Choose all of the following observables for which it will be convenient to use the coupled representation to find the probabilities of different outcomes in a given quantum state:

I. L_z
II. S_z
III. J_z
IV. $\vec{J}^2$

a) III only
b) IV only
c) I and II only
d) III and IV only
e) All of the above

1

In the product space of a spin 2 and a spin 1/2 particles, the total spin quantum number for the system is 5/2, and its z component is $\frac{\hbar}{2}$, i.e., the state in the coupled representation is $\left|\frac{5}{2}\ \frac{1}{2}\right\rangle$. Which one of the following is a possible expansion of this coupled state in the uncoupled representation?

a) $\sqrt{\frac{2}{5}}|2\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle+\sqrt{\frac{3}{5}}|2\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

b) $\sqrt{\frac{2}{5}}|2\ 1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle+\sqrt{\frac{3}{5}}|2\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

c) $\sqrt{\frac{2}{5}}|2\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle+\sqrt{\frac{3}{5}}|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

d) $\sqrt{\frac{2}{5}}|2\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle+\sqrt{\frac{3}{5}}|2\ 0\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$

e) $\sqrt{\frac{2}{5}}|1\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle+\sqrt{\frac{3}{5}}|2\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

2

In the product space of a spin 2 and a spin 1/2 particles, the total spin quantum number for the system is 5/2, and its z component is $\frac{\hbar}{2}$, i.e., the state in the coupled representation is $\left|\frac{5}{2}\ \frac{1}{2}\right\rangle$. This state in the uncoupled representation is

$\left|\frac{5}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{5}}|2\ 1\rangle\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle + \sqrt{\frac{3}{5}}|2\ 0\rangle\left|\frac{1}{2}\ \ \frac{1}{2}\right\rangle$. What are the possible values of $\vec{S}^2$ for the spin ½ particle and what are their probabilities?

a) $\frac{3}{4}\hbar^2$, probability 2/5 and $6\hbar^2$, probability 3/5
b) $\frac{3}{4}\hbar^2$, probability 3/5 and $2\hbar^2$, probability 2/5
c) $6\hbar^2$, 100% probability
d) $\frac{3}{4}\hbar^2$, 100% probability
e) None of the above

3

In the product space of a spin 2 and a spin 1/2 particles, the total spin quantum number for the system is 5/2, and its z component is $\frac{\hbar}{2}$, i.e., the state in the coupled representation is $\left|\frac{5}{2}\ \frac{1}{2}\right\rangle$. This state in the uncoupled representation is

$\left|\frac{5}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{5}}|2\ 1\rangle\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle + \sqrt{\frac{3}{5}}|2\ 0\rangle\left|\frac{1}{2}\ \ \frac{1}{2}\right\rangle$. If you measure the z component of the angular momentum of the spin 2 particle, what are the possible values and their probabilities?

a) $-\frac{\hbar}{2}$, probability 2/5 and $\frac{\hbar}{2}$, probability 3/5
b) $\hbar$, probability 2/5 and 0, probability 3/5
c) $\frac{\hbar}{2}$, 100% probability
d) $2\hbar^2$, 100% probability
e) None of the above

4

An electron in a hydrogen atom with $l = 1$ occupies the angular momentum state $\sqrt{\frac{1}{3}}|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1\ 1\rangle\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle$.

From the Clebsch Gordan table:

$|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

$|1\ 1\rangle\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

If you measure $\vec{L}^2$, what values might you obtain and what are their probabilities?

a) $2\hbar^2$, 100% probability
b) $2\hbar^2$, 1/3 probability
c) $\frac{3}{4}\hbar^2$, 100% probability
d) $\frac{15}{4}\hbar^2$, 100% probability
e) None of the above

5

An electron in a hydrogen atom with $l = 1$ occupies the angular momentum state $\sqrt{\frac{1}{3}}|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1\ 1\rangle\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle$.

From the Clebsch Gordan table:

$|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

$|1\ 1\rangle\left|\frac{1}{2}\ \ -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

If you measure S_z, choose all of the following values you might obtain and along with their probabilities:

I. $\frac{\hbar}{2}$, 1/3 probability
II. $-\frac{\hbar}{2}$, 2/3 probability
III. $\frac{\hbar}{2}$, 100% probability

a. I only b. II only c. I and II only d. III only e. none of the above

6

An electron in a hydrogen atom with $l = 1$ occupies the angular momentum state $\sqrt{\frac{1}{3}}|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$.
From the Clebsch Gordan table:
$|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$
$|1\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$. If you measure $\vec{J}^2$, what values might you obtain and what are their probabilities?
a) $\frac{15}{4}\hbar^2$, 100% probability
b) $\frac{15}{4}\hbar^2$, 1/3 probability and $\frac{3}{4}\hbar^2$, 2/3 probability
c) $\frac{15}{4}\hbar^2$, 4/9 probability and $\frac{3}{4}\hbar^2$, 5/9 probability
d) $\frac{15}{4}\hbar^2$, 8/9 probability and $\frac{3}{4}\hbar^2$, 1/9 probability
e) None of the above

7

An electron in a hydrogen atom with $l = 1$ occupies the angular momentum state $\sqrt{\frac{1}{3}}|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$.
From the Clebsch Gordan table:
$|1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$
$|1\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$. If you measure J_z, the z component of the total angular momentum, what values might you obtain and what are their probabilities?
a) $-\frac{\hbar}{2}$, probability 2/3 and $\frac{\hbar}{2}$, probability 1/3
b) 0, probability 1/3, and $\hbar$, probability 2/3
c) $\frac{\hbar}{2}$, 100% probability
d) $\frac{3\hbar}{2}$, probability 1/3 and $\frac{\hbar}{2}$, probability 2/3
e) None of the above

8

Consider the following statements for the product space of two spin systems: $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ cannot be written as a diagonal matrix in the uncoupled representation because

I. $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ does not commute with the operators $\hat{S}_{1z}$ and $\hat{S}_{2z}$ whose eigenstates are the basis vectors in the uncoupled representation.

II. $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ is the Hamiltonian operator which can never be diagonal no matter what basis you choose.

III. We are dealing with two spin-1/2 system. $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ would be diagonal if we had two spin-one system.

a) (I) only b) (II) only c) (III) only d) (I) and (III) only
e) none of the above

9

Choose all of the following statements that are correct about $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ for the product space of two spin system.

I. $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ is an off-diagonal matrix if the basis vectors are the simultaneous eigenstates of $\hat{S}_1^2$, $\hat{S}_{1z}$, $\hat{S}_2^2$, and $\hat{S}_{2z}$.

II. It is possible to put $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ into a diagonal matrix form in a suitable basis but the basis vectors will not be the eigenstates of $\hat{S}_1^2$, $\hat{S}_{1z}$, $\hat{S}_2^2$, and $\hat{S}_{2z}$.

III. The basis vectors can be chosen to be simultaneous eigenstates of $\hat{S}_{1x}$ and $\hat{S}_{1z}$.

a) (I) only b) (II) only c) (I) and (II) only
d) (I) and (III) only e) All of the above.

10

Consider the product space of two spin-1/2 systems. The raising and lowering operators for each spin, e.g., for the first spin are given as $\hat{S}_{1+} = \hat{S}_{1x} + i\hat{S}_{1y}$ and $\hat{S}_{1-} = \hat{S}_{1x} - i\hat{S}_{1y}$. Choose all of the following expressions that are correct for $\hat{S}_1 \cdot \hat{S}_2$. These expressions will be helpful in writing $\widehat{H} = C\hat{S}_1 \cdot \hat{S}_2$ in the uncoupled or coupled representation.

I. $\hat{S}_1 \cdot \hat{S}_2 = \hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} + \hat{S}_{1z}\hat{S}_{2z}$

II. $\hat{S}_1 \cdot \hat{S}_2 = \frac{\hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{1+}\hat{S}_{2-}}{2} + \hat{S}_{1z}\hat{S}_{2z}$

III. $\hat{S}_1 \cdot \hat{S}_2 = \frac{\hat{S}^2 - \hat{S}_1^2 - \hat{S}_2^2}{2}$

a) (I) only b) (I) and (II) only c) (I) and (III) only
d) (II) and (III) only e) All of the above

11

Consider the product space of two spin-1/2 systems. The raising and lowering operators for each spin, e.g., for the first spin are given as $\hat{S}_{1+} = \hat{S}_{1x} + i\hat{S}_{1y}$ and $\hat{S}_{1-} = \hat{S}_{1x} - i\hat{S}_{1y}$. Choose all of the following expressions for $\hat{S}_1 \cdot \hat{S}_2$ that will make $\widehat{H} = C\hat{S}_1 \cdot \hat{S}_2$ diagonal in the uncoupled representation.

I. $\hat{S}_1 \cdot \hat{S}_2 = \hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} + \hat{S}_{1z}\hat{S}_{2z}$

II. $\hat{S}_1 \cdot \hat{S}_2 = \frac{\hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{1+}\hat{S}_{2-}}{2} + \hat{S}_{1z}\hat{S}_{2z}$

III. $\hat{S}_1 \cdot \hat{S}_2 = \frac{\hat{S}^2 - \hat{S}_1^2 - \hat{S}_2^2}{2}$

a) (I) only b) (II) only c) (III) only d) Both (I) and (II)
e) None of the above

12

Consider the product space of two spin-1/2 systems. The raising and lowering operators for each spin, e.g., for the first spin are given as $\hat{S}_{1+} = \hat{S}_{1x} + i\hat{S}_{1y}$ and $\hat{S}_{1-} = \hat{S}_{1x} - i\hat{S}_{1y}$. Choose all of the following expressions for $\hat{S}_1 \cdot \hat{S}_2$ that will make $\hat{H} = C\hat{S}_1 \cdot \hat{S}_2$ diagonal in the coupled representation.

I. $\hat{S}_1 \cdot \hat{S}_2 = \hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} + \hat{S}_{1z}\hat{S}_{2z}$

II. $\hat{S}_1 \cdot \hat{S}_2 = \frac{\hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{1+}\hat{S}_{2-}}{2} + \hat{S}_{1z}\hat{S}_{2z}$

III. $\hat{S}_1 \cdot \hat{S}_2 = \frac{\hat{S}^2 - \hat{S}_1^2 - \hat{S}_2^2}{2}$

a) (I) only b) (II) only c) (III) only d) (I) and (II) only
(e) None of the above.

13

Define the raising and lowering operators for a single spin-1/2 system as $\hat{S}_+ = \hat{S}_x + i\hat{S}_y$ and $\hat{S}_- = \hat{S}_x - i\hat{S}_y$. Which one of the following gives the correct values for $\hat{S}_x$ and $\hat{S}_y$?

a) $\hat{S}_x = (\hat{S}_+ + \hat{S}_-)/2$, $\hat{S}_y = (\hat{S}_+ - \hat{S}_-)/2\mathrm{i}$
b) $\hat{S}_x = (\hat{S}_+ - \hat{S}_-)/2$, $\hat{S}_y = (\hat{S}_+ - \hat{S}_-)/2\mathrm{i}$
c) $\hat{S}_x = (\hat{S}_+ + \hat{S}_-)/2$, $\hat{S}_y = (\hat{S}_+ + \hat{S}_-)/2\mathrm{i}$
d) $\hat{S}_x = (\hat{S}_+ - \hat{S}_-)/2$, $\hat{S}_y = (\hat{S}_+ + \hat{S}_-)/2\mathrm{i}$
e) None of the above

14

Consider the product space of two spin-1/2 systems. Which one of the following is correct?

a) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\uparrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\uparrow\rangle_2) = 0$
b) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\uparrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\uparrow\rangle_2) = \hbar^2|\uparrow\rangle_1|\uparrow\rangle_2$
c) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\uparrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\uparrow\rangle_2) = 2\hbar^2|\uparrow\rangle_1|\uparrow\rangle_2$
d) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\uparrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\uparrow\rangle_2) = \hbar^2|\downarrow\rangle_1|\uparrow\rangle_2$
e) None of the above

15

Consider the product space of two spin-1/2 systems. Which one of the following is correct?

a) $\hat{S}_{1-}\hat{S}_{2+}|\downarrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\downarrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = 0$
b) $\hat{S}_{1-}\hat{S}_{2+}|\downarrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\downarrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = \hbar^2|\downarrow\rangle_1|\downarrow\rangle_2$
c) $\hat{S}_{1-}\hat{S}_{2+}|\downarrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\downarrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = \hbar^2|\downarrow\rangle_1|\uparrow\rangle_2$
d) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\uparrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\uparrow\rangle_2) = 2\hbar^2|\downarrow\rangle_1|\downarrow\rangle_2$
e) None of the above

16

Consider the product space of two spin-1/2 systems. Which one of the following is correct?

a) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = 0$
b) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = \hbar^2|\downarrow\rangle_1|\uparrow\rangle_2$
c) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = 2\hbar^2|\downarrow\rangle_1|\uparrow\rangle_2$
d) $\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\hat{S}_{1-}|\uparrow\rangle_1)(\hat{S}_{2+}|\downarrow\rangle_2) = 2\hbar^2|\uparrow\rangle_1|\downarrow\rangle_2$
e) None of the above

17

Consider the product space of two spin-1/2 systems. Which one of the following scalar products is correct?

a) $\langle\downarrow|_1\langle\uparrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\langle\downarrow|_1\hat{S}_{1-}|\uparrow\rangle_1)(\langle\uparrow|_2\hat{S}_{2+}|\downarrow\rangle_2) = 0$
b) $\langle\downarrow|_1\langle\uparrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\langle\downarrow|_1\hat{S}_{1-}|\uparrow\rangle_1)(\langle\uparrow|_2\hat{S}_{2+}|\downarrow\rangle_2) = \hbar^2|\downarrow\rangle_1|\uparrow\rangle_2$
c) $\langle\downarrow|_1\langle\uparrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\langle\downarrow|_1\hat{S}_{1-}|\uparrow\rangle_1)(\langle\uparrow|_2\hat{S}_{2+}|\downarrow\rangle_2) = \hbar^2$
d) $\langle\downarrow|_1\langle\uparrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = (\langle\downarrow|_1\hat{S}_{1-}|\uparrow\rangle_1)(\langle\uparrow|_2\hat{S}_{2+}|\downarrow\rangle_2) = 2\hbar^2$
e) None of the above.

18

Consider the product space of two spin-1/2 systems. Which one of the following scalar products is correct?

a) $\langle\uparrow|_1\langle\downarrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = 0$
b) $\langle\uparrow|_1\langle\downarrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = \hbar^2$
c) $\langle\uparrow|_1\langle\downarrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = \hbar^2|\uparrow\rangle_1|\downarrow\rangle_2$
d) $\langle\uparrow|_1\langle\downarrow|_2\hat{S}_{1-}\hat{S}_{2+}|\uparrow\rangle_1|\downarrow\rangle_2 = 2\hbar^2$
e) None of the above.

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