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BEYOND USER-SPECIFICITY FOR EMG DECODING USING MULTIRESOLUTION MUSCLE SYNERGY ANALYSIS

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ABSTRACT

Electromyographic (EMG) processing is a vital step towards converting noisy muscle activation signals into robust features that can be decoded and applied to applications such as prosthetics, exoskeletons, and human-machine interfaces. Current state of the art processing methods involve collecting a dense set of features which are sensitive to many of the intra- and intersubject variability ubiquitous in EMG signals. As a result, state of the art decoding methods have been unable to obtain subject independence. This paper presents a novel multiresolution muscle synergy (MRMS) feature extraction technique which represents a set of EMG signals in a sparse domain robust to the inherent variability of EMG signals. The robust features, which can be extracted in real time, are used to train a neural network and demonstrate a highly accurate and user-independent classifier. Leave-one-out validation testing achieves mean accuracy of $81.9 \pm 3.9\%$ and area under the receiver operating characteristic curve (AUC), a measure of overall classifier performance over all possible thresholds, of 92.4 \pm 8.9%. The results show the ability of sparse MRMS features to achieve subject independence in decoders, providing opportunities for large-scale studies and more robust EMG-driven applications.

INTRODUCTION

Over the last three decades, quantifying and interpreting muscular activity has become a vital step in the advancement of prosthetics and exoskeletons [1,2], and recently in new applications of teleoperation [3] and human-machine interfaces [4]. Electromyography (EMG) has emerged as the frontrunner for detecting this activity due to its ability to collect signals noninvasively from the surface of the skin. Despite recent advances in EMG technology to eliminate electromagnetic noise from external sources, EMG signal processing presents many challenges due to both intra- and inter-user variability. Intra-subject variability is caused by factors such as muscle (motor unit) density changes, EMG sensor placement, fatigue, and joint orientation. Inter-subject variability is caused by factors such as age, muscle density, scar tissue, and different muscle synergies across users [5]. These challenges have so far contributed to state of the art EMG processing and decoding methods designed around a small select group of subjects to eliminate the influence of many of these factors. As a result, a majority of applications involving EMG processing are individualized to specific users and require intense training phases before they can be used effectively [1].

In order for EMG-based applications to become more accessible for larger studies and use, robust and user-independent EMG processing techniques must be developed. A typical decoding sequence in state of the art methods consists of four steps:

- 1. Preprocessing to de-noise and normalize the EMG signal.
- 2. Extracting information, or features, from the signal.
- 3. Training the decoder with a machine learning technique.
- 4. Testing the decoder in a real-time application.

EMG Feature Exraction

The extraction step is vital to the success of the decoder, as it must convert the raw signal into a descriptive input that allows the machine learning algorithm to distinguish robust patterns for decoding. Many such inputs have been considered in the literature, most of which can be classified into three categories: statistical moment, time domain, and frequency domain features [6]. Statistical moments, such as variance, skewness, and kurtosis, provide information about the general structure of the EMG signal. These have shown to be effective descriptors to descriminate in cases of constant, isometric movements, but far less reliable in dynamic environments, where EMG signal structures are constantly changing with motion [6]. A few other statistical measures have been proposed based on information theory, such as various entropy measures. These have shown to be reliable descriptors for classifying motions within a single user's motion patterns, but they are also reliant on periods of time invariance and their computational complexity prevents them from being applied in real time [7].

Time domain features, such as mean absolute value and root mean square are based on signal amplitudes. While these features have shown to be effective representations for the relationship between EMG and force, they are hindered by their inherent inter-subject variance [6]. Using amplitude-based features typically requires a normalization step to measure muscular contraction levels with respect to the maximum voluntary contraction [8], making the resulting decoder very sensitive to changes in muscle mass or other factors which would effect the activation levels of muscles. In addition, the amount of information a single amplitude signal provides is dependent on the amount of smoothing used to obtain stable features. More smoothing (i.e. a lowpass filter with a smaller cutoff frequency) gives more robust features, but at the expense of less time resolution, greater delay and often no real time capabilities [9].

Frequency domain features, such as zero crossings, slope sign changes, and waveform length, provide information about the rate of motor unit action potentials. While these have shown to be useful descriptors for predicting motion from time varying EMG signals [3], they suffer from an inherent inability to consider time as part of the descriptor. A workaround for this limitation is to use a sliding window to calculate frequency domain features in a given interval of time. This incorporates both time and frequency in the descriptor, but also adds limitations to both the time and frequency resolutions dependent on the size of the window [10]. Small windows will give better resolution in time, meaning changes in frequency can be better associated with a specific time interval, but are limited in distinguishing low frequency information. The opposite is true for larger windows, which achieve better resolution in frequency at the expense of determining the moment in time a given frequency exists. This complexity has kept time domain features the most popular in current literature, and contributed to the high amount of subjectdependent decoders despite several attempts at achieving userindependence [2, 11, 12]. The apparent but rather unexplored solution to achieving robust and descriptive EMG features may lie in the wavelet transform.

Wavelet Transform

The wavelet transform provides means to extract descriptive frequency information while maintaining good resolution in both time and frequency. Moreover, this information is provided in a sparse domain which makes the features more robust against EMG variability than the dense sets of features listed above. Using a multiresolution approach [13], the wavelet transform provides features at varying resolutions in time, with inversely proportionate frequency resolution at each level. This approach has been used extensively in both image and audio processing for applications in recognition, de-noising and compression [14, 15], but it has not yet met its potential in the EMG processing field.

A few recent studies have incorporated wavelets into EMG processing. Ahsan et. al. [16] utilize wavelets to effectively denoise EMG signals for further processing. Other works have incorporated wavelet features as inputs for a classifier. Liu et. al. [17] use multiresolution analysis along single EMG channels and extract the singular value decomposition as input to support vector machines. A similar approach is used in [18], where the wavelets are used to create autoregressive models and trained in a neural network. Xiao et. al. [19] extract entropy measures from wavelet transforms to directly distinguish between pronation and supination motions. In [20], wavelet coefficients from multiple EMG channels were thresholded to distinguish between sitting and standing motions. Wavelets were also used with principal component analysis and sequential forward selection for dimensionality reduction to train a neural network and Bayesian classifier to control a computer mouse from EMG signals [4].

All of the above approaches yield great user-dependent classification results, but none of them encapsulate the full power of wavelets. The wavelet transform, as with the fourier transform, is separable when applied across multiple dimensions [13]. This separation makes wavelets well-suited for 2D processing, and also enables wavelets to provide an even more robust measurement of EMG signals: through a 2D multiresolution analysis of multiple EMG signals. Such an analysis is able to capture changes in muscle synergies at multiple time resolutions, making it well suited for detecting the sparse sets of robust muscle synergies common across the general population, as suggested in [5].

Contribution

The contribution of this paper is twofold. Firstly, it introduces a novel multiresolution muscle synergy (MRMS) feature extraction method that represents EMG synergies across muscles in a sparse domain. These features are extracted using wavelet multiresolution analysis across two dimensions—time and muscles—in order to robustly represent the synergies of muscles at multiple resolutions. To the best of the authors' knowledge, no other paper has considered using multiresolution analysis to obtain feature descriptors which capture information about muscle synergy. The second contribution of this paper is the demonstration of subject-independent decoders using traditional machine learning techniques based on the robust features provided by the multiresolution muscle synergies. The analysis naturally encapsulates the traditional first pre-processing step, removing any subject-dependency on MVC or delays caused by instense denoising. As a result, the features can be extracted from a signal in real time and across subjects without modifying any parameters. To date, no known EMG decoder has been able to achieve both real-time capabilities and consistent results when tested on users who were not involved in the training process.

The rest of this paper is organized as follows. The Method Section describes the process of extracting the MRMS features, as well as the neural network setup used to train a decoder. The Results Section outlines the testing procedure and presents the results of testing on unseen users. Finally, the Conclusion Section summarizes and discusses the main findings of this study.

METHOD

Multiresolution muscle synergy analysis is performed using the discrete wavelet transform (DWT) and Mallat's algorithm [13] along two dimensions. With real time performance in mind, a select set of features are extracted from each level of the resulting wavelet coefficients to provide a set of features describing only the most recent muscle synergies at each resolution. The resulting features are converted into a sparse vector for each time sample to be used as input to a backpropagation neural network. The neural network is then able to detect synergy patterns associated with different motions across subjects to form a robust EMG decoder.

Multiresolution Muscle Synergy Analysis

MRMS analysis consists of a specialized 2D DWT that runs along the dimensions of time and muscles. If N EMG channels are recorded for M time samples, the data can be arranged in a MxN matrix Z representing EMG signal over time and across channels. This matrix can in turn be transformed into multiresolution wavelet coefficients using the 2D DWT. The procedure for 1D DWT has been well documented, and the interested reader can refer to [4, 13] for details. A general review is supplied here for completeness.

The wavelet transform is a spectral estimation technique similar to the fourier transform in that the signal is decomposed into an infinite set of scaled functions [4]. The main difference is that wavelets are finite in the time domain and scaleable, leading to a multi-scale decomposition. Multiresolution analysis can be obtained by applying a signal x through two sets of filters at level *i*. Filter g[n] is the discrete wavelet function, and acts as a high-pass filter. Filter h[n], is traditionally the biorthogonal counterpart to g, acting as a low-pass filter [4]. Convolving x with h[n] = h[-n] and downsampling results in a level i + 1 subspace of x, V_{i+1} , with approximation coefficients a_{i+1} . V_{i+1} has an orthogonal complement W_{i+1} that is the downsampled convolution of x with $\bar{g}[n] = g[-n]$, producing level i + 1 detail coefficients d_{i+1} . Repeating this process on V_{i+1} results in V_{i+2} and its orthogonal complement W_{i+2} . Thus W_{i+2} is orthogonal to W_{i+1} , and after infinite levels the original signal can be represented in a sparse domain of detail coefficients from each level.



Figure 1. The 1 Level 2D wavelet transform.

The 2D wavelet transform, as described in [13] is separable, meaning the process above can be applied to one dimension (rows) and then the other (columns), as shown in Fig. 1. The result is four distinct combinations of approximation and detail coefficients along each dimension. In the case of matrix Z, the decomposition along the rows represents information about signal structure, and decomposition along the columns represents muscle synergies. Along the time domain, only the detail coefficients are important, as they represent the frequency changes, so the focus of the multiresolution muscle synergy features is on obtaining a robust and sparse set of DA_i and DD_i . In accordance with the literature, with EMG sampled at 2KHz, 7 levels of decomposition are used along the rows to obtain detail coefficients roughly in the range of the majority of the energy of EMG signals (6-500Hz) [21]. A buffer of the latest 1000 samples is kept to perform the decomposition. The Daubechies wavelet with 4 vanishing moments (db4) is selected for the decomposition due to its good resolution in both time and frequency. After decomposition, only the most recent set of coefficients at each level are kept as features for the latest sample. This process is shown in Fig. 2 on a 6 level decomposition, where the graved regions indicate discarded coefficients for the current sample. The regions that are kept correspond to the latest 4 coefficients at each resolution, giving each datapoint a relative history ranging from 2ms-256ms at resolutions ranging from 2ms-64ms, with a denser proportion of information provided at more recent time due to the nature of the level resolutions. This time interval and density is also consistent with findings that indicate EMG signals are activated 50-100ms before motion, depending on the muscle in question [8]. Note that both the first decomposition and end approximation coefficients are completely discarded to remove noise and potential amplitude-dependent features, respectively.

After obtaining the relevant detail coefficients along each row, the transform is applied across the second dimension to encode muscle synergies at each resolution. The Haar wavelet is selected for this part of the decomposition due to its derivative-like behavior, and both approximation and detail coefficients are kept to maximize the amount of information provided by the muscle synergies at each resolution. The result is a (6 * 4)N sparse feature vector X containing multiresolution muscle synergies which can be calculated in 4 - 7ms.



Figure 2. Example 1D 6 level DWT. The top plot shows the original signal, and each consecutive plot below shows the detail coefficients Dn resulting from the wavelet transform at level n. The shaded region represents discarded coefficients when extracting features at time 1000ms.

Neural Network Classifier

A backpropagation neural network classifier is used to train on X. The neural network is chosen because it is well suited for multi-class problems and detecting patterns in sparse domains. The two major critiques against neural networks, slow training time and overfitting to training data [9], are made obsolete by training the system to be subject-independent with the robust set of MRMS features and adding large regularization parameters to the logit backpropagation algorithm, respectively. A one hiddenlayer network is formed with 18 nodes in the hidden layer. Using one layer helps prevent the neural network from overfitting to training data, making it more likely to detect the sparse set of user-independent synergies suggested in [5]. The network is trained using logit backpropagation with heavily regularized nonlinear conjugate gradient descent ($\lambda = 10$) [22] used to optimize the weights connecting each node, resulting in an optimized linear combination of weighted sums of sigmoids:

$$y_k(X) = \sigma\left(\sum_{j=1}^{18} w_{kj}^{(2)} \sigma\left(\sum_{i=1}^{L} w_{ji}^{(1)} + w_{j0}^{(1)}\right) + w_{k0}^{(2)}\right)$$
(1)

where L = (6 * 4)N is the size of *X*, σ is the sigmoid function:

$$\sigma(a) = \frac{1}{1 + e^{-a}} \tag{2}$$

each w is an optimized weight connecting nodes in the network, k = 1, ..., K and K is the total number of outputs. An additional benefit of the neural network is its computationally efficient prediction time, which combined with the computation time of the MRMS features allows predictions within 10ms of obtaining the signal, leaving room for additional high-level controllers without presenting a noticeable delay to the user.

Table 1. SELECTED MUSCLES AND PRIMARY FUNCTIONS [8].

Muscle	Primary Function
Extensor Digitorum	Wrist\Finger Extension
Extensor Carpi Ulnaris	Wrist Extension, Ulnar Deviation
Flexor Carpi Ulnaris	Wrist Flexion\Abduction, Ulnar Deviation
Flexor Carpi Radialis	Wrist Flexion, Radial Deviation
Pronator Teres	Forearm Pronation
Flexor Digitorum Superficialis	Wrist\Finger Flexion

RESULTS

The MRMS features are evaluated by their ability to provide robust features to develop a user-independent classifier. The backpropagation neural network described in the previous section is used as a decoder whose performance is tested with a preexisting database $B = \{b_1, \dots, b_N\}, N = 10$ (7 males, 3 females, age 22 ± 3 , 9 right-handed, 1 left-handed) associating EMG signals from six forearm muscles with five discrete hand motions (grasping, wrist extension, wrist flexion, forearm pronation, and index finger pointing). The muscles recorded by the EMG and their primary functions are given in Table 1. During data collection, subjects were instructed to alternate between resting and each the five motions for 10 seconds per motion. EMG electrodes (Trigno Wireless, Delsys Inc) recorded the EMG signals at 2KHzduring all motions. To evaluate user-independent performance, leave-one-out validation was performed with *B*. For each $b_i \in B$, the neural network is first trained on the set $\{b_i \in B, j \neq i\}$ to include data from all subjects except b_i . Then the trained network is tested on data from b_i . The metrics used to evaluate classifier performance are discrete prediction accuracy and Area Under the Curve (AUC) measurements for each subject [23].

AUC measurements are a measure of overall classifier performance when the output is probabilistic rather than deterministic, as in the case of the neural network. Detailed in [23], a Receiver Operator Characteristic (ROC) plot depicts overall classifier performance over all possible thresholds. To generate a ROC plot, a threshold is used to binarize the classifier output for each class. The threshold iterates from 0 to 1 in *n* steps, and at each iteration, a point on the plot is calculated as (1 - specificity, sensitivity), where $specificity = \frac{tn}{tn+fp}$ and $sensitivity = \frac{tp}{tp+fn}$. In words, specificity is the number of correct negative classifications tn over the total number of negative examples (tn + fp), and *sensitivity* is the number of corritive classifications tp over the total number of positive examples (tp + fn). Thus, the ROC plot provides a visual reference how



Figure 3. Example ROC plot and visualization of AUC measurements during leave-one-out validation.

well the classifier can detect a condition when it is present versus how well it can detect the absense of a condition when it is not present. AUC is the area under the ROC plot, with 1 a perfect score, 0.5 equivalent to random guessing, 0.85 a moderate classifier, and 0.95 regarded as a highly effective classifier.

An example ROC plot from leave-one-out testing on subject 10 is shown in Fig. 3 for reference. This plot is selected to demonstrate how the AUC measurement provides an informative measure for classifier performance. In the plot, both grasping and extension come closest to reaching perfect classification (i.e. always detecting the motion when it is present and never predicting it when it is not, at a given threshold). Index pointing is the lowest curve indicating a weaker performance with respect to other motions (i.e. does not detect the motion as often and more regularly detects the motion when it is not present). This is reflected in the AUC for each motion, as grasping and extension are nearly 1, and index pointing receives the lowest AUC at 0.886, which is a moderate performance classifier.

Table 2 shows the results of leave-one-out validation on all 10 subjects. The mean AUC over all subjects and all motions is 92.4 \pm 8.9, verifying the effectiveness of the decoder on data from unseen subjects. In 85% of the individual cases, the resulting classifier would be considered better than moderately effective. To provide a metric more commonly used in the literature, accuracy is measured by thresholding the output of the classifier on a random threshold between 0.1 and 0.5 for each subject and each motion. The mean accuracy is 81.9 ± 3.9 , when evaluated on every datapoint in the database. The high accuracy and low standard deviation show the robustness of the MRMS features across subjects. To consider the effectiveness of the classifier only in cases when motion was present, a normalized confusion matrix is shown in Fig. 4. The confusion matrix indicates that flexion and index pointing are confused with pronation for some subjects, but overall motions are detected between 65-95% with no confusion on the randomly selected threshold. These results, and the real time capabilities of the extraction method, suggest



Figure 4. Normalized confusion matrix for all subjects tested with leaveone-out validation.

that a higher level controller and/or visual feedback could smooth and correct any discontinuities when used in applications.

CONCLUSION

This paper introduces a method for robust EMG feature extraction by mapping EMG signals to a sparse multiresolution muscle synergy domain. The sparse features are less sensitive to inter- and intra-subject variabilities, making them well-suited for inputs to user-independent EMG to motion decoders. Additionally, the multiresolution analysis removes the need for preprocessing, allowing the decoder to perform in real time with less than 10ms delay. The robustness of the features are evaluated on the performance of a backpropagation neural network on a database of EMG signals mapped to a discrete set of hand and wrist motions. Using leave-one-out validation to test on subjects that were not included in the training phase of the neural network, the decoder achieves mean accuracy of $81.9 \pm 3.9\%$ and AUC $92.4 \pm 8.9\%$ over all motion classifications, validating its effectiveness. These results demonstrate the robustness of the features to extract meaningful and generalizable features from EMG signals, opening opportunities for the first user-independent EMG driven applications.

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Subject #	1	2	3	4	5	6	7	8	9	10	Mean	SD
Overall Accuracy (%)	82.5	82.6	86.0	82.8	87.4	80.6	85.2	80.5	75.1	76.7	81.9	3.9
Grasp AUC (%)	95.9	99.0	98.7	99.5	99.4	94.9	56.8	94.2	99.6	99.4	93.7	13.1
Extension AUC (%)	97.6	95.9	98.9	98.2	92.8	98.8	98.3	99.2	99.3	99.4	97.8	2.1
Flexion AUC(%)	90.2	97.9	98.8	99.1	93.6	97.5	97.1	90.0	83.0	96.2	94.3	5.2
Pronation AUC (%)	86.1	98.7	95.4	95.6	89.0	90.9	98.9	86.7	75.1	92.6	90.9	7.1
Index AUC (%)	80.2	76.6	71.4	95.1	92.8	90.8	93.2	89.3	74.7	85.9	85.0	8.6

Table 2. LEAVE ONE OUT VALIDATION RESULTS

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