PHY 342 HW Ch.6b

q6.4
The virial theorem for the Coulomb potential is $\langle T\rangle = -\frac{1}{2}\langle V\rangle$. Use this result to show that

$$\langle\psi_{nlm}|\frac{1}{r}|\psi_{nlm}\rangle = \frac{1}{n^2 a}.$$

q6.5
Combine Eqs. (6.57) and (6.65) to obtain Eq. (6.66).

q6.6
Consider a linear perturbation $H' = \alpha x$ in the SHO. Show that the first-order energy correction E_n^1 is zero for every state, and explain briefly. [Bonus] Solve the perturbed SHO exactly by combining the linear and quadratic potentials, e.g. by variable substitution $x' = x + a/m\omega^2$ to confirm the exact energy contains no linear term in a.

q6.7
Consider the strong Zeeman splitting of $n = 2$ levels of hydrogen. Find the energy of each split level in terms of $\mu_B B_0$ and sketch them. How many distinct levels are there?

q6.8
Suppose the eigenstates $|a\rangle$,$|b\rangle$,$|c\rangle$ are 3-fold degenerate and a perturbation H' acts on this system. Indicate true or false for each statement below if the states $|a\rangle$,$|b\rangle$,$|c\rangle$ are a "good" basis for the perturbed system:
(1) $H'_{aa} = H'_{bb} = H'_{cc}$;
(2) $H'_{ab} = H'_{bc} = H'_{ca} = 0$;
(3) $|a\rangle$,$|b\rangle$,$|c\rangle$ are orthogonal to each other.

q6.9
Consider the Hamiltonian $H^0 + H' = E_0 \begin{bmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{bmatrix}$, where $\epsilon \ll 1$.
The basis states for the matrix in order $|a\rangle$,$|b\rangle$,$|c\rangle$ are eigenstates of the unperturbed $H^0(\epsilon = 0)$. Indicate true or false for each statement below:
(1) The eigenenergies of H^0 are E_0, E_0, and $2E_0$;
(2) $|a\rangle$ is a good state for H';
(3) $|c\rangle$ is a good state for H';
(4) The perturbation matrix in the degenerate subspace is $H' = E_0 \begin{bmatrix} -\epsilon & 0 \\ 0 & 0 \end{bmatrix}$.