QM2 Concept Test 10.1

Consider the Hamiltonian $\hat{H}^0 + \hat{H}' = V_0 \begin{pmatrix} 1-\varepsilon & \varepsilon & 0 \\ \varepsilon & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix}$, where $\varepsilon \ll 1$.

The basis vectors for the matrix $|a\rangle$, $|b\rangle$, and $|c\rangle$ are the energy eigenstates of the unperturbed Hamiltonian $\hat{H}^0$ ($\varepsilon = 0$). Choose all of the following statements that are correct.

1) $|a\rangle$ is a "good" state for the perturbation $\hat{H}'$.
2) $|c\rangle$ is a "good" state for the perturbation $\hat{H}'$.
3) The perturbation matrix in the degenerate subspace of $\hat{H}^0$ is $V_0 \begin{pmatrix} -\varepsilon & \varepsilon \\ \varepsilon & 0 \end{pmatrix}$.

A. 1 only B. 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above

QM2 Concept Test 10.2

Consider the Hamiltonian $\hat{H}^0 + \hat{H}' = V_0 \begin{pmatrix} 1-\varepsilon & \varepsilon & 0 \\ \varepsilon & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix}$, where $\varepsilon \ll$ 1. The basis vectors for the matrix $|a\rangle$, $|b\rangle$, and $|c\rangle$ are the energy eigenstates of the unperturbed Hamiltonian $\hat{H}^0$ $(\varepsilon = 0)$. Choose all of the following statements that are correct.

1) The first order correction to the energy for the state $|a\rangle$ is $-\varepsilon V_0$.
2) The first order correction to the energy for the state $|b\rangle$ is zero.
3) The first order correction to the energy for the state $|c\rangle$ is zero.

A. 1 only B. 3 only C. 1 and 2 only D. 1 and 3 only E. All of the above.

QM2 Concept Test 10.3

A perturbation $\hat{H}'$ acts on a hydrogen atom with the unperturbed Hamiltonian $\hat{H}^0 = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$. To calculate the perturbative corrections, we use $|n, l, m_l, s, m_s\rangle$ (the eigenstates of ($\hat{H}^0$,$\hat{L}^2$, and $\hat{L}_z$) as the basis vectors. Choose all of the following statements that are correct.

1) If $\hat{H}' = \alpha\hat{L}_z$, where α is a suitable constant, we can calculate the first order corrections as $E^1 = \langle n, l, m_l, s, m_s|\hat{H}'|n, l, m_l, s, m_s\rangle$.
2) If $\hat{H}' = \alpha\delta(r)$, the first order correction to energy is $E^1 = \langle n, l, m_l, s, m_s|\hat{H}'|n, l, m_l, s, m_s\rangle$.
3) If $\hat{H}' = \alpha\hat{J}_z$ (z component of $\vec{J} = \vec{L} + \vec{S}$) we can calculate the first order correction as $E^1 = \langle n, l, m_l, s, m_s|\hat{H}'|n, l, m_l, s, m_s\rangle$.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only E. All of the above

QM2 Concept Test 10.4

A perturbation $\hat{H}'$ acts on a hydrogen atom with the unperturbed Hamiltonian $\hat{H}^0 = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$. To calculate the perturbative corrections, we use the coupled representation $\left|n, l, s, j, m_j\right\rangle$ as the basis vectors. Choose all of the following statements that are correct.

1) If $\hat{H}' = \alpha \hat{L}_z$, where α is a suitable constant, we can calculate the first order corrections as $E^1 = \langle n, l, s, j, m_j | \hat{H}' | n, l, s, j, m_j \rangle$.
2) If $\hat{H}' = \alpha\delta(r)$, the first order correction to energy is $E^1 = \langle n, l, s, j, m_j | \hat{H}' | n, l, s, j, m_j \rangle$.
3) If $\hat{H}' = \alpha \hat{J}_z$ (z component of $\vec{J} = \vec{L} + \vec{S}$) we can calculate the first order correction as $E^1 = \langle n, l, s, j, m_j | \hat{H}' | n, l, s, j, m_j \rangle$.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above

QM2 Concept Test 10.5

A perturbation $\hat{H}'$ acts on a hydrogen atom with the unperturbed Hamiltonian $\hat{H}^0 = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$. Choose all of the following statements that are correct.

1) If $\hat{H}' = \alpha(\hat{L}_z + \hat{S}_z)$ we can calculate the first order correction as $E^1 = \langle n,l,s,j,m_j|\hat{H}'|n,l,s,j,m_j\rangle$.
2) If $\hat{H}' = \alpha(\hat{L}_z + \hat{S}_z/2)$ we can calculate the first order correction as $E^1 = \langle n,l,s,j,m_j|\hat{H}'|n,l,s,j,m_j\rangle$.
3) If $\hat{H}' = \alpha(\hat{L}_z + \hat{S}_z/2)$ we can calculate the first order correction as $E^1 = \langle n,l,m_l,s,m_s|\hat{H}'|n,l,m_l,s,m_s\rangle$.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above

QM2 Concept Test 10.6

Suppose the eigenstates $|a\rangle$, $|b\rangle$, and $|c\rangle$ of $\hat{H}^0$ are 3-fold degenerate with energy E^0. A perturbation $\hat{H}'$ acts on this system and the perturbation matrix is $\hat{H}' = V\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ if $|a\rangle$, $|b\rangle$, and $|c\rangle$ are used as basis vectors. Note: The matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ has eigenvectors $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ with eigenvalues 2 and 0, respectively. Choose all of the following statements that are correct.

1) The perturbation $\hat{H}'$ splits the original energy level E^0 into three distinct energy levels because the eigenvalues of the matrix are 0, 1, and 2.
2) $|a\rangle$ is a “good” state for this system.
3) $\frac{|b\rangle+|c\rangle}{\sqrt{2}}$ and $\frac{|b\rangle-|c\rangle}{\sqrt{2}}$ are “good” states for this system.

A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only E. All of the above.

QM2 Concept Test 10.7

Suppose the unperturbed Hamiltonian $\hat{H}^0$ is two-fold degenerate, i.e., $\hat{H}^0\psi_a^{\,0} = E_1^{\,0}\psi_a^{\,0}$, $\hat{H}^0\psi_b^{\,0} = E_1^{\,0}\psi_b^{\,0}$, $\langle\psi_a^{\,0}|\psi_b^{\,0}\rangle = 0$. A perturbation $\hat{H}'$ acts on this system and a Hermitian operator $\hat{A}$ commutes with both $\hat{H}^0$ and $\hat{H}'$. Choose all of the following statements that are correct.

1) $\hat{H}^0$ and $\hat{H}'$ must commute with each other.
2) If $\psi_a^{\,0}$ and $\psi_b^{\,0}$ are degenerate eigenstates of $\hat{A}$, they must be "good" states for finding perturbative corrections to the energy and wavefunction due to $\hat{H}'$.
3) If $\psi_a^{\,0}$ and $\psi_b^{\,0}$ are non-degenerate eigenstates of $\hat{A}$, they must be "good" states.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 1 and 3 only