quantify those forces. Now, we introduce the concept of *field*. We are all familiar with the force of gravity, which causes an object to fall to earth. Scientists also talk of a *gravitational field* which just means that something is there that causes objects to fall to earth, even if there is not an object. The same is true for charges, and this is called an *electric field*. If only one charge is present, there is no force, but there is still an electric field, that is capable of causing any charge that happens to come around to experience a force. The electric field caused by a charge q_1 is given by

$\mathrm{E} = \frac{q_1}{4\pi r^2}$ (1.2).

Thus, we see that the force on a charge q_2 is given by

$\mathrm{F} = q_2\mathrm{E}$ (1.3).

Note that this equation works no matter what the source of electric field, force is always charge times electric field.

Now, we can use the basic principle of energy,

Energy = Force x Distance (1.4),

that is, the energy expended to do something equals the force applied times the distance over which the force is applied. Then, in electrical work,

Energy, electrical = qEh (1.5),

where *h* is the distance that a charge is moved through an electric field. Note that this is completely analogous to the energy expended to move a mass through a gravitational field:

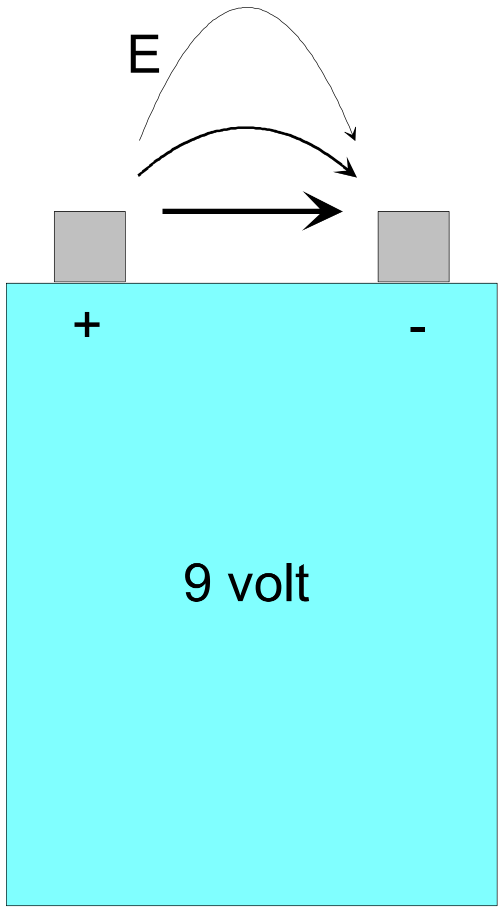
Energy, gravitational = mgh (1.6).

In electrical phenomena, we define a term called the voltage potential,

$V = \mathrm{E}h$ (1.7).

This is the quantity you are familiar with from everyday life, such as a 9 *volt* battery. So, an electric field is determined by the voltage across a distance, volts/meter.

So far, all the equations above have used scalar quantities, that is, having no direction. Of course, we know that forces have a direction, so are a vector quantity, and thus so are electric fields. Furthermore, we can surmise that electric fields are not constant in space:



Thus, the electric field between the terminals of a 9 volt battery are not constant and do not have a constant direction. Now, we must rewrite equation 1.7 as

$$V = \int_{+}^{-} \mathbf{E} \cdot \mathbf{dl}, \quad (1.8)$$

that is, along any path from the positive to the negative terminal, taking the dot product of the electric field with the differential of length, the integration with yield 9 volts. Note that this is for any path taken. This is completely analogous to if you walk up a hill, no matter what path you take, you expend the same energy against the gravitational field, since you have changed the same amount of height. Going forward in this course, we will only be doing electrostatic problems where the electric fields are in straight lines, so we will not have to worry about the vectors, but equation 1.8 is shown so that we understand the vector nature of fields.

HOMEWORK 1.1: A single positive charge exists in space. What is the voltage potential a distance *R* from that charge? Hint: The voltage potential is zero at an infinite distance from the charge.

Current, and Power

Current (*I*) is the movement of charge, or, the charge per unit time that crosses a plane. Now, we know that power is the energy supplied per unit time. If in the 9 volt battery above, we connect the terminals across a load, allowing a current to flow, the power supplied to the load equals the charge per unit time leaving the positive terminal, times the voltage between the terminals:

$$power = \frac{energy}{time} = \frac{charge}{time} x \frac{energy}{charge} \qquad (1.9).$$

Charge/time is current, and energy/charge is given by equations 1.5 and 1.7, so we have that power (*W*) is given by current times voltage:

$$W = IV \qquad (1.10).$$

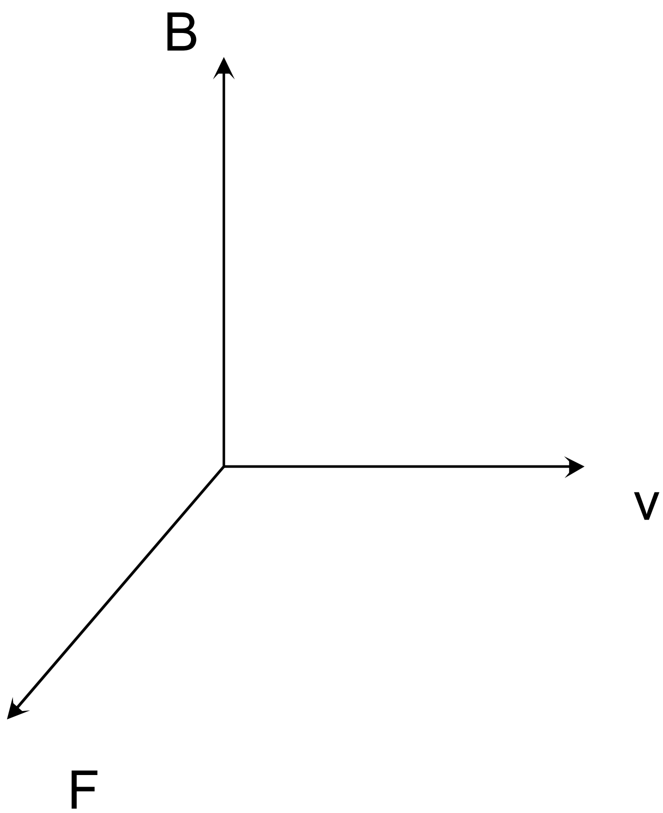
Magnetostatics

Stationary charges produce electric fields. When charges are in motion, they also produce magnetic fields. We are all familiar with magnets. Inside a magnet, the motion of electrons orbiting atoms results in a magnetic field. Since current is the movement of electrons through a wire, it can produce a magnetic field as well. We'll get into that later, but first, let's understand magnetic forces.

Just as magnetic fields only result from charges in motion, so too magnetic fields only exert forces on charges in motion. A stationary charge in a magnetic field will feel nothing from it. But, if the charge has a velocity, a force will be exerted upon it. That force is given by

$$\mathbf{F}_{magnetic} = q\mathbf{vxB}. \qquad (1.11)$$

Recall that the direction of a cross product is given by the "right hand rule," and can be found by pointing your fingers in the direction of **v**, rotating them toward **B**; then your thumb points in the direction of **vxB**:



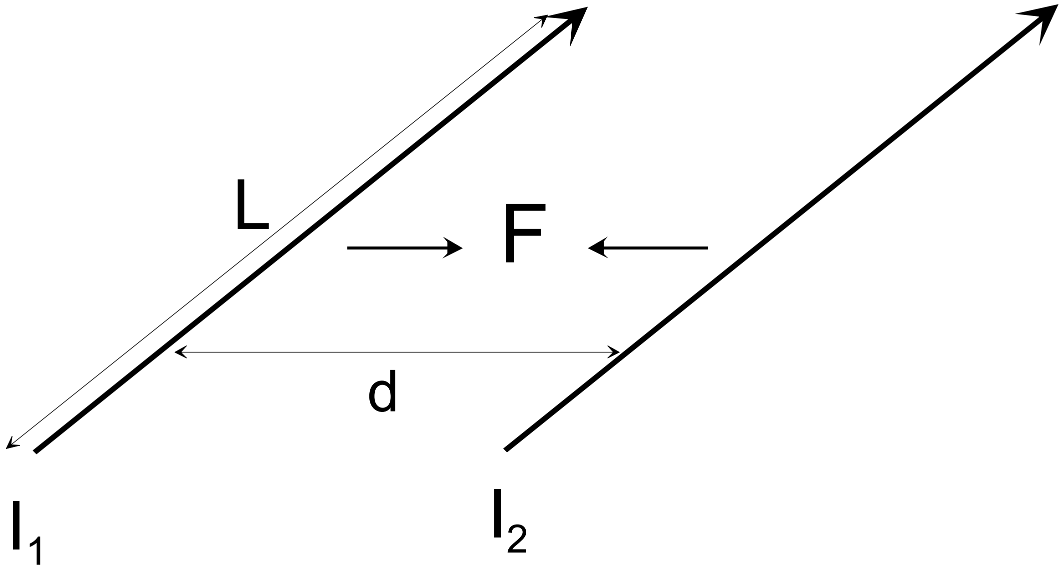
Homework 1.2: A charge is moving straight from left to right in the plane of the paper. A magnetic field is suddenly turned on and points into the paper. What shape path does the charge then take? Hint: think of the direction of force on a satellite in space.

Now, as said, moving charges produce magnetic fields. How do we know this? Well, analogously to Coulomb's experiment with stationary charges, we can discuss the first experiments with moving charges or currents. The current through a wire is the amount of charge moving per unit time across a point in the wire:

$$I = \frac{dq}{dt} \qquad (1.12).$$

Note that while we know that electrons have negative charge, positive current is for positive charges moving through the wire in the direction of current, by definition. This may sound odd, but in fact positive charge carriers exist in electronics, so this is an appropriate definition. The unit of current is an ampere, and is defined as the amount or current in a wire that has 1 coulomb passing a point in 1 second.

It was found, just as charges produce a force on each other, current-carrying wires produce a force on each other:

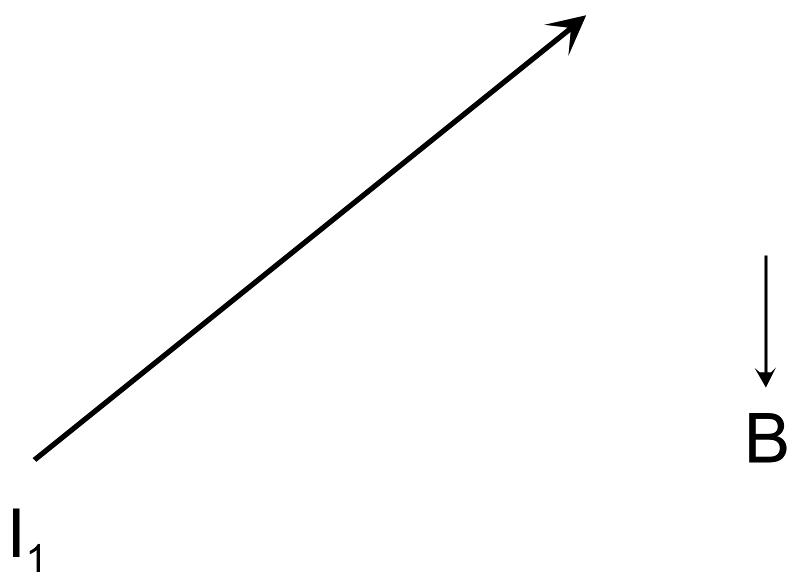


This force was determined to vary as the distance between the wires:

$$\mathbf{F} = \frac{\mu I_1 I_2}{2\pi d} L, \qquad (1.13)$$

where *L* is the length of the wires, and μ is called the *permeability.* For air, μ=4πx10^{-7} henries/meter, where a henry is the unit of inductance. Note here that this looks a lot like equation 1.1 for stationary charges and electric fields. However, it is different in that the force goes inversely as the distance, not distance squared, and also we must multiply by the length of the wire. The reason for this is that the longer the wire, the more moving charges in it, and the greater the force.

Now, just as a single stationary charge in space produces an electric field, as single wire in space produces a magnetic field:



Now, examine the direction of the magnetic field produced by the wire designated 1. It points down. If we replace wire 2, it will experience a force to the left. Does this make sense? Think about it, by convention current as shown is positive charges moving in the direction shown, that is the direction of their velocity. Hence, by the right hand rule of cross product and equation 1.11, the force on wire 2 will be to the left.

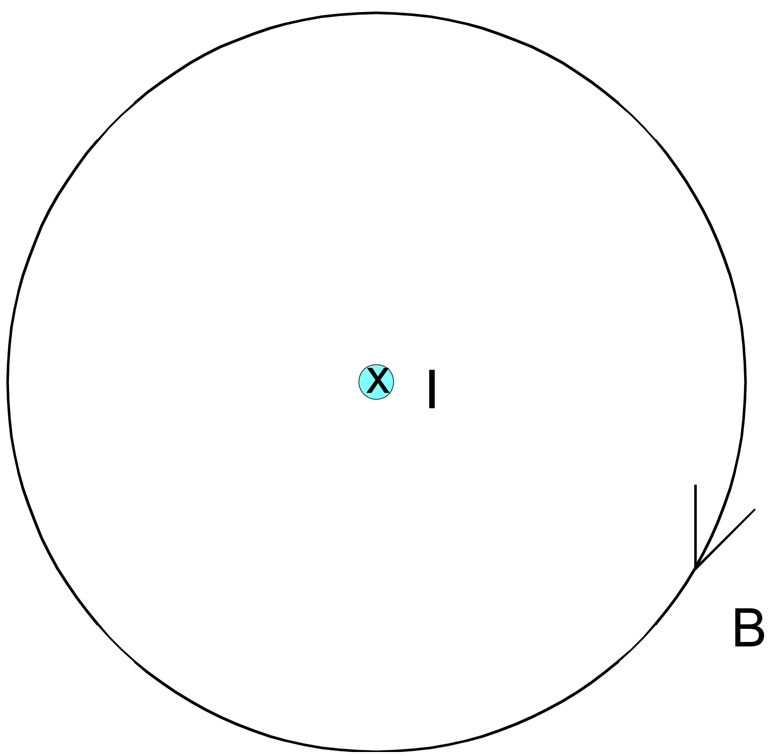
What is the magnetic field produced by wire 1? Well, analogous to equation 1.2, we write that it is

$$\mathrm{B} = \frac{\mu I_1}{2\pi d}. \qquad (1.14)$$

Then, if we place wire 2 back, the force on it will be

$$\mathrm{F} = I_2 \mathrm{B} L. \qquad (1.15)$$

Now, by symmetry, we can reason that wire 1 produces a magnetic field everywhere in space (since no matter where wire 2 is placed, a force will exist), and that also no matter where wire 2 is placed, the force will point toward wire 1. Therefore, the magnetic field is a circle:



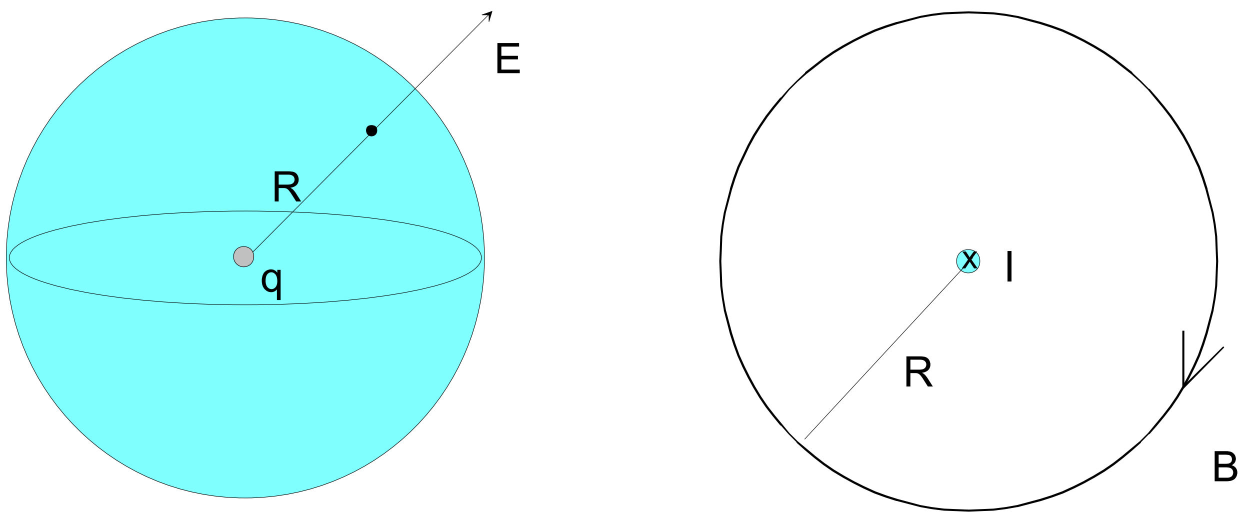
Here for simplicity of viewing, the current is shown as a wire directed into the page, and then B is in the page.

Note here a fundamental difference between electric and magnetic fields: whereas electric fields point from positive to negative charges, and thus have a beginning and end, magnetic fields loop onto themselves, and have no beginning or end. This is true even if magnetic fields are not circular, but have a more complicated shape; they still loop onto themselves.

Homework 1.3: Two wires 0.1 meter apart and 1 meter in length carry 1 ampere each. What is the force in newtons between them?

Statics Mathematical Formalism

We see then that a duality exists between electrostatics and magnetostatics:



$4\pi R^2 E = q/\varepsilon$ $2\pi RB = \mu I$

Now, here is where a postulate is made for both cases. Note for electrostatics that the surface area of the sphere times the electric field equals charge divided by permittivity. For magnetostatics the circumference of the circle times the magnetic field equals current times permeability. It was postulated and found true that for any closed surface, the surface integral of electric field on that surface (that is, over its area), equals the charge/ε contained within, regardless of the shape of the surface or the position of the charges. Likewise, it was postulated and found true that for any closed curve drawn around a current, the line integral of the magnetic field on that curve (that is, over its length) equals the current times μ contained within it, regardless of the shape of the curve or the position of the currents:

$$\iint \mathbf{E} \cdot \mathbf{dS} = \frac{q}{\epsilon} \qquad (1.16)$$

$$\int \mathbf{B} \cdot \mathbf{dl} = \mu I \qquad (1.17).$$

These are respectively called Gauss's Law and Ampere's Law, after the scientists that determined them. In (1.16), S is area, and **dS** points perpendicular from the surface. Thus, for the point charge at the origin, it points away from the origin, in the same direction as **E**. Likewise for (1.17), for a circular path around a wire at its center, B and dl point in the same direction, and hence for both the dot product is simply the product of the magnitudes, and we recover equations 1.2 and 1.14.   Equations 1.16 and 1.17 will be used in the next lecture to find formulas for capacitance of parallel plate capacitors, and inductance of solenoidal coils.

II Ohm's Law, Resistors, Capacitors, Inductors, and Circuit Analysis

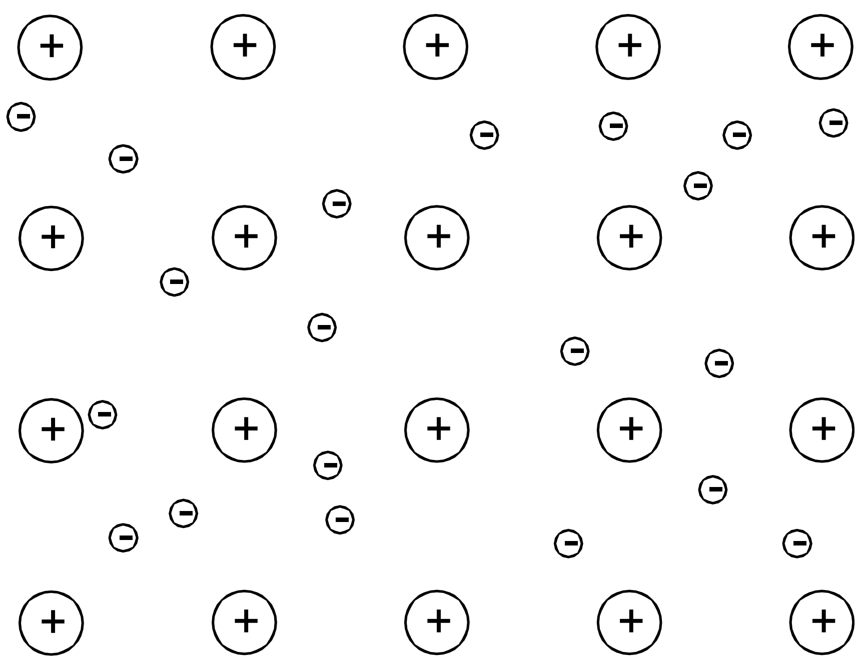
Here we begin the concept of electrical circuits, or connections of electrical elements. A simple electrical circuit may be a voltage source (our 9 volt battery) connected to an element. If the element is a *resistor*, the current flow through it is independent of time. If the element is a *capacitor* or *inductor*, the current flow depends upon how long after the connection is made. The reason for this is that capacitors or inductors can perform energy storage, via the production of electric or magnetic fields.

Resistance

Resistance is defined the ratio of voltage over current for an element that has no inductance or capacitance. Thus, it is just what the name sounds like, an element with higher resistance has more "resistance" to current flow:

$I = V/R$ (2.1).

We see that resistance is in units of volts/amps, termed ohms. What physically is a resistor? Well, most lumps of matter, including me. Upon connecting a voltage supply to my hands and raising the voltage, I was able to observe that the current went up linearly, and my resistance was about 100,000 ohms. To see that for most matter, current goes up linearly with voltage, let's examine what happens microscopically:



Shown here is a conducting solid, consisting of fixed atoms and mobile electrons. If I apply a voltage to this solid, it produces an electric field, via equation 1.7. This electric field in turn produces a force on the electrons, via equation 1.3. If *h* is the length of the solid, then, we have that the forces on the electrons is

$F = e\left(\frac{V}{h}\right) = m\frac{dv}{dt}$ (2.2).

Here we have completed the equation with Newton's Law, F=*ma*. Since the force is constant,

$v = \left(\frac{eV}{mh}\right)t$ (2.3).

Now, if this were the complete picture, the electrons would continue to increase velocity forever and the current would also increase. That is not what happens; the reason is that the electrons bounce off the fixed atoms (this is not actually what happens, but to understand what actually happens requires quantum mechanics). So, the electrons accelerate, then hit something, losing their velocity, then accelerate again, etc.; this result in them having an average velocity. Some statistical analysis of this phenomena results in the average velocity of the electrons being given by

$v_{ave} = \left(\frac{eV}{mh}\right)\tau$ (2.4),

where τ is called the scattering time.

Now, clearly current is related to the movement of charge, so the greater their velocity, the greater the current. The greater the density of charge (ρ), the larger the current is also. Finally, the greater the cross sectional area of the resistor (*A*), the larger the current. So, we can write that

$I = \rho v_{ave} A$ (2.5),

where ρ is the density of mobile charge (charge/volume) in the solid. Combining 2.4 and 2.5, we have

$I = \left(\frac{\rho e\tau}{m}\right)\left(\frac{A}{h}\right)V$ (2.6),

and we see that current goes up linearly with voltage. It is seen that resistance depends both upon microscopic quantities and the macroscopic dimensions of the wire.

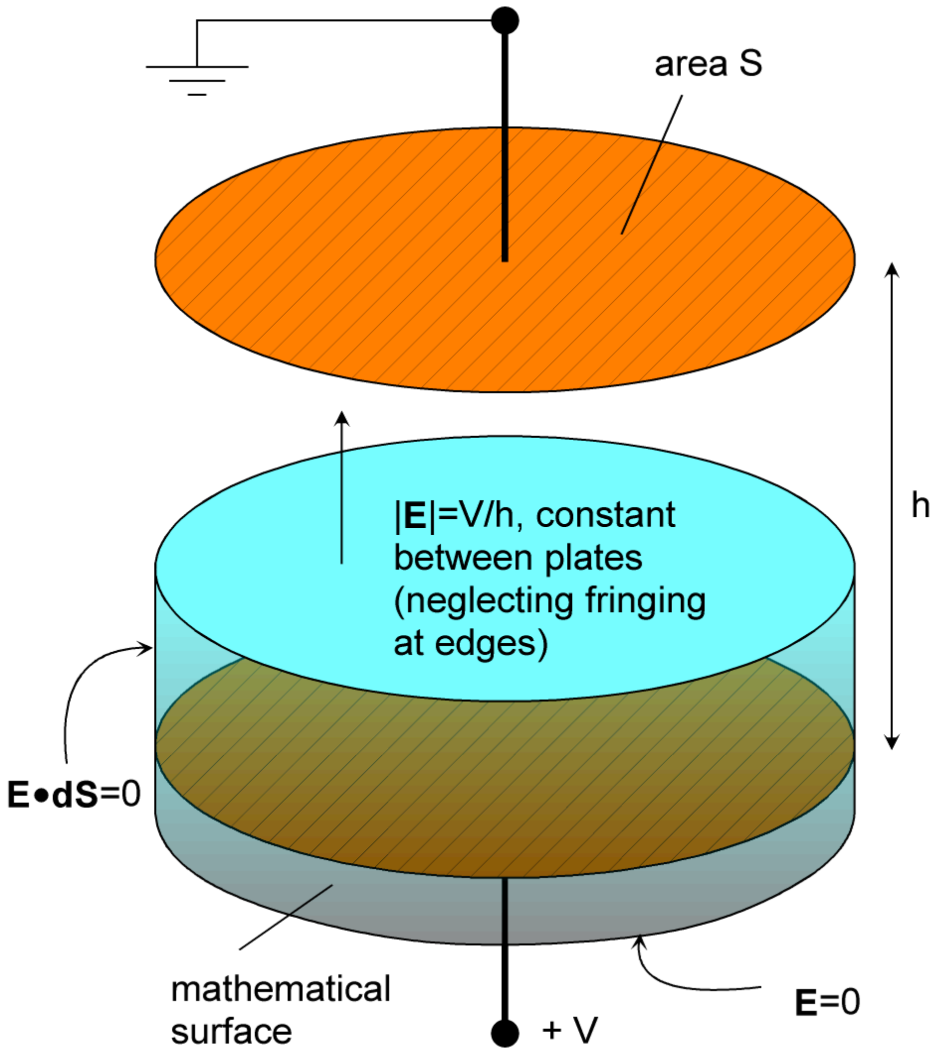
<u>Homework 2.1:</u> Copper has a mobile charge density about $8.5\text{x}10^{27}$ electrons/m3 and a scattering time of about $3\text{x}10^{-14}$ seconds. A power cable has a cross section of 10^{-4} m^2 (1x1 cm) and a length of 10 meters. It is to convey 20 amps. What is the voltage drop across the cable?

<u>Capacitance</u>

A *capacitor* is an element that when a voltage is placed upon it, retains a separation of charge in it. Actually, capacitance (*C*) is a definition, that is what it sounds like, the capacity of an element to store charge at a given voltage:

$C = Q/V$ (2.7).

The most common capacitor consists of two parallel plates:



Here, we two parallel plates are shown, one at voltage *V* and the other at zero voltage. This results in positive charge *Q* on the positive plate and negative charge –*Q* on the other plate. To calculate what *Q* is, and hence the capacitance from equation 2.7, we use Gauss's Law (1.16). Shown in the figure is a mathematical surface. Now, the electric field between the plates points from the positive to the zero plate, and by equation 1.7 is given by *V/h*. So, the mathematical surface has no component of $\mathbf{E} \cdot \mathbf{dS}$ on it's sides; also, since E=0 outside the capacitor, there is no component underneath the positive plate. So

$$\iint \mathbf{E} \cdot \mathbf{dS} = \left(\frac{V}{h}\right) A = \frac{Q}{\epsilon} \qquad (2.8),$$

where *A* is the area of the capacitor plates. Hence, we have that

$$C = \left(\frac{Q}{V}\right) = \frac{\epsilon A}{h} \qquad (2.9).$$

As stated above, while for resistors the relationship between current and voltage is not a function of time, here for capacitors (and inductors, below), it is. That can be seen qualitatively from thinking about the sudden hooking up of a voltage source to the capacitor: initially, the current will be high, as the capacitor charges up, but eventually the current will drop to zero, when the capacitor is fully charged. Thus the current is a function of time. The relationship between voltage and current on a capacitor is therefore a differential equation, and can be easily derived from equation 2.7:

$$Q = CV \qquad (2.10),$$

$$\frac{dQ}{dt} = I = \frac{d(CV)}{dt} = C\frac{dV}{dt} \qquad (2.11).$$

Thus there is non-zero current going into a capacitor when the voltage across it is changing.

We will use eq. 2.11 below in analyzing circuits with capacitors. Before we leave capacitors specifically, it is helpful to understand that a capacitor is an energy storage device, and we can use these equations to determine the amount of energy stored. We can employ equation 1.5 to write

$$energy, capacitor = \frac{QEh}{2} = QV/2 \qquad (2.12).$$

Why the ½? Because, if the charge on both sides of a capacitor "drops" to the midpoint, the pluses and minuses annihilate each other and the energy is released. So, each charge only has to move across half the distance separating the plates. We can use equations 2.10 and 2.12 to rewrite the energy stored as

$$energy, capacitor = CV^2/2 \qquad (2.13).$$

Homework 2.2: You make a capacitor by rolling up a sandwich of two metal films separated by a 0.01 mm thick insulator. The insulator has a permittivity 10,000,000 times that of air. The capacitor area is 20x500 cm. You place 100 volts on the capacitor. What is the energy stored in kWh? (A kWh is equivalent to 1000 watts or 1000 joules/second running for one hour.)

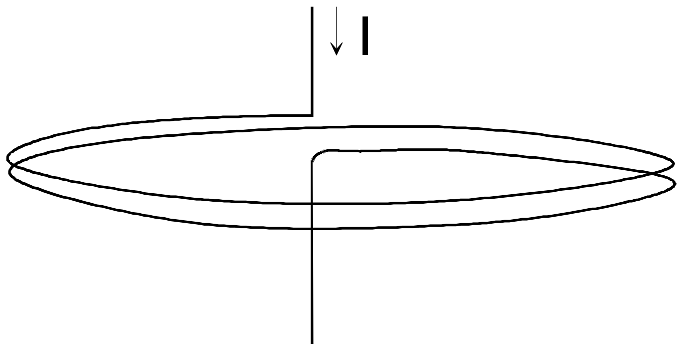
Inductors

For inductors, there is no easy definition like eq. 2.7. To a certain extent, the easiest definition for an inductor is the analog to equation 2.11 for capacitors, switching *I* and *V*:

$$V = L\frac{dI}{dt} \qquad (2.14).$$

Thus, an inductor is an element where the voltage across it is proportional to the time rate of change of current. The proportionality constant, *L*, is its inductance.

For circuit analysis, 2.14 is all we need to know about inductors. But, we can delve into them a little more, and understand why they obey 2.14. First, as you may know, the classic inductor is a coil or wire:



Now, from what we know about magnetic fields, think about what is happening as we put current through the coil. As the charges move through the wire, they will produce a magnetic field as we found in the first lecture. If the current is constant and therefore the magnetic field is constant, everything is fine, and the voltage across the inductor is zero. Now, we must understand that, just as in a capacitor energy is stored via the production of an electric field, also when a magnetic field is produced, energy is stored. How do we know this? Because, we know that a magnetic field can induce forces on charges that happen to be moving by. So, the magnetic field can deliver energy to the charges. Therefore, the magnetic field has stored energy. If the current to the inductor increases, thus increasing the magnetic field, power must be supplied to the inductor, since the stored energy increases. Now, from section 1, we learned that power=current x voltage. Therefore, a voltage must be supplied to the inductor. So, we have that when current is changing across an inductor, and thus the magnetic field and energy storage is changing, voltage must also be supplied to it, and we have equation 2.14.

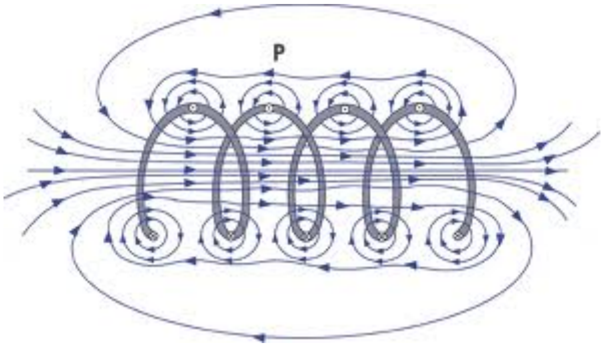
How do we determine the inductance L? Well, clearly, if a coil produces a larger magnetic field, L is larger. So, we can define inductance as the ratio of magnetic field to current. It's a little different than that,

$$L \approx \frac{BS}{I} \qquad (2.15),$$

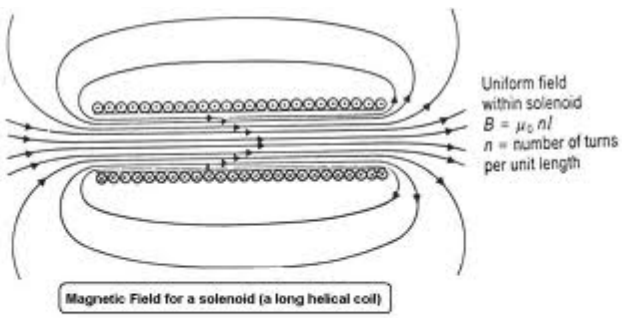
where S is the area of the coil (note that if you put I on the left, you get a nice mnemonic). Now, this equation is not quite correct, as the magnetic field is not always constant across the coil area. In addition, for the classic coiled inductor with N coils, the inductance is larger for each coil. So, the correct equation is

$$L = N\frac{\iint \mathbf{B}\cdot\mathbf{dS}}{I} \qquad (2.16).$$

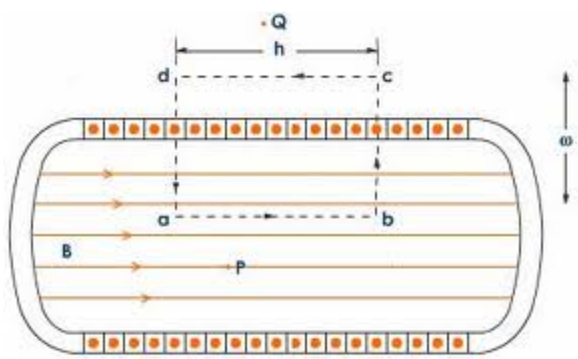
For a many-coiled inductor or solenoid, we can reason out the direction of the magnetic field:



As we know, the magnetic field circles the wires of the winding. It should be apparent that as you move toward the axis of the solenoid, the magnetic fields add up such that they point along the axis of the solenoid. As the winding becomes tighter and as coils are added, this becomes more pronounced:



Note that the magnetic field outside the solenoid is small and can be approximated as zero. Then, we can use Ampere's law (eq. 1.17), that the integral of B times length along any curve equals the current contained within:



Here, performing the integral of eq. 1.17 along the dotted path shown, there is only a component along line a-b, which has length *h*:

$\int \mathbf{B} \cdot \mathbf{dl} = \mathrm{B}h = \mu I = n\mu I$ (2.17),

where *n* is the number of coils contained within the path. Although we see that the magnetic field becomes "curved" near the ends of the solenoid, we can approximate that

$B = \frac{\mu NI}{z}$ (2.18),

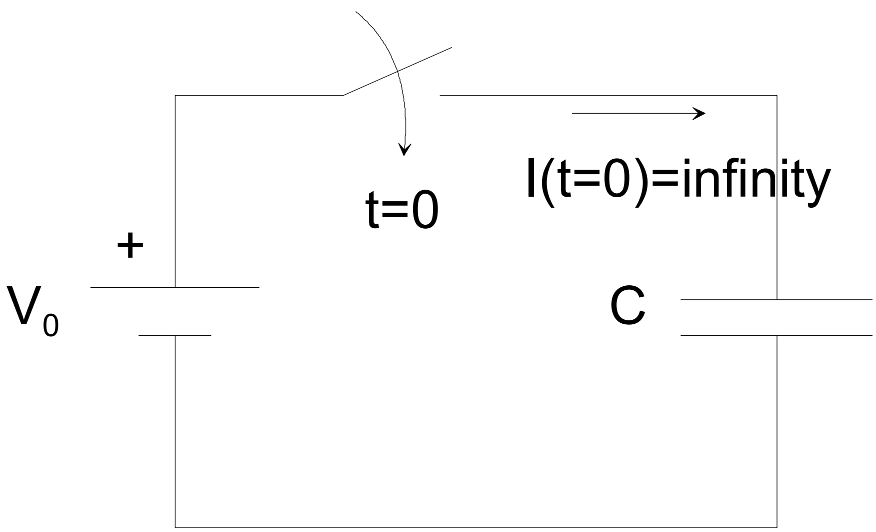
where N is the total number of coils and z is the total length of the solenoid. Then, using equation 2.15, we have that

$L = N\frac{\iint \mathbf{B}\cdot\mathbf{dS}}{I} = \frac{N^2\mu A}{z}$ (2.19).

So the inductance of a coil goes up as the square of the number of windings, and also with area, and inversely with the length. The latter indicates that very tightly coiled inductors have higher inductance.

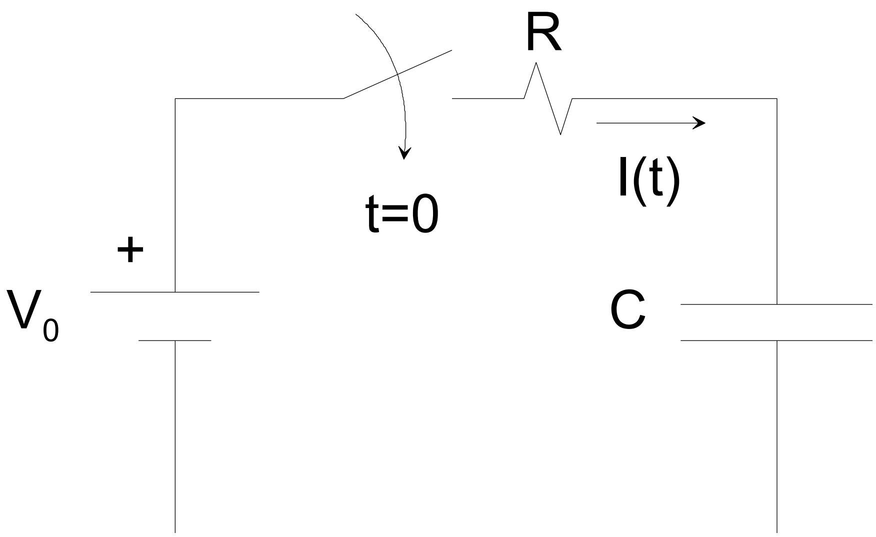
Circuit Analysis

Now, let's place resistors, capacitors, and inductors in a circuit with a voltage source and see what happens. Before beginning, let's examine special cases that illustrate problems to avoid in circuits. First, let's cover the thought problem above of hooking up a battery to a capacitor:



At t=0, then, the capacitor, which was uncharged previously, is hooked to the battery. Thus it immediately has a voltage V_0 across it. That implies, by equation 2.11 that, given an instantaneous change in voltage, the current into the capacitor is infinite. Which, of course, it must be since to change voltage instantaneously, it must be charged instantaneously, which implies infinite current. So, if you were to actually do this experiment, you would have a bad day and possibly start a fire!

Rather, the charging must be done with a resistor in series:



Now, since the capacitor's voltage cannot change instantaneously (otherwise we would have infinite current!), at *t*=0, all the battery's voltage initially appears across the resistor. Then, as time progresses, the capacitor eventually charges up to the battery voltage, and current stops. We can formalize this mathematically by writing the differential equation for the circuit at all times. To do this, note that

$$voltage\ across\ resistor = V_R = IR \qquad (2.20),$$

and

$$I = C\frac{dV_C}{dt} \qquad (2.21).$$

But,

$$V_C = V_0 - V_R \qquad (2.22),$$

so, we can write

$$V_C = V_0 - IR = V_0 - RC\frac{dV_C}{dt} \qquad (2.23).$$

Note what we have done here: by reasoning through, and collecting terms, we have written a differential equation for the circuit <u>with one variable</u>, here V_C. Once we solve this differential equation, we can find how the other variables, for example *I* vary with time, for example by using equation 2.21. So, it's a type of algebra, where one wants to eventually write a single equation with a single variable, but having derivatives.

Now, solving 2.23 is just like solving any other differential equation: one knows the answer ahead of time! One can be aided here by knowing that any first order differential equation has an exponential solution:

$$V_C = A + Be^{t/\tau} \qquad (2.24).$$

One plugs our assumed solution 2.24 into 2.23, and if it is correct, can then solve algebraically for *A*, *B*, and τ:

$$V_C = A + Be^{t/\tau} = V_0 - RC\frac{dV_C}{dt} = V_0 - RC\frac{d}{dt}(A + Be^{t/\tau}) \qquad (2.25).$$

Solving,

$$A + Be^{t/\tau} = V_0 - RC\frac{d}{dt}\left(A + Be^{\frac{t}{\tau}}\right) = V_0 - \frac{RC}{\tau}\left(Be^{\frac{t}{\tau}}\right) \qquad (2.26).$$

Now, examining, we see that in order for the solution to work,

$$A = V_0 \qquad (2.27),$$

and

$$\tau = -RC \qquad (2.28).$$

So, we have that the solution is

$$V_C = V_0 + Be^{-t/(RC)} \qquad (2.29).$$

There is a parameter left, *B*. Of course, solving a first order differential equation, there is always a parameter left, which is determined by the *boundary conditions*, which in this case, is a boundary in time: at $t=0$, $V_C=0$. Therefore, $B=-V_0$, and

$$V_C = V_0[1 - e^{-t/(RC)}] \qquad (2.29).$$

We see that, as stated, the voltage across the capacitor starts at zero, and rises asymptotically to the battery voltage with a time constant *RC*.

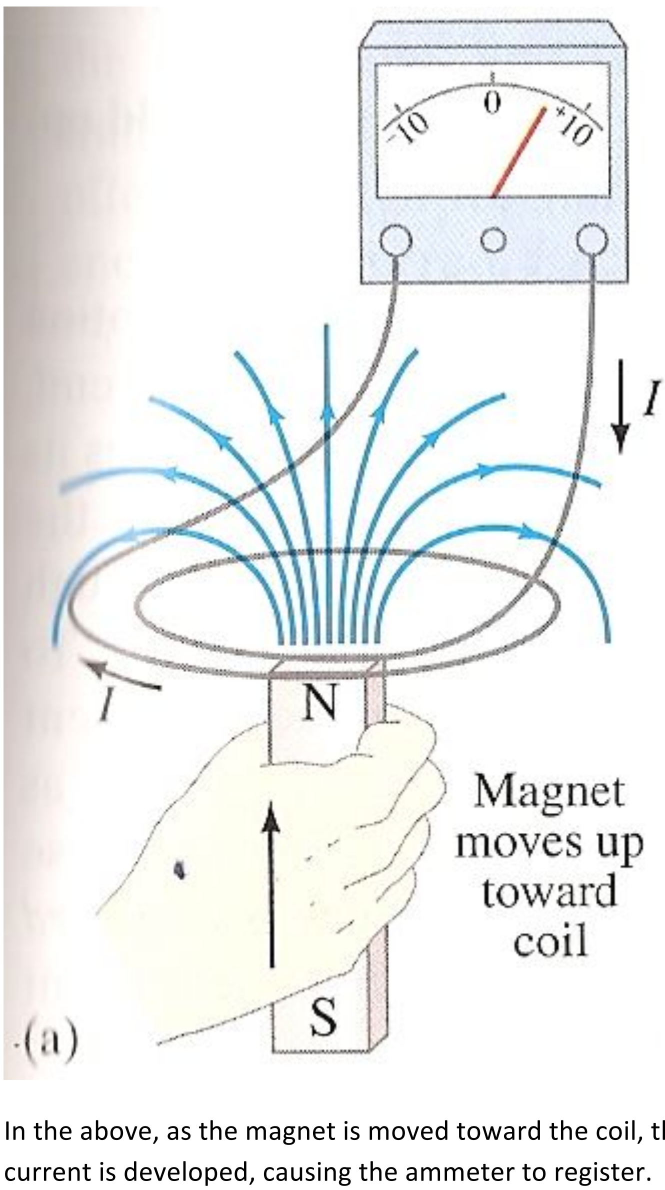
<u>Homework 2.3: You actually could hook a voltage source up directly to a capacitor, because voltage sources always have an internal resistance that is unavoidable and governed by the connections inside, etc. For the capacitor of homework 2.2, assuming the 100 volt source has an internal resistance of 1 ohm, how long does it take for the capacitor to charge to 90 volts?</u>

For circuits with inductors as well as capacitors and resistors, one performs the same operations to solve for the circuit voltages and currents. In lecture 4, we will use such circuit analysis to understand things like power factor in loads with inductance.

III Electromagnetic Theory, Faraday's Law

In the previous lecture, we stated that when the current into an inductor (or any coil) is increased, a voltage must be supplied. The reasoning was that since an increase in current causes an increase in magnetic field, which stores energy, power must be supplied. Since power = voltage x current, voltage must be supplied as well as current to increase the stored energy in the coil.

Here, by corollary, we can also reason that for a coil just sitting there, if it's magnetic field is changed, it must change its stored energy, and current and voltage is developed, passing or taking energy from the circuit it is hooked into. Let's look at some of the original experiments that showed this phenomena:



In the above, as the magnet is moved toward the coil, the magnetic field inside the coil changes and a current is developed, causing the ammeter to register. Actually both a current and voltage are developed, but only the current is shown on the ammeter. If one hooked up a voltmeter instead, a voltage would be shown. If instead a load or resistor was hooked up to the coil, both voltage and current would appear across it, thus supplying power to the load. The power comes from the mechanical power required to move the magnet toward the coil (hey, we made a generator!).

Now, we have danced around this dynamic phenomenon associated with inductors and moving magnets, etc., and made some qualitative remarks about developed voltages, etc. In the last lecture, we quantified part of this with equation 2.14. We supported equation 2.14 with a qualitative remark about how if the current increased into an inductor, increasing its magnetic field, voltage must appear across it as well since power must be supplied to increase the stored energy.

It turns out, that we cannot *derive* equation 2.14. It is a fact of nature, like *F=ma*. Really, equation 2.14 is a result of a more basic relationship, which is: a changing magnetic field will produce a changing electric field. The last statement is true regardless of whether there are any coils or wires or even matter around! With coils and moving magnets (or moving coils), we see the effects of this basic principle of the universe. To ask why this happens is like asking why there is a gravitational field; it is part of the makeup of our universe. Physicists may delve into the basis for this effect, but as engineers we are simply required to understand it and employ the equations that govern it.

So, while we could start with equation 2.14, for example, and try to derive equations that govern the moving magnet experiment above, it is really more appropriate to start with the fundamental equation governing the underlined statement above. Now, I'm going to write it in its complete mathematical form; for those of you that understand vector calculus it may assist you in understanding; for those that don't, do not worry as we are going to quickly go into special cases that get rid of the vector nature:

$$\nabla\mathbf{x}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (3.1).$$

Those of you in the know will recognize Maxwell's 3rd equation. Again, this equation, showing how a changing magnetic field produces a changing electric field, is immutable, a fact of the universe. The various experiments including the one showed above, resulted in intermediate equations governing voltages, changing magnetic fields, etc., and eventually led scientists to understand that 3.1 is the basic equation governing all that.

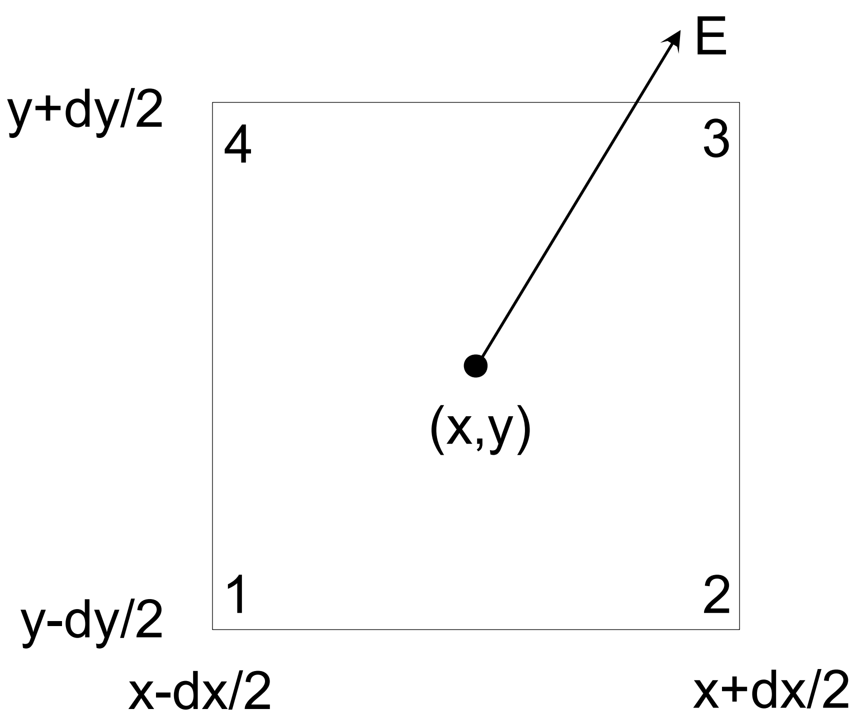
We can find alternate and simpler forms of this equation, for particular cases, for example, the case of a magnetic field only in the z direction. Since

$$\nabla \times \mathbf{E} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathrm{E}_x & \mathrm{E}_y & \mathrm{E}_z \end{vmatrix} = \left(\frac{\partial \mathrm{E}_z}{\partial y} - \frac{\partial \mathrm{E}_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial \mathrm{E}_x}{\partial z} - \frac{\partial \mathrm{E}_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial \mathrm{E}_y}{\partial x} - \frac{\partial \mathrm{E}_x}{\partial y}\right)\mathbf{a}_z \qquad (3.2),$$

If only a component of **B** exists in the z direction, this implies that a changing magnetic field will only produce components of electric field in the *x* and *y* directions. So, in the above experiment with the magnet in the coil, if we approximate that the magnetic field is only in the z direction, moving it toward the coil only produces an electric field perpendicular to the magnetic field or in the plane of the shown coil. Thinking about this, we start to get a hint of the behavior, that it is only changes in magnetic field perpendicular to a coil that matter. Writing 3.1 with only magnetic fields in the z direction, we have

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \qquad (3.3).$$

Now, to proceed, we have to do something complicated mathematically, called Stoke's theorem. I'm going to walk through it; do not worry about having to recreate the following derivation, but simply understand the result. Equation 3.3 says that if a magnetic field is pointing perpendicular to a plane, and it varies with time, it creates electric fields that vary both in time and across the plane. What we are really interested in is what happens to a coil in that plane. So, we have to connect what happens along the length of the coil with equation 3.3. Let's examine a plane with an electric field, zooming in on a very small infinitesimal area:



Now, let's integrate $\mathbf{E} \cdot \mathbf{dl}$ around the loop, going from point 1 to 2 to 3 to 4. This is just the integral of E_x along 1-2, E_y along 2-3, $-E_x$ along 3-4, and $-E_y$ along 4-1. But, for example Ex along line 1-2 does not equal Ex in the center of the loop. Rather, it is given by

Along line 1-2: $E_x = E_x(center) - \frac{\partial E_x}{\partial y}(\frac{dy}{2})$ (3.4)

Along line 2-3: $E_y = E_y(center) + \frac{\partial E_y}{\partial x}(\frac{dx}{2})$ (3.5)

Along line 3-4: $E_x = E_x(center) + \frac{\partial E_x}{\partial y}(\frac{dy}{2})$ (3.6)

Along line 4-1: $\mathrm{E}_y = \mathrm{E}_y(center) - \frac{\partial \mathrm{E}_y}{\partial x}(\frac{dx}{2})$ (3.7)

So, integrating $\mathbf{E} \cdot \mathbf{dl}$ around the loop, we have first for the horizontal lines:

$\oint \mathbf{E} \cdot \mathbf{dl},$ 1-2 and 3-4= $[\mathrm{E}_x(center) - \frac{\partial \mathrm{E}_x}{\partial y}\left(\frac{dy}{2}\right)]dx - [\mathrm{E}_x(center) + \frac{\partial \mathrm{E}_x}{\partial y}(\frac{dy}{2})]dx$=$-\frac{\partial \mathrm{E}_x}{\partial y}dxdy$ (3.8).

Along the vertical lines,

$\oint \mathbf{E} \cdot \mathbf{dl},$ 2-3 and 4-1= $[\mathrm{E}_y(center) + \frac{\partial \mathrm{E}_y}{\partial x}\left(\frac{dx}{2}\right)]dy - [\mathrm{E}_y(center) - \frac{\partial \mathrm{E}_y}{\partial x}(\frac{dx}{2})]dy$=$\frac{\partial \mathrm{E}_y}{\partial x}dxdy$ (3.9).

And, we have that

$\oint \mathbf{E} \cdot \mathbf{dl},$ 1-2-3-4 =$(\frac{\partial \mathrm{E}_y}{\partial x} - \frac{\partial \mathrm{E}_x}{\partial y})dxdy$ (3.10).

Hey, look at equation 3.3!! Without the *dxdy*, equation 3.10 is the left hand side. So, plugging in from eq. 3.3,

$\oint \mathbf{E} \cdot \mathbf{dl},$ 1-2-3-4 =$\left(\frac{\partial \mathrm{E}_y}{\partial x} - \frac{\partial \mathrm{E}_x}{\partial y}\right) dxdy = -\frac{\partial B_z}{\partial t}dxdy$ (3.11).

Now, if we make the loop larger, we just add up all the terms of the right:

$\oint \mathbf{E} \cdot \mathbf{dl},$ (a large loop) =adding all the $(-\frac{\partial B_z}{\partial t}dxdy)$ in the loop (3.12).

But, adding all the $(-\frac{\partial B_z}{\partial t}dxdy)$ in the loop simply means performing the area integral in the loop:

$\oint \mathbf{E} \cdot \mathbf{dl} = -\iint \frac{\partial \mathrm{B}_z}{\partial t}dxdy = -\frac{\partial}{\partial t}\iint \mathrm{B}_z dxdy$ (3.13).

We define

$\emptyset \equiv \iint \mathrm{B}_z dxdy$ (3.14),

as the magnetic flux enclosed by the coil. Now, from 1.8, $V = \oint \mathbf{E} \cdot \mathbf{dl}$, so the voltage developed across a loop of wire equals the time rate of change of the magnetic flux enclosed by it:

$V = -\frac{d\emptyset}{dt}$ (3.15).

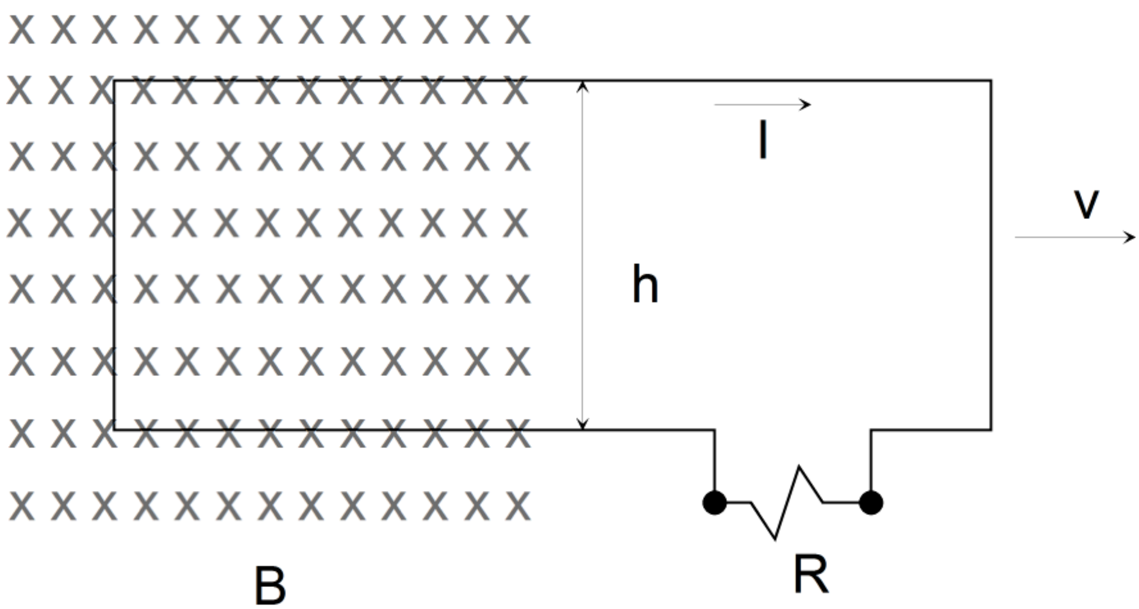
Note that if there are multiple loops in the coil, we simply add the voltages of all the loops:

$V_{N\ coils} = -N\frac{d\emptyset}{dt}$ (3.16).

In lecture 5, we will use eq. 3.15 to calculate the power generated by a rotating coil in a magnetic field. In lecture 4, we will use eq. 3.16 to show how a transformer works. Note that while it is confusing to talk about the voltage developed across a closed loop of wire, as in the experiment above, eq. 3.15 applies to a loop that is coiled, or has a small break in the end to tap current off of.

Homework 3.1: A circular wire is perpendicular to a constant magnetic field B. Its radius r shrinks with time. What voltage is developed in the wire?

We've shown that if a magnet is moved relative to a coil, voltage is developed in the coil. If a resistor is placed across the ends of the coil, current will flow through it, and hence power is delivered to it. Let's examine this "generator":



Here the magnetic field points into the page. The magnetic flux enclosed by the coil is given by $\phi = Bhx$, where x is the horizontal distance from the end of the coil to where the magnetic field ends. Therefore,

$$\frac{d\emptyset}{dt} = \mathrm{B}h\frac{dx}{dt} = \mathrm{B}hv = -V \qquad (3.17).$$

Note that whether or not the resistor is connected, the voltage that appears across the coil opening is given by 3.17. Now, since the ends of the coil are connected across a resistor, current will flow, equal to *V/R*.

So far, we have ignored the minus sign in 3.15. It is actually important to know which direction the current will flow in. To see, consider that in our analysis for B pointing in the z direction, we calculated $\oint \mathbf{E} \cdot \mathbf{dl}$ in the counter-clockwise direction. So, if B points in the positive z direction and increases, E points clockwise. Here, B points in the negative z direction but contained flux is decreasing, so E (and hence the induced current) also points in the clockwise direction, as shown.

In our electrical "generator" above, power is supplied to the load resistor. That power must come from that which is pulling the coil to the right, which means, there must be a force opposing that pulling. That force is the force of the magnetic field on the moving current. We know that forces are at right angles to the moving charge or current and the magnetic field. Therefore, the force opposing the pulling is on the coil arm to the left. From equation 1.15, that force is

$F = IhB$ (3.18).

Since energy = force x distance, power = force x velocity. Thus, the power supplied by the agent pulling the coil equals

$P_{mech} = IhBv$ (3.19).

We've learned that electrical power is current times voltage:

$P_{elec} = IV = IBhv$ (3.20).

Thus, the electrical power supplied to the load resistor equals the mechanical power expended pulling the coil through the magnetic field. This is of course what an electrical generator means, the conversion of mechanical power into electrical power. In lecture 5, we will continue to use Faraday's Law to describe a rotating coil generator.

<u>Homework 3.2: In the above generator, one has a 50 ohm load, and wants to supply 200 watts of power. h=20 cm. B=1 tesla. What force is required to pull the coil at? What mass is that equivalent to having to lift in the earth's gravity?</u>

IV Electric Power, DC and AC Power Transmission, and Three-Phase Power

As mentioned, we will cap this 5 lecture series with a description of rotating electrical generators in lecture 5. While perhaps out of order, before we do that we can describe a lot about how that generated electrical power reaches the consumer.

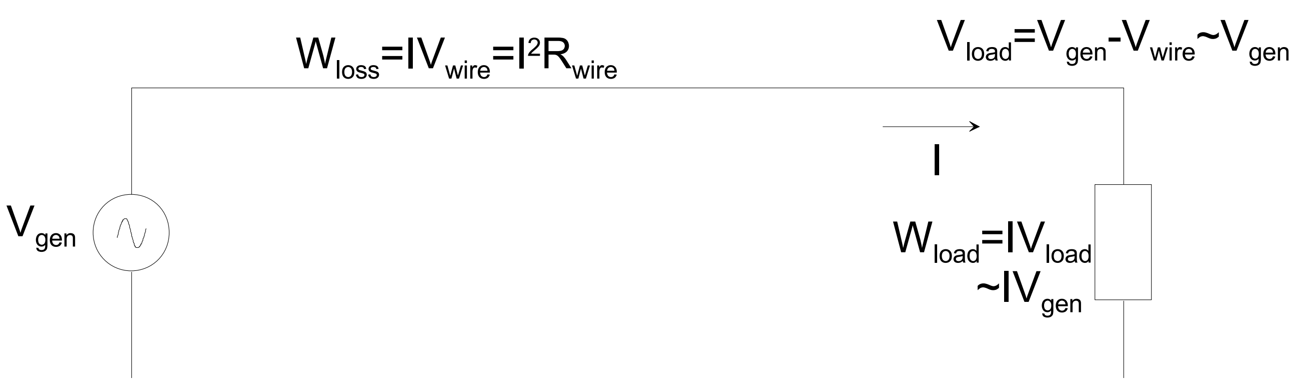
AC and DC electric power

In the previous lecture, an electrical "generator" was described that produces a steady, termed Direct Current or DC electrical output. The word generator is in quotes here, since clearly when the coil reaches the edge of the magnetic field area, it stops working, and must be reset back to the left. Generally, one gets the idea that a rotating device would avoid this problem. Also, generally, one gets the idea that such a rotating device may produce first one polarity than the other, which is called Alternating Current or AC. Actually, one can arrange for a rotating device to produce DC, but the point is that one can produce either.

We generally know that what comes out of the outlets is AC, while not perhaps having a complete understanding of it. Usually, and in the case of our electrical outlets, "AC" implies that the voltage (and current) obeys a sine function:

$$V_{AC} = V_{peak}\sin(\omega t) \qquad (4.1).$$

At this point, we should discuss why electric power is transmitted to us in AC form. It does seem perhaps a bit of a complication, why not just do it DC? This is actually a very interesting historical note on the development of electric power, with Edison promoting DC and Tesla promoting AC transmission. Most of the consideration is the losses in the wires due to wire resistance. Now, it is beyond the scope of this course, but it can be shown that DC transmission at a given voltage has lower wire loss. But, that is only part of the story. What actually governs electric power transmission is that *higher voltages have lower wire loss*. To see this, examine this figure:



As can be seen, the power delivered to load is given by

$$W_{load} = IV_{load} \qquad (4.2).$$

The wire loss is given by

$W_{loss} = IV_{drop\ across\ wire}$ (4.3).

Note here that the voltage drop across the wire is very small, so we can approximate that

$V_{load} = V_{generated} - V_{drop\ across\ wire} \cong V_{generated}$ (4.4).

But, the voltage drop across the wire is given by

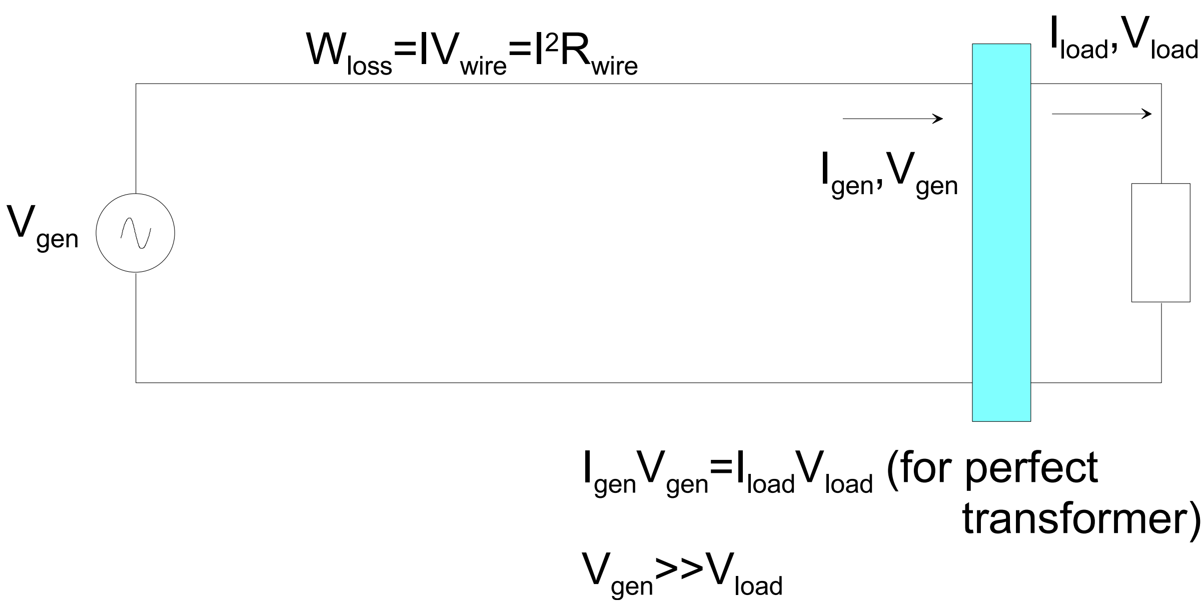
$V_{drop\ across\ wire} = IR$ (4.5),

where *R* is the wire resistance. Therefore, the wire loss is given by

$W_{loss} = I^2R = W_{load}^2 R/V_{generated}^2$ (4.6).

Thus we see that the losses in the transmission line goes inversely as the square of the voltage on the line.

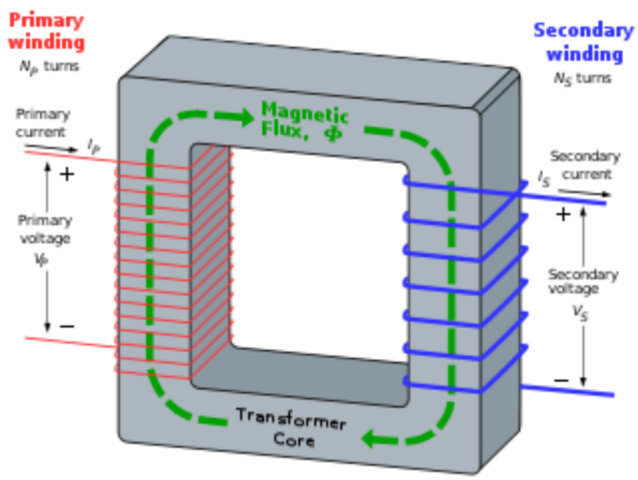
For this reason, transmission voltages are very high, in the hundreds of thousands of volts typically. We still have not explained why this favors AC transmission. The reason is that one does not desire hundreds of thousands of volts at the power outlet! Rather, one wishes to transmit power at very high voltage, but then *step down* the voltage just before the end use:



Note here that equation 4.6 still applies, as when the voltage is stepped down, the current is stepped up, so that the same power is delivered to load. Since the power loss in the wires is determined by the transmission current, which is low since the generation voltage is high, the wire loss is low.

Transformers-

Now, the difference between DC and AC here, is that it is difficult to step down DC, but for AC, there is a very simple device called a *transformer*. You have seen these devices, which are typically cylindrical objects located on telephone poles near your house. Using the principles from the last lecture, we can understand their operation:



Here a simple transformer is shown. It is similar to an inductor, like two coupled inductors. We did not discuss inductor *cores* in the lecture 2; here, just think of the core as "containing" the magnetic field lines. Thus, the same magnetic field strength goes through both windings. So,

$\frac{d\emptyset}{dt}$ is the same in both windings. (4.7)

Now, by equation 3.16, the voltages of the coils are given by:

$$V_p = N_p \frac{d\emptyset}{dt}, V_s = N_s \frac{d\emptyset}{dt} \quad (4.8),$$

and thus,

$$V_s = \frac{N_s}{N_p} V_p \quad (4.9).$$

So, we have stepped down the voltage from a high value to a lower value. But, note!: this device only works at AC, since $d\emptyset/dt$ is zero for DC.

So, AC won the engineering battle for electric power transmission, because transformers could be used to step down the voltage, allowing high line voltages (and low line currents and hence low line loss), and low load voltages (so we don't kill ourselves).

Note that today DC-DC voltage converters are available. However, they have higher costs and internal power losses, so we still use AC transmission.

Homework 4.1: As we know, 110 volts is a standard load voltage in our houses. A standard transmission voltage is 600,000 volts. The transmission distance is 100 kilometers, and the cable has the same parameters of problem 2.1, except the area is 10x10 cm. It conveys 10 MW (million watts). What is the loss in the line in watts?

AC power-

Now, in homework 4.1, I allowed you to still use eq. 1.10 to calculate power. But, you may have wondered, if it is AC, what voltage do I use? The voltage is changing in time, so … . Here, we will analyze the power delivered by an AC generator:

$$V_{AC} = V_p \sin(\omega t) \qquad (4.10).$$

Now, later, we will consider that the current is *out of phase* with the voltage, but for now assume the current has the same time relationship:

$$I_{AC} = I_p \sin(\omega t) \qquad (4.11).$$

Then the *time-varying power* is given by

$$P_{AC} = I_p V_p sin^2(\omega t) \qquad (4.12).$$

We see that the power delivered is always positive, but varies in time. We can find the *time-averaged power* by integrating over one cycle, and dividing by the tim

$$P_{AC,ave} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} I_p V_p sin^2(\omega t) dt \qquad (4.13).$$

Here, we've noted that the *period* of the cycle (*T*), or when it repeats itself, is when $\omega T = 2\pi$, as for any sine function. Noting that

$$sin^2(\omega t) = 0.5 + \sin(2\omega t) \qquad (4.14),$$

and noting that the integral of the sine in 4.14 will be zero over a cycle (up and down), we can easily then write that

$$P_{AC,ave} = \frac{\omega}{2\pi} I_p V_p (0.5) \frac{2\pi}{\omega} = \frac{I_p V_p}{2} \qquad (4.15).$$

We see that if we define something called

$$V_{RMS} \equiv \frac{V_P}{\sqrt{2}} \qquad (4.16),$$

$$I_{RMS} \equiv \frac{I_P}{\sqrt{2}} \qquad (4.17),$$

$$P_{AC,ave} = I_{rms} V_{rms} \qquad (4.18).$$

"RMS" here means "Root mean squared," because

$V_{rms} = \sqrt{< V^2(t) >}$ (4.19),

where <> means taking the time average of, as we did in equation 4.13 for power. Don't worry about that, just remember the result is that it is the peak voltage divide by root 2.

So, when we say that the wall output has 120 volts, what we are referring to is the "RMS" voltage. And, when we use an AC voltmeter, it has been designed to show the RMS voltage. Conveniently then, using an AC voltmeter and ammeter, taking the product of what we see yields the average power.

We can show that Ohm's law (2.1) is true for AC voltages and currents if we use the RMS values. Noting that

$I(t) = \frac{V(t)}{R}$ (4.20)

at any instant of time, then

$I_P = \frac{V_P}{R}$ (4.21), and hence

$I_{rms} = \frac{V_{rms}}{R}$ (4.22).

<u>Inductive and Capacitive Loads-</u>

If the load is an inductor, then on an instantaneous basis 2.14 applies:

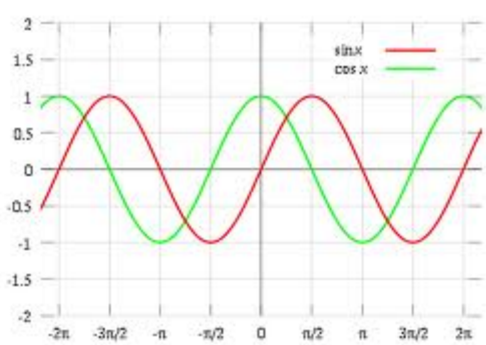
$V(t) = L\frac{dI(t)}{dt}$ (4.23).

And if

$I(t) = I_p \sin(\omega t)$ (4.24), then

$V(t) = LI_p \cos(\omega t)$ (4.25).

We see that current and voltage are then *out of phase* with each other:



Now, instantaneous power is still given by their product, and it

$$W(t) = LI_p^2 \sin(\omega t) \cos(\omega t) \quad (4.26).$$

Since

$$\sin(\omega t) \cos(\omega t) = 0.5 \sin(2\omega t) \quad (4.27),$$

instantaneous power goes positive and negative, and averages out to zero. This makes perfect sense, as the inductor cannot dissipate power, it can only store it and then give it back to the generator.

For capacitors we see by eq. 2.21 that the same thing happens in reverse, if voltage is sine current is cosine. Since

$$\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right) \quad (4.28),$$

we say that for capacitive or inductive loads, current and voltage are *90 degrees out of phase*, since $\frac{\pi}{2}$ in radians is 90 degrees. For either, there is no power dissipation, rather power is simply cycled to and from the load.

Loads with resistance and inductance and capacitance, and power factor-

While determining the formula for the current when applying an AC voltage to such a load may sound complicated, it is simplified by simply writing that if

$$V_{AC} = V_p \sin(\omega t) \quad (4.29),$$

$$I_{AC} = I_p \sin(\omega t + \emptyset) \quad (4.30),$$

where $\emptyset$ is called the *phase angle* of the load. We see that $\emptyset = 0$ occurs for purely resistive loads, and $\emptyset$ = 90 or -90 degrees occurs for purely capacitive or inductive loads. For an arbitrary angle, the average power dissipation can be calculated using

$$\sin(\omega t + \emptyset) = \sin(\omega t) \cos(\emptyset) + \cos(\omega t) \sin(\emptyset) \quad (4.31).$$

If we examine 4.13, we see that only the first term contributes, and

$$P_{AC,ave} = \frac{I_p V_p}{2} \cos(\emptyset) \quad (4.32).$$

$\cos(\emptyset)$ is called the *power factor* and we see ranges from 0 for purely capacitive or inductive loads to 1 for purely resistive loads. Now, typically, inductive loads are seen since motors, for example, behave more like inductors. Frequently in those situations, a *capacitor bank* is added to the load to compensate for the inductive load and bring the power factor as close to 1 as possible. The reason why, is that from equation 4.32, we see that

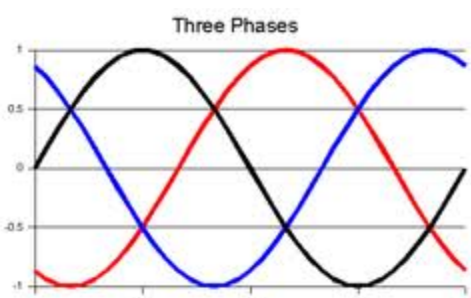
$$I_p = \frac{2P_{AC,ave}}{V_p \cos(\emptyset)} \quad (4.33).$$

So, for the same power dissipation in the load, the lower the power factor, the higher the current requirement. Since transmission line power loss goes as the square of the current, transmission losses go as the one over the square of the power factor. Thus it is desirable to “correct” for low power factors in a factory, for example, which are usually due to inductive loads, by adding capacitors to the main junction.

Three-phase electric power-

You may have noticed that electric power transmission lines typically consist of three wires:

This is because it is inefficient to generate a single sine wave. While we will not go into the details of three-phase power generation in the next lecture, you should have the feeling that in a rotating generator having just one rotating coil, part of the cycle it is doing nothing. We might as well have at least two coils at right angles, so that when one is at the part of the cycle where it is doing nothing, the other is; engineers settled on three, spaced 120 degrees apart. Thus, the three sine waves look like this:



Mathematically, the voltages on the three lines are given by

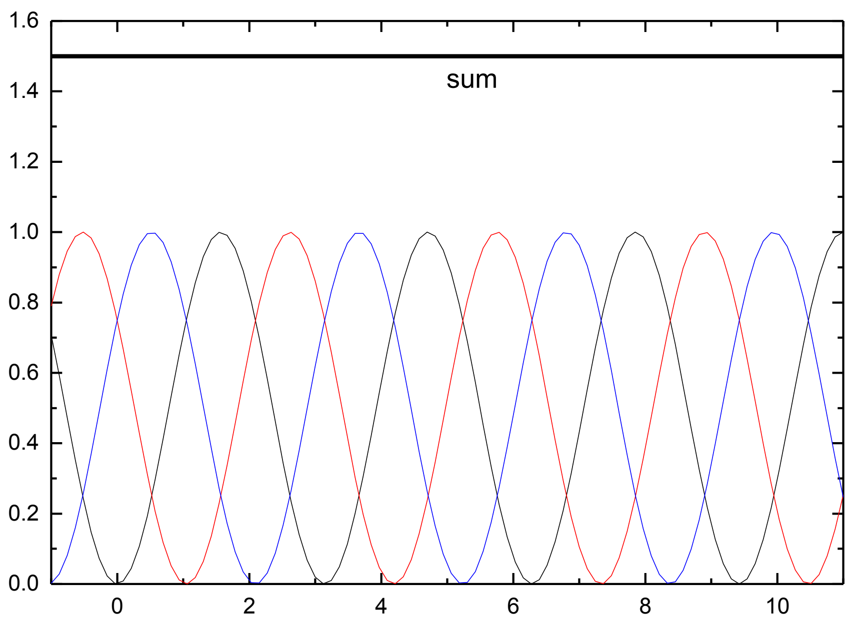
$V_1 = V_p \sin(\omega t)$ (4.34),

$V_2 = V_p \sin(\omega t + 2\pi/3)$ (4.35),

$V_3 = V_p \sin(\omega t + 4\pi/3)$ (4.36).

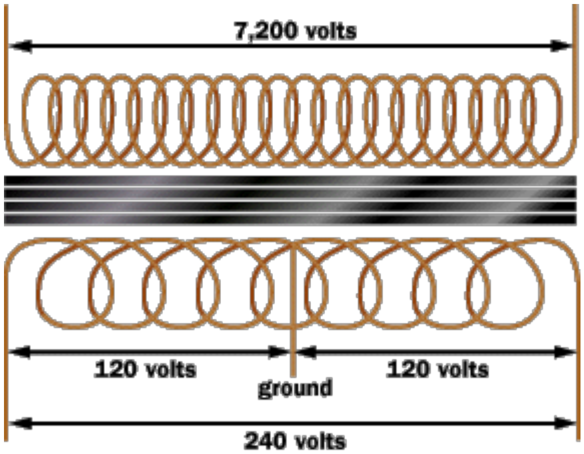
An important property of three-phase power is that one can show the power delivered to a resistive load is constant at all times. We're not going to get into how exactly the three lines are connected to a load; suffice to say there are circuit techniques for doing so. Since the load is resistive, current in each line is in phase with voltage, and the power delivered goes as the sum of the squares of 4.34-4.36:

$$W \propto \sin^2(\omega t) + sin^2(\omega t + 2\pi/3) + sin^2\,(\omega t + 4\pi/3) \qquad . \qquad (4.37)$$

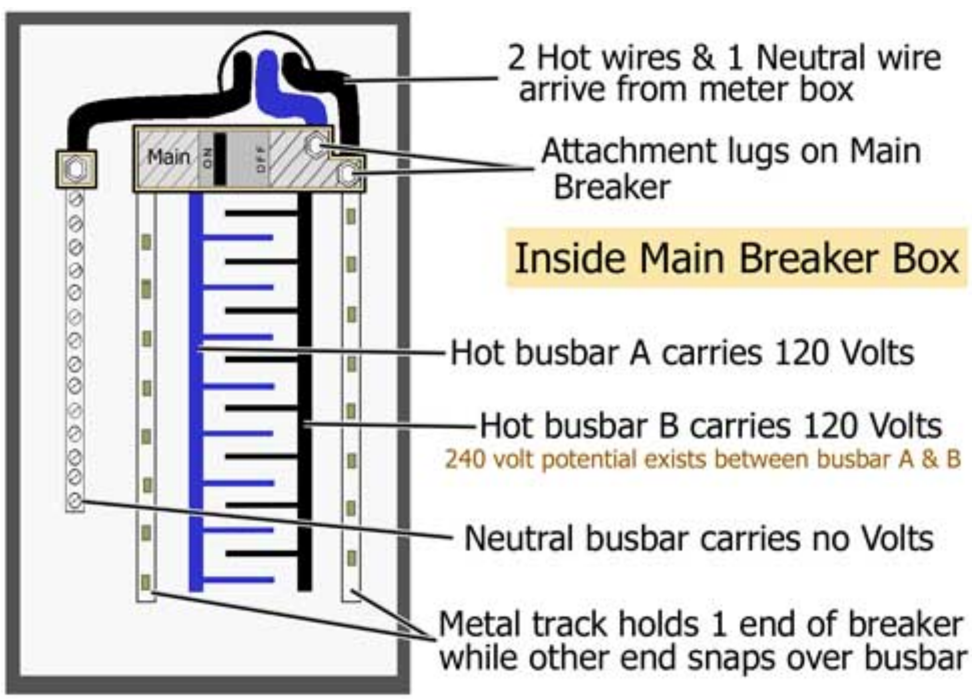


<u>Homework 4.2: Show mathematically that eq. 4.37 equals 1.5.</u>

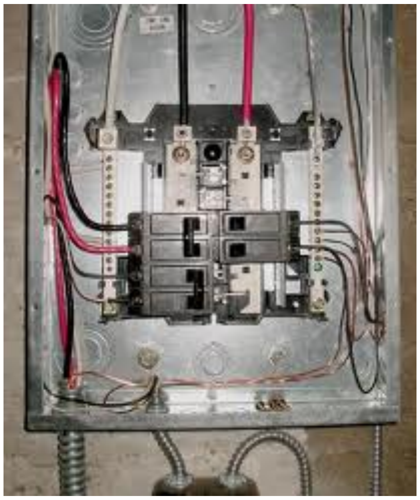
Finally, let's describe how power gets to the outlets in our homes and businesses. What comes out of the outlet is usually *single phase*, that is, one wire is *hot* having a single sine wave, and the other is grounded. Let's show how three-phase becomes single phase in the house. The typical way is just to take one of the phases of the transmission line and put it through a transformer:



Note here that the output coil of the transformer has a *center tap* which is grounded. Thus, what you see going into your house is three wires, one which is ground and two that are hot and 180 degrees out of phase, basically V_{hot}, 0, $-V_{hot}$. The reason for this is that some appliances in your house require higher power and thus higher voltages, and so use the two outside wires. The *breaker box* in your home has this circuit arrangement:

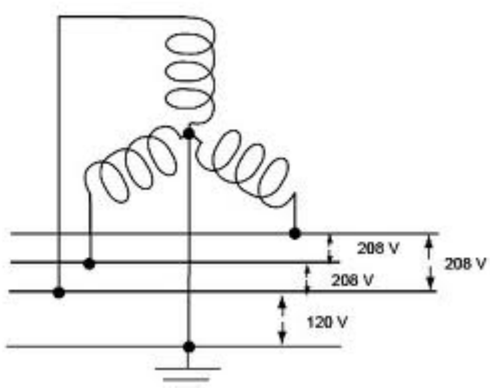


Then, breakers are installed:



You see from above that breakers next to each other get opposite hot phases. Thus, in the picture above, the large red and black wires on the two upper left breakers are going together to the electric range, for example, while the smaller single wires are going to 120 V outlets.

Some commercial places use three-phase step down transformers and three phase distribution inside the facility:



Here only the output coils are shown for brevity. As can be seen, the commercial place then has three-phase distribution within it, allowing powering of three-phase loads, with three hot wires and a ground. Between any of the hot wires and ground, one gets 120 volts, and thus one can also supply ordinary outlets. Thus, each hot line is

$V_1 = 120\sqrt{2}\sin(\omega t)$ (4.38),

$V_2 = 120\sqrt{2}\sin(\omega t + 2\pi/3)$ (4.39),

$V_3 = 120\sqrt{2}\sin(\omega t + 4\pi/3)$ (4.40).

Remember, peak voltage is root 2 times RMS. Then, where a higher voltage is required, one takes the voltage between two of the hot lines:

$V_{208} = 120\sqrt{2}[\sin\left(\omega t + \frac{2\pi}{3}\right) - \sin(\omega t)]$ (4.41).

We can calculate the RMS voltage of this signal:

$V_{208,rms} = sqr\{\frac{120*120*2}{2\pi}\int_0^{2\pi}\left[\sin\left(\omega t + \frac{2\pi}{3}\right) - \sin(\omega t)\right]2dt\}$ (4.42)

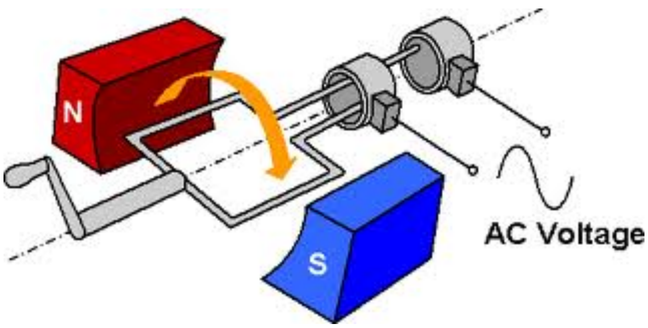
One can show that the integral is (3/2)2π, and plugging in the rest, and thus

$V_{208,rms} = 120\sqrt{3} = 208$ (4.43).

Thus, in a facility with three-phase internal distribution, one may obtain single phase 120 volt for ordinary outlets, but to get higher voltage by using two hot lines, one gets 208 volt instead of 240 volt in a common house with two phases. One must pay attention to this, as some appliances designed for 240 volts will not operate at 208 volts.

V Rotating Electrical Generators

In III, we showed an electrical generator based upon moving a wire coil relative to a magnetic field in a straight line. Of course, such a generator would be problematic in practice, as one would have to go back and forth awkwardly. The solution of course is to rotate a coil in a magnetic field or vice versa rotate a magnet relative to a fixed coil. Now, since the coil must have connections to the load, spinning it would seem problematic and it is, so most electrical generators have a fixed coil and a spinning magnet. However, it is easier to do the math for a spinning coil, so we will treat that first:



Here a moving-coil generator is shown. The magnetic field points from north to south, and the coil is spun as shown. Note the use of *brushes* to make the connection to the coil; basically the brush is an electrical contact that can slide on the spinning rings shown.

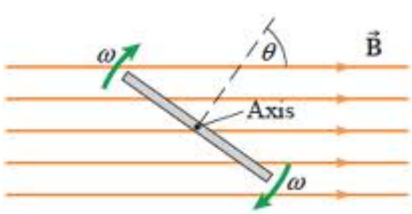
Now, from 3.15, the voltage developed around the coil is equal to the time rate of change of magnetic flux:

$$V = -\frac{d\emptyset}{dt} \quad (5.1),$$

where the magnetic flux ϕ is given by the integral of the magnetic field across the area of the loop:

$$\emptyset \equiv \iint B_z dxdy \quad (5.2).$$

Now, here we must think about equation 5.2 a bit. When we derived it, it was for a coil perpendicular to the magnetic field. In the rotating generator above, the magnetic field is not always perpendicular. Thus, the relevant magnetic field is the component perpendicular to the coil. In the picture below, that component is given by $Bsin(\theta)$:



Think about it, when θ=0, the perpendicular component is just B; when θ=90 degrees, the perpendicular component is zero. Thus,

$\emptyset = BSsin(\theta)$ (5.3),

where *S* is the area of the coil. Now, since the coil is spinning at a uniform rate, the rotation angle is proportional to time:

$\theta = \omega t$ (5.4),

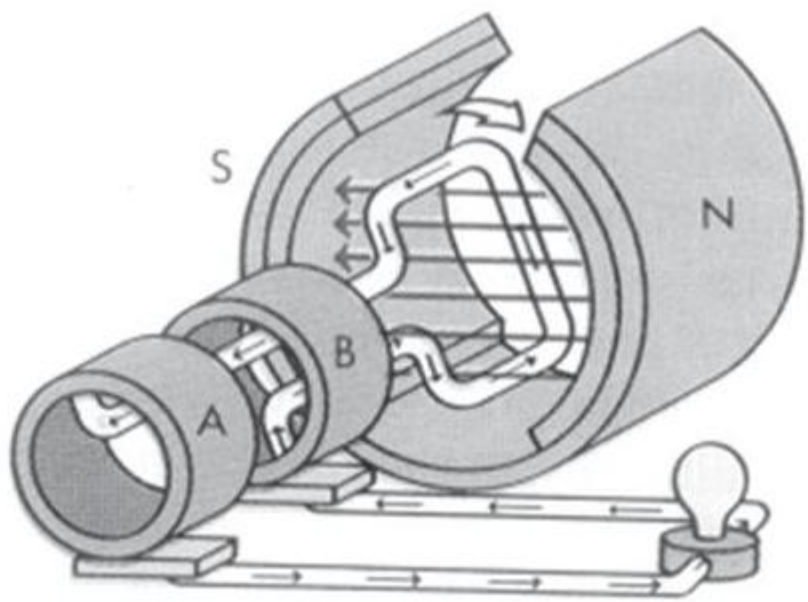
where ω is the radial frequency. Thus,

$\emptyset = BSsin(\omega t)$ (5.5), and

$V = -\frac{d\emptyset}{dt} = -BS\omega cos(\omega t)$ (5.6).

So we see that the voltage generated is AC as shown. Furthermore, the magnitude of the voltage is proportional to the magnetic field, area of the coil, and rotating frequency.

Now, there is a minus sign in 5.6, let's figure that out. Remember in lecture 3, when we did the math, we determined that if B points in the positive z direction and increases, E points clockwise in the x-y plane. Therefore, if we draw the generator at a certain point in time:



We can see in this picture, the current developed is in the right direction since the coil is rotating to the right, and thus the flux is increasing. If we positioned ourselves at the south pole of the magnet (the "z" direction), and looked "down" on the coil, we would see that the current was going clockwise. So, don't worry so much about the minus sign in 5.6; just keep straight which direction the current is in, when a load is attached.

Homework 5.1: In the picture above, after the coil spins past the point of maximum perpendicular to the magnetic field, what direction is the current in, the same or opposite? What does this mean about the magnitude of the current when the coil plane is perpendicular to the magnetic field?

Now, remember that when the coil spins, the voltage developed (equation 5.6) is independent of whether there is any load (current draw) or not. Therefore, the current through the coil is just given by equation 5.6 divided by the load resistance:

$I = \frac{V}{R} = -\frac{BS\omega}{R}\cos\omega t$ (5.7).

Since current flows when a load is attached, then, there is a force on the coil given by equation 1.15:

$\mathrm{F} = IL\mathrm{B}$ (5.8).

Homework 5.2: In the picture above, is the force on the top arm of the coil pointed up or down?

By the right hand rule, the force on the up and down parts of the coil in the picture above are along the rotation axis and thus do not affect the rotation. But, the forces on the sideways parts of the coil do experience force that resists the applied rotation. Note that when the coil is at the top of the rotation, the force is perpendicular to the rotation direction and does not affect the rotation. It is only when the coil plane is along the axis of the magnetic field that the force impedes rotation. That makes sense, as when the coil plane is perpendicular to the magnetic field, no current flows. Thus the force impeding rotation goes as $\mathrm{F} = ILBcos(\theta) = IL\mathrm{B}cos(\omega t)$ (5.9).

Remembering that energy is force times distance, and therefore

Power (W) = force x velocity (5.10).

The velocity of the coil is the circumference of the circle described by the sideways part of the coil ($2\pi r$), divided by the rotation period *T*. But, since the rotation frequency (*f=1/T)* is given by

$f = \omega/(2\pi)$ (5.11),

we have that

$velocity = \frac{2\pi r}{\frac{2\pi}{\omega}} = \omega r$ (5.12).

Thus, noting that this force exists on both sideways arms of the coil (x2), the mechanical power that must be supplied to rotate the coil is given by

$W_{mech} = 2ILB\omega rcos(\omega t)$ (5.13).

But,

$2Lr = S$ (5.14), so

$W_{mech} = IBS\omega cos(\omega t)$ (5.13).

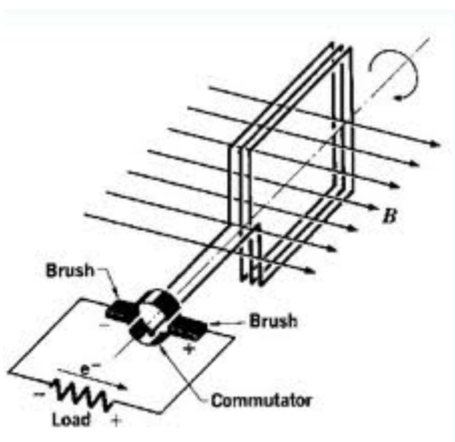
But, note! By equation 5.6 this is just

$$W_{mech} = IV = W_{elec} \qquad (5.14),$$

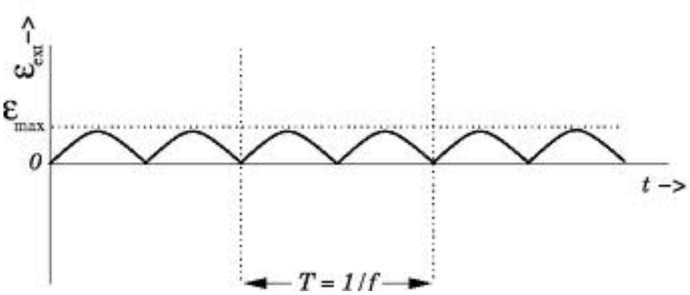
and the instantaneous mechanical power to rotate the coil exactly equals the instantaneous electrical power delivered to the load.

This essentially completes the formal portion of the course. At this point, we can discuss various related aspects without needing to go into mathematical formalism. First, note that in the figure above, if we don't rotate the coil, but supply current in the opposite direction, a force exists on the coil in the direction of rotation. Thus, it is then a motor. Do we need to supply voltage as well as current? Yes!, since the coil will still induce a voltage in the same direction. Thus, current must be supplied in the opposite direction of voltage, and instead of a load, a voltage source must be hooked up. **A motor is just a generator in which voltage is applied and current supplied in the direction opposite to generated current.**

Can we take a rotating coil generator and produce a DC generator? Sure! We install a *commutator* or split-ring contact:

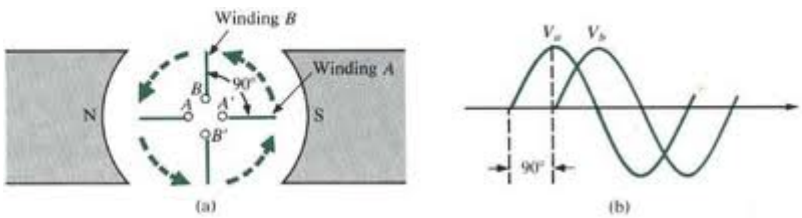


We see that after the coil passes the midplane and current changes direction, the contacts to the load "flip" from one side of the coil to the other. Thus the current supplied is also in the same direction, like this:

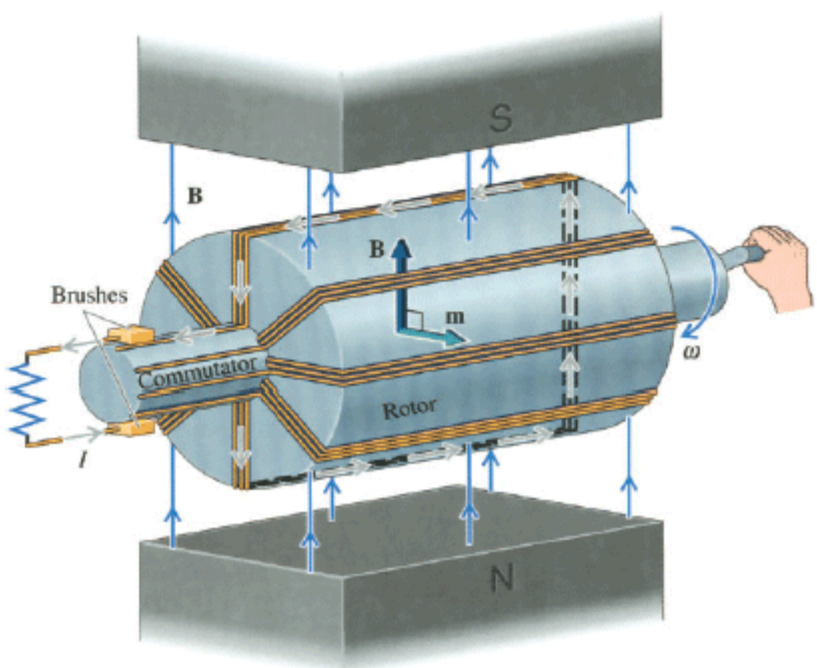


Now, this "DC" current and voltage vary as shown, and further the rings can wear out, so in fact sometimes it is better to make DC if necessary by rectifying and smoothing AC (that is beyond the scope of this course).

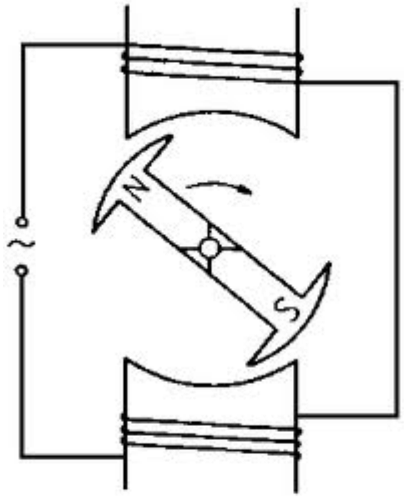
What about 3-phase? As we discussed in lecture 4, and as you can see from above, half the time the generator is doing nothing, so adding coils at different angles would seem to be a good thing. We could make a 2-phase generator:



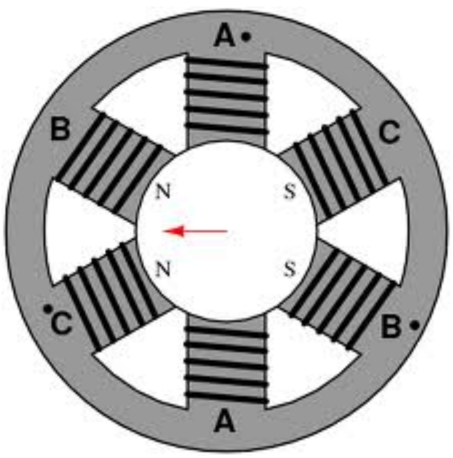
Then, we get power out of coil A when the power out of coil B is zero. Why don't we do this? Well, part of the answer is that as we found in lecture 4, for a 3-phase system the power delivered to the load is constant vs. time. Therefore, the mechanical power that must be supplied to a 3-phase generator is also constant vs. time! And that's nice for whatever is supplying the power. For a 2-phase system, power is not constant vs. time, therefore we use 3-phase, and place three coils at 60 degrees relative to each other:



As we pointed out above, while rotating the coil is easy to describe mathematically, it has difficulties since it requires the brush contacts. In addition, if you think about it the strength of the magnetic field can be higher if it is on the inside rotating:



Looking at this picture, when the magnet is rotated the flux is increasing on the top part of the coil when it is decreasing in the bottom, so the currents add. Going to three phase, it looks like this:



So, as the magnet is rotated, it induces voltage in the coils 120 degrees out of phase.

Finally, it is possible to use multiple rotating magnets, or *multi-pole* generators:

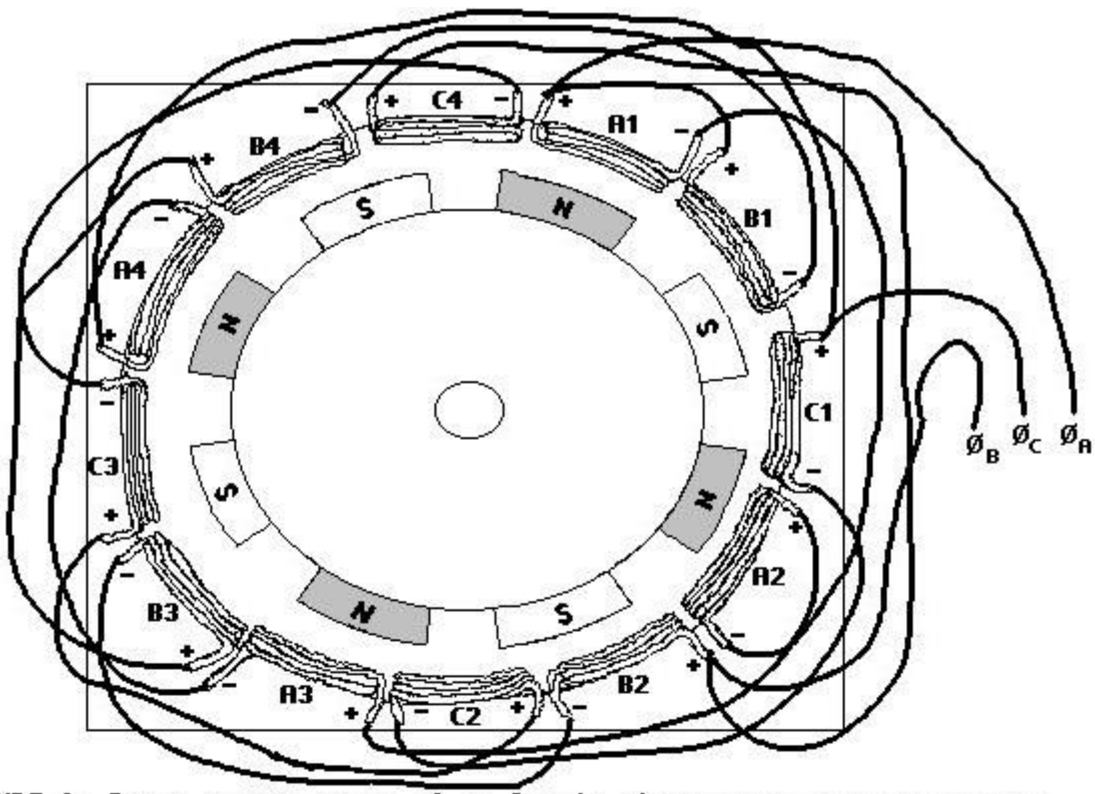


FIGURE 6: Basic construction of an 8-pole, three-phase motor/generator. Cut a circle from the center of a block. Trim the circle to allow magents and coils to clear. Mount coils in the block and wire as shown. Mount magnets on the circle, alternating poles. Mount axle so circle spins in block. Use.

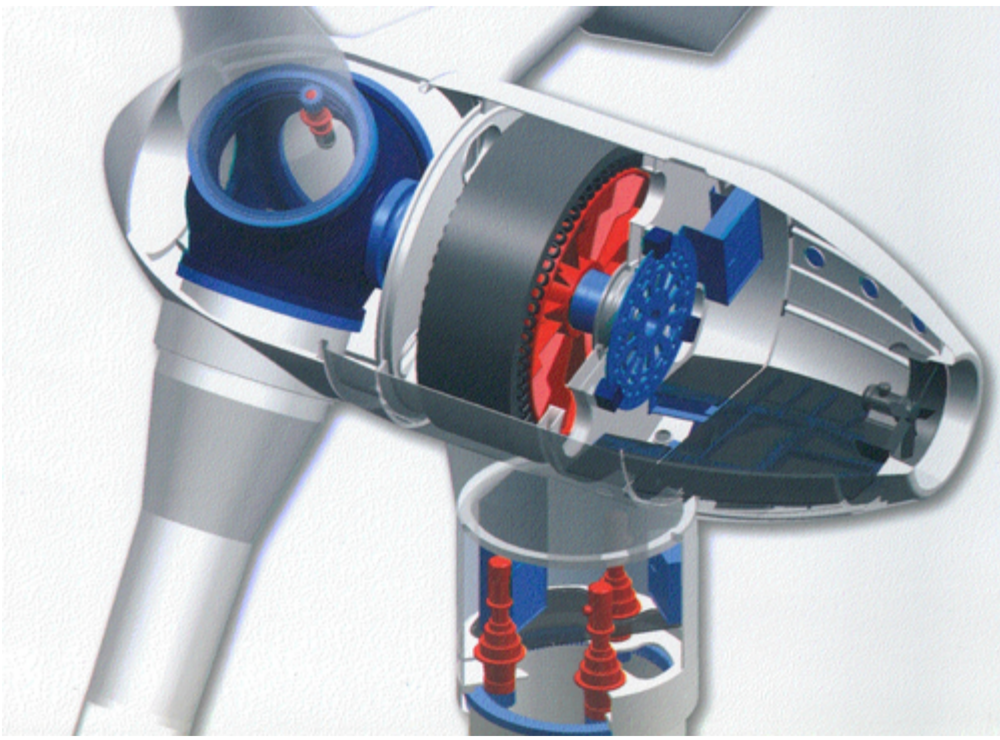
Here each phase has 4 coils (A1, A3, A3, and A4, for phase A). There are north and south magnet poles for each coil, hence this is an 8-pole generator. In the picture above, all the A's have the north poles passing by them, and hence together they generate the same voltage in phase. Now, think about it, in the above picture, a quarter-cycle later, the north poles are again under the A coils. **So, a multi-pole generator generates at a higher frequency, by the number of poles.** Alternatively, if you want to keep the electrical frequency constant (e.g., 60 Hz for US systems), you can use lower rotation speed with multi-pole generators:

Poles	RPM for 60 Hz
2	3,600
4	1,800
6	1,200
8	900
10	720
12	600
14	514.3
16	450

18	400
20	360
40	180

Usually, a 6-pole generator is used in most US power plants, using a rotation speed of 1200 rounds per minute.

In a wind turbine, the main shaft turns at a slow speed, and thus typically in the past a gearbox was used to turn the generator shaft at a much higher speed. However, recently it has been determined that the gearbox is potentially a source of failure, so *direct-drive* wind turbines are being pursued, by using generators with a high number of poles:



Here each of the poles can be seen in the generator; there may be on the order of a hundred to allow slow rotation speeds. As can be perceived in the diagram, having so many poles increases the required diameter of the generator, and results in larger size and mass of the wind turbine, while eliminating the need of a gearbox. The industry seems headed for direct drive, but is still reviewing these design trade-offs.