QM1 Concept Test 4.1

1. For a spin-1/2 system, the spin quantum number $s = 1/2$ and the quantum number for the z component of the spin is $m_z = \pm 1/2$. The matrix representations for $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$, which are the eigenstates of $\hat{S}_z$, are $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, respectively. Choose all of the following statements that are true.

(I) $\left|\frac{1}{2},\frac{1}{2}\right\rangle = |1/2\rangle = |\uparrow\rangle_z$

(II) $\left|-\frac{1}{2},-\frac{1}{2}\right\rangle = |-1/2\rangle = |\downarrow\rangle_z$

(III) $\hat{S}_z|\downarrow\rangle_z = \hbar/2|\downarrow\rangle_z$

(a) (I) only
(b) (II) only
(c) (I) and (II) only
(d) (I), (II) and (III)

1

QM1 Concept Test 4.2

2. The matrix representations for $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$, which are the eigenstates of $\hat{S}_z$, are $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, respectively. Which one of the following sets of scalar (inner) products is correct?
(all $|\uparrow\rangle$ denote $|\uparrow\rangle_z$ and all $|\downarrow\rangle$ denote $|\downarrow\rangle_z$)

(a) $\langle\uparrow|\uparrow\rangle = 1,\ \langle\uparrow|\downarrow\rangle = 1,\ \langle\downarrow|\uparrow\rangle = 1,\ \langle\downarrow|\downarrow\rangle = 1$
(b) $\langle\uparrow|\uparrow\rangle = 1,\ \langle\uparrow|\downarrow\rangle = 0,\ \langle\downarrow|\uparrow\rangle = 0,\ \langle\downarrow|\downarrow\rangle = 1$
(c) $\langle\uparrow|\uparrow\rangle = 1, \langle\uparrow|\downarrow\rangle = -1, \langle\downarrow|\uparrow\rangle = -1, \langle\downarrow|\downarrow\rangle = 1$
(d) $\langle\uparrow|\uparrow\rangle = 1,\ \langle\uparrow|\downarrow\rangle = 0,\ \langle\downarrow|\uparrow\rangle = 0, \langle\downarrow|\downarrow\rangle = -1$

2

QM1 Concept Test 4.3

3. The matrix representations for $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$, which are the eigenstates of $\hat{S}_z$, are $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, respectively. Which one of the following is the matrix representation of $\hat{S}_z$ if we choose the basis vectors in the order $|\uparrow\rangle_z, |\downarrow\rangle_z$ to construct the matrix?

(a) $\frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

(b) $\frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

(c) $\frac{\hbar}{2}\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}$

(d) $\frac{\hbar}{2}\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

3

QM1 Concept Test 4.4

4. The matrix representation for $|\uparrow\rangle_z$, which is an eigenstate of $\hat{S}_z$, is $\begin{pmatrix}1\\0\end{pmatrix}$. Which of the following statements are correct about $|\uparrow\rangle_z$?

(I) $|\uparrow\rangle_z = \frac{1}{\sqrt{2}}(|\uparrow\rangle_x + |\downarrow\rangle_x)$

(II) $|\uparrow\rangle_z = \frac{1}{\sqrt{2}}(|\uparrow\rangle_x - |\downarrow\rangle_x)$

(III) $|\uparrow\rangle_z = \frac{1}{\sqrt{2}}(|\uparrow\rangle_y - |\downarrow\rangle_y)$

A. I only B. II only C. III only D.I and III only E. none of the above

4

QM1 Concept Test 4.5

5. At time $t=0$, the initial state of a spin-1/2 particle is $|\uparrow\rangle_z$. Choose all of the following statements that are correct.

(1) If we perform a measurement of the x or y component of the spin at $t=0$, we will obtain zero for both.

(2) If we measure $\hat{S}_y$, we can get either $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$ with equal probability.

(3) If we measure $\hat{S}^2$, we can only get $\frac{3\hbar^2}{4}$.

A. 1 only B. 2 only C. 3 only D. 1 and 3 only
E. 2 and 3 only

5

QM1 Concept Test 4.6

6. Suppose the uniform magnetic field $\vec{B}_0$ is along the z-direction. Which one of the following matrices corresponds to the Hamiltonian of an electron spin in the magnetic field in the basis of eigenstates of $\hat{S}_z$? The Hamiltonian is $\hat{H} = -\gamma\vec{S}\cdot\vec{B}$, where the gyromagnetic ratio γ is a constant.

A. $-\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ B. $-\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

C. $-\gamma B_0 \hbar\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ D. $-\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

E. None of the above

6

QM1 Concept Test 4.7

7. Suppose the uniform magnetic field $\vec{B}_0$ is along the x-direction. Which one of the following matrices corresponds to the Hamiltonian of an electron spin in the magnetic field in the basis of eigenstates of $\hat{S}_z$? The Hamiltonian is $\hat{H} = -\gamma\vec{S}\cdot\vec{B}$, where the gyromagnetic ratio γ is a constant.

A. $-\gamma B_0 \hbar\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ B. $-\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

C. $-\gamma B_0 \hbar\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ D. $-\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

E. $-\gamma B_0 \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

7