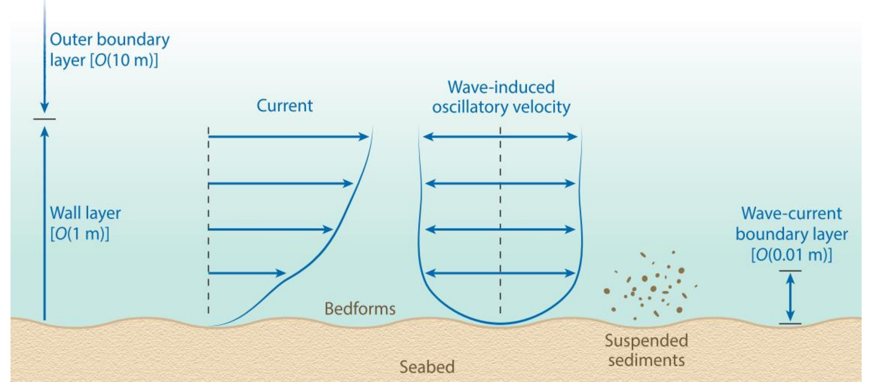
Toward modeling flocculation in turbulence-resolving simulations for cohesive sediment transport

Tian-Jian Hsu[1], **Jorge Penaloza-Giraldo**[1], Liangyi Yue[2], Andrew J Manning[1,2,3], Eckart Heinz Meiburg[4] and Bernhard Vowinckel[5]

(1) University of Delaware, Civil and Environmental Engineering, Newark, DE, United States, (2) Stanford University, Stanford, CA, United States (3) HR Wallingford Ltd, Coasts & Oceans Group, Wallingford, United Kingdom, (4) University of California Santa Barbara, Santa Barbara, CA, United States, (5) Technische Universität Braunschweig, Leichtweiß-Institut für Wasserbau, Braunschweig, Germany

1. Motivation

- Fine-grained sediments, commonly known as cohesive sediments, are ubiquitous in estuaries and on continental shelves.
- Flocs are vehicles of nutrients, organic carbon and pollutants, and hence their fate is the key to the understanding of coastal ecosystem, morphodynamics, water quality, and carbon cycle.
- Cohesive sediment transport is a tightly coupled system among flows, sediment resuspension and deposition, and flocculation.



Source: Trowbridge and Lentz (2018)

Inhomogeneous turbulent flow for fine sediment transport

Coupling between Flocculation and Large Eddy Simulation (LES) or Reynolds-averaged models (RANS)

- The LES study by Liu et al. (2019) exclude the bottom boundary layer.
- RANS study by Sherwood et al. 2018) focus on estuarine scale processes.

Direct Numerical Simulation (DNS) of turbulence in wave bottom boundary layer and fine sediment transport (Ozdemir et al. 2010, Yue et al. 2020).

- Neglect flocculation.
- Assume constant Settling velocity
- Exchange of sediments in wave-current boundary layer is poorly understood (e.g., Hill et al. 2001).

Couple DNS with a size-class based flocculation model

2. Size class-based flocculation model – Verney et al (2011)

$$\frac{dn_k}{dt} = \underbrace{G_{aggr}(k) + G_{break\_shear}(k)}_{\text{Gain number concentration } (m^{-3}\,s^{-1})} - \underbrace{L_{aggr}(k) - L_{break\_shear}(k)}_{\text{Loss number concentration } (m^{-3}\,s^{-1})}$$

Aggregation (A)

$$G_{aggr}(k) = \frac{1}{2}\sum_{i+j=k} \alpha A(i,j) n_i n_j \ ,$$

$$L_{aggr}(k) = \sum_{i=1}^{N} \alpha A(i,k) n_i n_k$$

$$A(i,k) = \frac{1}{6} G (d_i + d_j)^3$$

DNS provides the accurate turbulent shear rate $G = \sqrt{\varepsilon/\nu}$

Breakup (B)

$$G_{break\_shear}(k) = \sum_{i+j=k} FDBS_{ki} B_i n_i$$

$$L_{break\_shear}(k) = \sum_{i=1}^{N} B_i n_i$$

$$B_i = \beta G^{3/2} d_i \left(\frac{d_i - d_p}{d_p}\right)^{3-f_d}$$

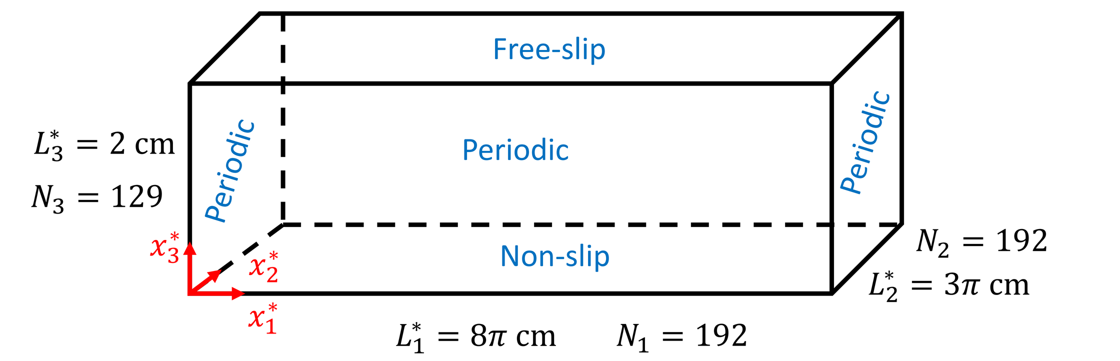
Binary breakup

$$FDBS_{ki} = \begin{cases} 2 & if\ m_j = m_i/2 \\ 0 & \text{otherwise} \end{cases}$$

VARIABLES AND PARAMETERS :

α = Collision efficiency (assumed constant) = 0.25
β = Fragmentation rate (assumed constant) = 0.028
f_d = Fractal dimension = 2.4

3. DNS for Open Channel Flow – Yue et al (2020)



$$Re_\tau = \frac{u_\tau^* L_3^*}{\nu} = 180$$

Characteristic velocity scale (Friction velocity) → $u_\tau^* = 0.009$ m/s

Characteristic length scale (Domain Heigh) → $L_3^* = 0.02$ m

Governing equations

$$\underbrace{v_i^* = u_i^* - w_s^* n_i}_{\text{Equilibrium Eulerian approximation (Balachandar and Eaton, 2009)}} \qquad \frac{\partial u_i^*}{\partial x_i^*} = 0 \qquad \frac{\partial \phi}{\partial t^*} + \frac{\partial(\phi v_j^*)}{\partial x_j^*} = K_s \frac{\partial^2 \phi}{\partial x_j^* \partial x_j^*}$$

$$\frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = -\frac{1}{\rho_f}\frac{\partial p^*}{\partial x_i^*} + \nu \frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*} + Rg\phi n_i + S_i^*$$

Boundary conditions

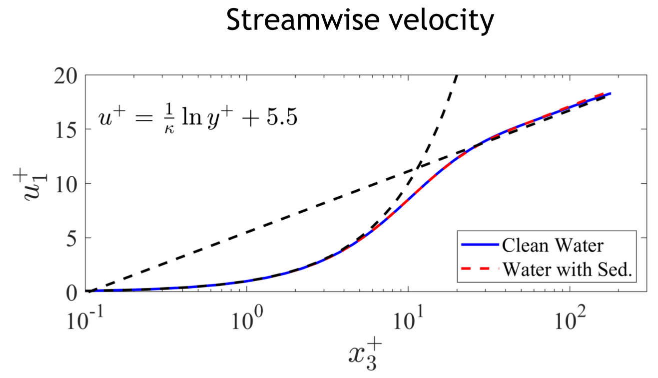
$$\frac{\partial u_i^*}{\partial x_3^*} = 0 \text{ at } x_3^* = L_3^* \qquad \phi w_s^* n_3 - K_s \frac{\partial \phi}{\partial x_3^*} = 0 \text{ at } x_3^* = L_3^*$$

$$u_i^* = 0 \text{ at } x_3^* = 0 \qquad \phi w_s^* n_3 - K_s \frac{\partial \phi}{\partial x_3^*} = q_e^* + q_d^* \text{ at } x_3^* = 0$$

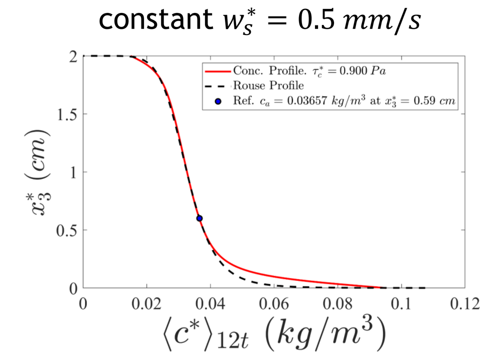
where sub-index $i = 1, 2, 3$ represents the streamwise x_1^*, the spanwise x_2^* and the vertical x_3^* directions, respectively. v_i^* is the velocity of sediment phase, u_i^* is carrier flow velocity, w_s^* the sediment settling velocity, n_i is a normalized vector representing the direction of the gravitational acceleration, t^* is time, p^* is pressure, ν is the kinematic viscosity, R is defined as $R = (s-1)$ with $s = \rho_s/\rho_f = 2.65$ is the specific gravity, ϕ is the volumetric concentration of sediment, K is the effective diffusivity of sediment, S_i^* is spatially averaged pressure gradient, which is the driving force of the model.

4. Ensemble averaged profiles

Streamwise velocity

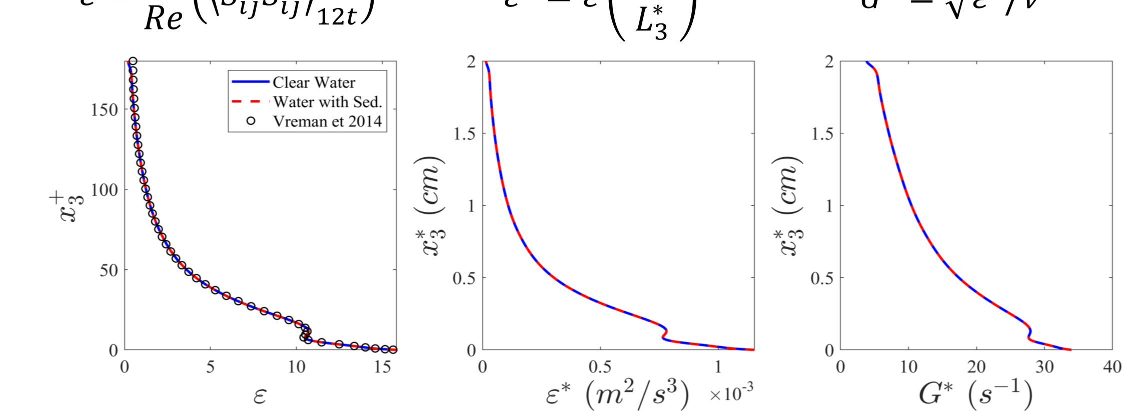


Sediment concentration with a constant $w_s^* = 0.5\ mm/s$



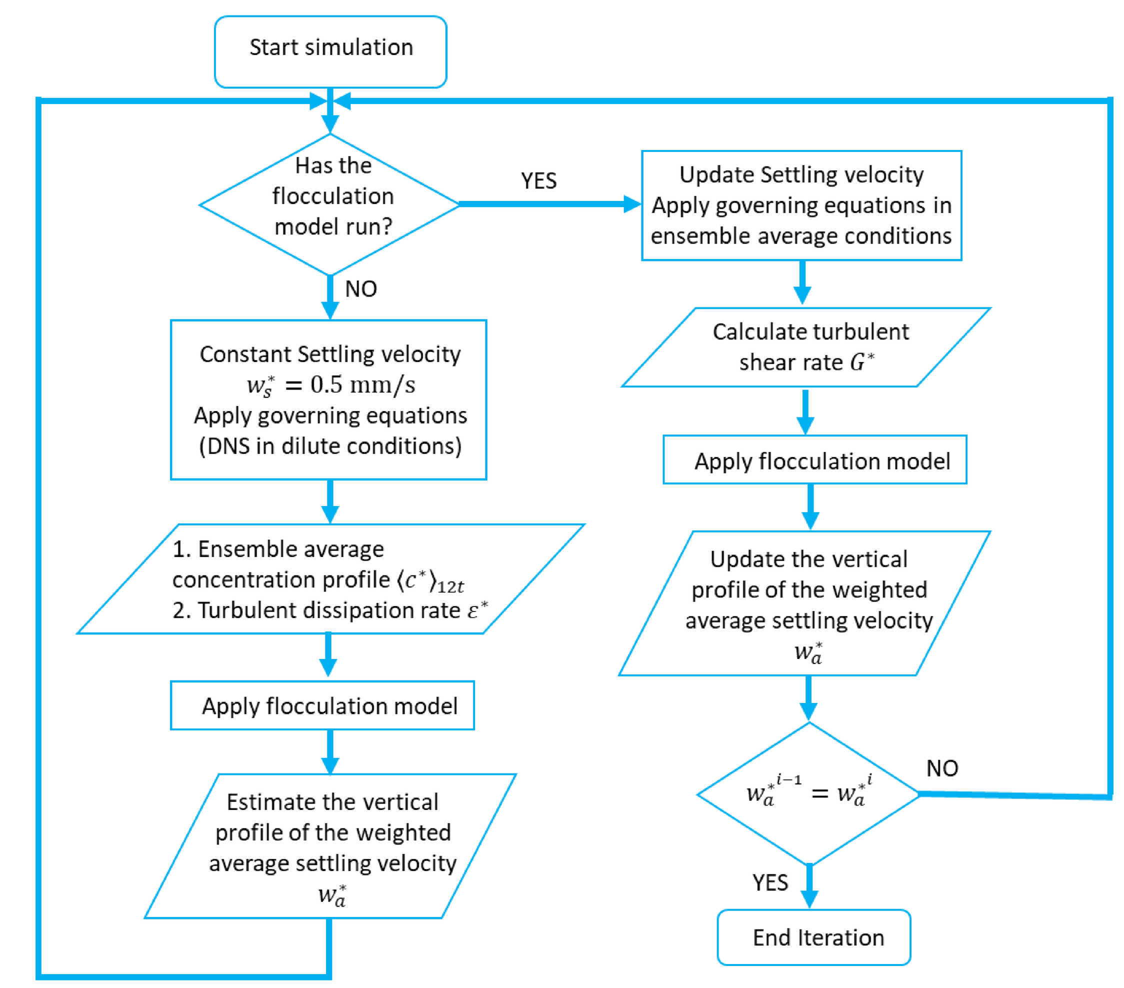
5. Turbulent shear rate G^*

$$\varepsilon = \frac{2}{Re}\left(\langle S'_{ij} S'_{ij} \rangle_{12t}\right) \qquad \varepsilon^* = \varepsilon \left(\frac{u_\tau^{*3}}{L_3^*}\right) \qquad G^* = \sqrt{\varepsilon^*/\nu}$$



$$S'_{ij} = \frac{1}{2}\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)$$

5. Weak coupling for dilute flow



Extracting eddy diffusivity in ensemble average equations

$$K_s \frac{d^2\langle c^* \rangle_{12t}}{dx_3^{*2}} + \frac{d(w_s^* \langle c^* \rangle_{12t})}{dx_3^*} - \frac{d\langle u_3^{*\prime} c'^* \rangle_{12t}}{dx_3^*} = 0$$

Integrating

$$K_s \frac{d\langle c^* \rangle_{12t}}{dx_3^*} + w_s^* \langle c^* \rangle_{12t} - \langle u_3^{*\prime}\, c'^* \rangle_{12t} = 0$$

Defining $-\langle u_3^{*\prime}\, c'^* \rangle_{12t}$

$$-\langle u_3'^* c_3'^* \rangle_{12t} = K_t \frac{d\langle c^* \rangle_{12t}}{dx_3^*}$$

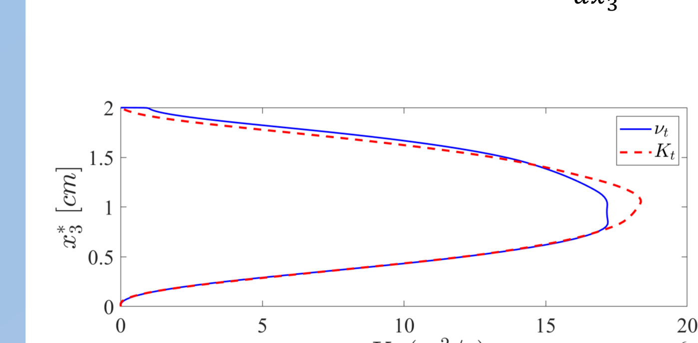
Replacing $-\langle u_3^{*\prime}\, c'^* \rangle_{12t}$

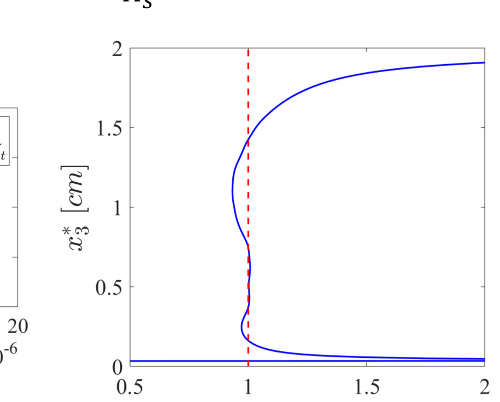
$$(K_t + K_s)\frac{d\langle c^* \rangle_{12t}}{dx_3^*} + w_s^* \langle c^* \rangle_{12t} = 0$$

Defining K_s and K_t

$$Sc_t = \frac{\nu_t}{K_t} = 1$$

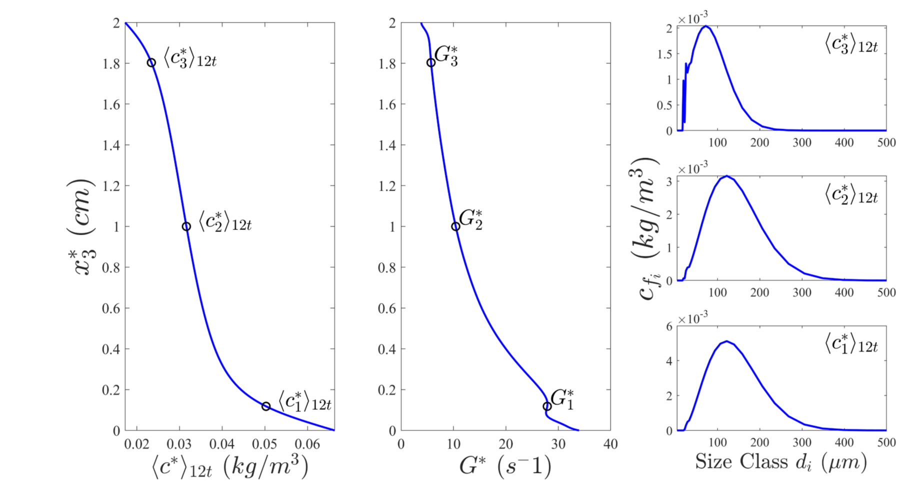
Note: $-\langle u_1'^* u_3'^* \rangle = \nu_t \frac{d\langle u_1^* \rangle}{dx_3^*}$ $\qquad Sc = \frac{\nu}{K_s} = 1$





Turbulence resolving simulation indicates $Sc_t \approx 1$, consistent with the equilibrium approximation.

6. Sediment concentration in different size class calculated by the flocculation model



$$W_e(x_3^*) = \left(\frac{c_{f_i}}{\langle c^* \rangle_{12t}|_{x_3^*}}\right)$$

W_e = Weight contribution
c_{f_i} = Concentration for each floc size class
$\langle c^* \rangle_{12t}$ = Ensemble averaged concentration profile

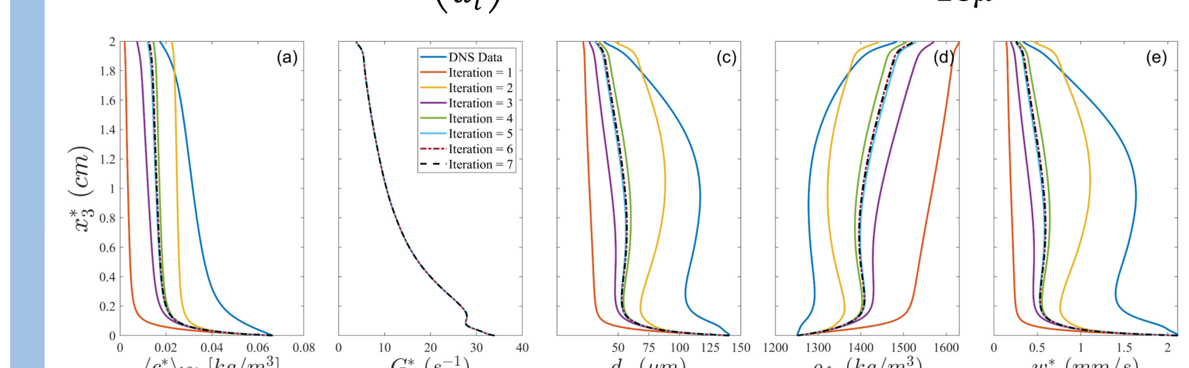
7. Vertical profiles of weighted average floc properties (first interaction)

$$\Phi_a(x_3^*) = \sum_{i=d_p}^{N=\max(d_i)} \left(\frac{c_{f_i}}{\langle c^* \rangle_{12t}|_{x_3^*}}\right)\Phi_i$$

Φ_i = Discrete property of the floc
c_{f_i} = Concentration for each floc size class
Φ_a = weighted average floc property

Discrete properties of the floc

$$\rho_{f,i} = \rho_w + (\rho_p - \rho_w)\left(\frac{d_p}{d_i}\right)^{3-f_d} \qquad w_{s,i}^* = \frac{(\rho_{f,i} - \rho_w)\, d_i^2}{18\mu}$$



8. Conclusions and Future work

- A loose coupling between Direct Numerical Simulation and size-class flocculation model in dilute condition predicts a large vertical variability of settling velocity between the viscous sublayer, buffer layer, and log layer.
- Large vertical variability of settling velocity below the log-layer (x_3^+) is due to large variabilities of turbulent shear and sediment concentration. Settling velocity in the log-layer can be approximated as a constant value.
- This loose coupling methodology will be extended for wave-current bottom boundary layer in the near future.

9. Reference

- Balachandar, S. and Eaton, J. K.: Turbulent Dispersed Multiphase Flow, Annu. Rev. Fluid Mech., 42(1), 111–133, doi:10.1146/annurev.fluid.010908.165243, 2009.
- Hill, P. S., Voulgaris, G. and Trowbridge, J. H.: Controls on floc size in a continental shelf bottom boundary layer, J. Geophys. Res. Ocean., 106(C5), 9543–9549, doi:https://doi.org/10.1029/2000JC900102, 2001.
- Liu, J., Liang, J.-H., Xu, K., Chen, Q. and Ozdemir, C. E.: Modeling Sediment Flocculation in Langmuir Turbulence, J. Geophys. Res. Ocean., 124(11), 7883–7907, doi:10.1029/2019JC015197, 2019.
- Ozdemir, C. E., Hsu, T.-J. and Balachandar, S.: Simulation of fine sediment transport in oscillatory boundary layer, J. Hydro-environment Res., 3(4), 247–259, doi:https://doi.org/10.1016/j.jher.2009.10.013, 2010.
- Sherwood, C. R., Aretxabaleta, A. L., Harris, C. K., Rinehimer, J. P., Verney, R. and Ferré, B.: Cohesive and mixed sediment in the Regional Ocean Modeling System (ROMS v3.6) implemented in the Coupled Ocean–Atmosphere–Wave–Sediment Transport Modeling System (COAWST r1234), Geosci. Model Dev., 11(5), 1849–1871, doi:10.5194/gmd-11-1849-2018, 2018.
- Trowbridge, J. H. and Lentz, S. J.: The Bottom Boundary Layer, Ann. Rev. Mar. Sci., 10(1), 397–420, doi:10.1146/annurev-marine-121916-063351, 2018.
- Verney, R., Lafite, R., Claude Brun-Cottan, J. and Le Hir, P.: Behaviour of a floc population during a tidal cycle: Laboratory experiments and numerical modelling, Cont. Shelf Res., 31(10), S64–S83, doi:10.1016/j.csr.2010.02.005, 2011.
- Yue, L., Cheng, Z. and Hsu, T.-J.: A Turbulence-Resolving Numerical Investigation of Wave-Supported Gravity Flows, J. Geophys. Res. Ocean., 125(2), e2019JC015220, doi:10.1029/2019JC015220, 2020.