QM1 Concept Test 7.9

The Hamiltonian of a spin-1/2 particle in a magnetic field $\vec{B} = B_0\hat{z}$ is $\hat{H} = -\gamma B_0 \hat{S}_z$, where $\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the z-component of the spin angular momentum operator. Which one of the following equations correctly represents the eigenvalue of the Hamiltonian $\hat{H} = -\gamma B_0 \hat{S}_z$?

A. $E_\pm = \mp \gamma B_0$
B. $E_\pm = \mp \frac{\gamma B_0}{2}$
C. $E_\pm = \mp \frac{\hbar}{2}\gamma B_0$
D. $E_\pm = \mp \frac{\hbar}{2}$
E. None of the above.

1

QM1 Concept Test 7.10

An electron in a magnetic field $\vec{B} = B_0\hat{z}$ is initially in the spin state $|\chi(0)\rangle = |\uparrow\rangle_z$. Which one of the following equations correctly represents the state $|\chi(t)\rangle$ of the electron? The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

A. $|\chi(t)\rangle = |\uparrow\rangle_z$
B. $|\chi(t)\rangle = e^{i\gamma B_0 t/2}|\uparrow\rangle_z$
C. $|\chi(t)\rangle = e^{i\gamma B_0 t/2}|\uparrow\rangle_z + e^{-i\gamma B_0 t/2}|\downarrow\rangle_z$
D. $|\chi(t)\rangle = ae^{i\gamma B_0 t/2}|\uparrow\rangle_z + be^{-i\gamma B_0 t/2}|\downarrow\rangle_z$
E. None of the above.

2

QM1 Concept Test 7.11

If the spin state of the quantum system at time t=0 is given by $|\chi(0)\rangle = a|\uparrow\rangle_z + b|\downarrow\rangle_z$, which one of the following equations correctly represents the state $|\chi(t)\rangle$ after time t? The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

A. $|\chi(t)\rangle = e^{i\gamma B_0 t/2}\left(a|\uparrow\rangle_z + b|\downarrow\rangle_z\right)$

B. $|\chi(t)\rangle = e^{-i\gamma B_0 t/2}\left(a|\uparrow\rangle_z + b|\downarrow\rangle_z\right)$

C. $|\chi(t)\rangle = e^{i\gamma B_0 t/2}\left((a+b)|\uparrow\rangle_z + (a-b)|\downarrow\rangle_z\right)$

D. $|\chi(t)\rangle = ae^{i\gamma B_0 t/2}|\uparrow\rangle_z + be^{-i\gamma B_0 t/2}|\downarrow\rangle_z$

E. None of the above.

3

QM1 Concept Test 7.12

The spin state of the quantum system at time t=0 is given by $|\chi(0)\rangle = a|\uparrow\rangle_z + b|\downarrow\rangle_z$. What is the probability of obtaining $\pm\hbar/2$ if we measure $\hat{S}_z$ at time $t = t_0$? The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

A. $\hbar/2$ with a probability $ae^{i\gamma B_0 t/2}$ and $-\hbar/2$ with a probability $be^{-i\gamma B_0 t/2}$

B. $\hbar/2$ with a probability $|a|^2 e^{i\gamma B_0 t/2}$ and $-\hbar/2$ with a probability $|b|^2 e^{-i\gamma B_0 t/2}$

C. $\hbar/2$ with a probability $|a|^2$ and $-\hbar/2$ with a probability $|b|^2$

D. $\hbar/2$ and $-\hbar/2$ with equal probability

E. None of the above.

4

QM1 Concept Test 7.13

Suppose the initial spin state of the quantum system is $|\chi(0)\rangle = \cos\alpha|\uparrow\rangle_z + \sin\alpha|\downarrow\rangle_z$. Which one of the following is the expectation value $\langle \hat{S}_z \rangle = \langle \chi(t)|\hat{S}_z|\chi(t)\rangle$? The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

Note: $\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

A. $\cos(2\alpha)\cos(\gamma B_0 t)\hbar/2$

B. $\sin(2\alpha)\sin(\gamma B_0 t)\hbar/2$

C. $\cos(2\alpha)\hbar/2$

D. $\sin(2\alpha)\hbar/2$

E. None of the above.

5

QM1 Concept Test 7.14

Suppose the initial spin state of a quantum system is $|\chi(0)\rangle = \cos\alpha|\uparrow\rangle_z + \sin\alpha|\downarrow\rangle_z$. Which one of the following is the expectation value $\langle \hat{S}_x \rangle = \langle \chi(t)|\hat{S}_x|\chi(t)\rangle$? The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

Note: $\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

A. $\cos(2\alpha)\cos(\gamma B_0 t)\hbar/2$

B. $\sin(2\alpha)\cos(\gamma B_0 t)\hbar/2$

C. $\cos(2\alpha)\hbar/2$

D. $\sin(2\alpha)\hbar/2$

E. None of the above.

6

QM1 Concept Test 7.15

Choose all of the following statements that are true about the expectation value $\langle \hat{S} \rangle$ in the generic spin state $|\chi(t)\rangle$. The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

A. $\langle \hat{S} \rangle$ must always depend on time because $[\hat{H}, \hat{S}] \neq 0$

B. $\langle \hat{S} \rangle$ must always depend on time because $\langle \hat{S} \rangle$ precesses about the z-axis.

C. $\langle \hat{S} \rangle$ is time-independent because the expectation value of an observable is its time-averaged value.

D. $\langle \hat{S} \rangle$ is time-independent only when the initial state is purely $|\uparrow\rangle_z$ or $|\downarrow\rangle_z$

E. None of the above

7

QM1 Concept Test 7.16

Choose all of the following statements that are true about the expectation value $\langle \hat{S} \rangle$ in the state $|\chi(t)\rangle$ when the initial state is **NOT** purely $|\uparrow\rangle_z$ or $|\downarrow\rangle_z$. The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

(1) The z component of $\langle \hat{S} \rangle$, i.e., $\langle \hat{S}_z \rangle$, is time-independent.

(2) The x and y components of $\langle \hat{S} \rangle$ change with time. When the magnitude of $\langle \hat{S}_x \rangle$ is a maximum, the magnitude of $\langle \hat{S}_y \rangle$ is a minimum and vice versa.

(3) The magnitudes of the maximum values of $\langle \hat{S}_x \rangle$ and $\langle \hat{S}_y \rangle$ are the same.

A. 1 only

B. 1 and 2 only

C. 1 and 3 only

D. 2 and 3 only

E. All of the above

8

QM1 Concept Test 7.17

Choose all of the following statements that are true about the expectation value $\langle \hat{S} \rangle$ in the state $|\chi(t)\rangle$ when the initial spin state is **NOT** purely $|\uparrow\rangle_z$ or $|\downarrow\rangle_z$. The external magnetic field is in the *z*-direction. The Hamiltonian operator is $\hat{H} = -\gamma B_0 \hat{S}_z$.

(1) The vector $\langle \hat{S} \rangle$ can be thought to be precessing about the z axis at a non-zero angle.
(2) The vector $\langle \hat{S} \rangle$ can be thought to be precessing about the z axis with a frequency $\omega = \gamma B_0$.
(3) All the three components of vector $\langle \hat{S} \rangle$ change as it precesses about the z axis.

A. 1 only
B. 1 and 2 only
C. 1 and 3
D. 2 and 3
E. All of the above

9

QM1 Concept Test 7.18

Choose all of the following statements that are correct about the raising operator $\hat{S}_+$ for a spin-1/2 particle. $|\uparrow\rangle_z$ is an eigenstate of $\hat{S}_z$ with eigenvalue $\frac{\hbar}{2}$.

(1) $\hat{S}_+ |\uparrow\rangle_z = 0$

(2) $\hat{S}_+ |\downarrow\rangle_z = \hbar |\uparrow\rangle_z$

(3) In the basis of eigenstates of $\hat{S}_z$, $\hat{S}_+ = \hat{S}_x + i\hat{S}_y = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

A. 3 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. all of the above

10

Concept Test 7.19

Suppose the particle is initially in an eigenstate of the x-component of spin angular momentum operator $\hat{S}_x$. Choose all of the following statements that are correct:

(1) The expectation value $\langle S_x \rangle$ depends on time.

(2) The expectation value $\langle S_y \rangle$ depends on time.

(3) The expectation value $\langle S_z \rangle$ depends on time.

A. 1 only B. 3 only C. 1 and 2 only D. 2 and 3 only E. all of the above

11

Concept Test 7.20

Suppose the particle is initially in an eigenstate of the z-component of spin angular momentum $\hat{S}_z$. Choose all of the following statements that are correct:

(1) The expectation value $\langle S_x \rangle$ depends on time.

(2) The expectation value $\langle S_y \rangle$ depends on time.

(3) The expectation value $\langle S_z \rangle$ depends on time.

A. None of the above B. 1 only C. 3 only D. 1 and 2 only E. all of the above

12