QM2 Concept Test 11.1

In a 3D Hilbert space, $\widehat{H}^0$ is the unperturbed Hamiltonian. $|a\rangle$ and $|b\rangle$ are the 2-fold degenerate energy eigenstates with energy E_1 and $|c\rangle$ is the energy eigenstate with energy E_2. If $|a\rangle$ and $|b\rangle$ are **not** "good" states for the perturbation $\widehat{H}'$, choose all of the following statements that must be correct.

1) $H'_{ac} = H'_{bc} \neq 0$, where $\langle i|\widehat{H}'|j\rangle = H'_{ij}$.

2) $H'_{ab} = (H'_{ab})^* \neq 0$

3) If $\widehat{H}'$ does not commute with $\widehat{H}^0$, we can never find a proper set of coefficients $\alpha_1, \beta_1, \alpha_2, \beta_2$ to diagonalize both $\widehat{H}^0$ and $\widehat{H}'$ completely using the basis vectors $\alpha_1|a\rangle + \beta_1|b\rangle$, $\alpha_2|a\rangle + \beta_2|b\rangle$, and $|c\rangle$.

A. 2 only B. 3 only C. 1 and 2 only D. 2 and 3 only E. all of the above

QM2 Concept Test 11.2

Choose all of the following statements that are correct.

1) If $\hat{H}^0$ and $\hat{H}'$ commute with each other , we can always find a basis to diagonalize the matrices for both $\hat{H}^0$ and $\hat{H}'$ simultaneously.
2) If $\hat{H}^0$ and H' do not commute with each other , we cannot find a basis to diagonalize the matrices for both $\hat{H}^0$ and $\hat{H}'$simultaneously.
3) In perturbation theory, $\hat{H}^0$ is chosen to be a diagonal matrix and the basis vectors are chosen as the orthonormal eigenstates of $\hat{H}^0$.

A. 1 only B. 3 only C. 1 and 2 only D. 1 and 3 only E. All of the above.

QM2 Concept Test 11.3

Suppose $\hat{H}^0$ and $\hat{H}'$ commute with each other. Choose all of the following statements that are correct.

1) If $\hat{H}^0$ is diagonal in a given basis and there is no degeneracy in the eigenvalue spectrum of $\hat{H}^0$ and $\hat{H}'$, then $\hat{H}'$must be diagonal in that basis.
2) If $\hat{H}^0$ is diagonal in a given basis and there is a degeneracy in the eigenvalue spectrum of $\hat{H}^0$, then $\hat{H}'$must be diagonal in that basis.
3) We can always find a special basis in which both $\hat{H}^0$ and $\hat{H}'$ are diagonal simultaneously.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. All of the above

QM2 Concept Test 11.4

Choose all of the following statements that are correct about the spin-orbit coupling term $\hat{H}'_{SO} = \left(\frac{e^2}{8\pi\varepsilon_0}\right)\frac{1}{m^2c^2r^3}\vec{S}\cdot\vec{L}$ in the Hamiltonian of the hydrogen atom (including the fine structure correction).

1) $\hat{H}'_{SO}$ commutes with $\hat{L}_z$.
2) $\hat{H}'_{SO}$ commutes with $\hat{J}_z = \hat{L}_z + \hat{S}_z$
3) $\hat{H}'_{SO}$ commutes with $\hat{L}^2$.

A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only E. All of the above

QM2 Concept Test 11.5

The fine structure correction for the hydrogen atom is $E_{fs} = E_{SO} + E_r = \frac{(E_n)^2}{2mc^2}\left(3 - \frac{4n}{j+{}^1/_2}\right)$, where $j = l + s, l + s - 1, \ldots |l - s|$ is the quantum number corresponding to the total angular momentum and $n = 1,2,3 \ldots$. Choose all of the following statements that are correct including fine structure.

1) E_{fs} is always negative for any possible value of n and j.
2) When $n = 2$, there are 2 distinct values of j.
3) There is no degeneracy left for the energy level with $n = 3, j = 3/2$ after we account for fine structure correction.

A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only E. All of the above.

QM2 Concept Test 11.6

Without considering the fine structure, the energy for a hydrogen atom is $E_n = \frac{-13.6\ eV}{n^2}$. Choose all of the following statements that are correct.

1) Ignoring spin, the energy level *n*=2 is four-fold degenerate corresponding to ($l = 1, m_l = 1$), ($l = 1, m_l = 0$), ($l = 1, m_l = -1$), and ($l = 0, m_l = 0$).
2) Including the fine structure, when the electron is in the state ($n = 2, j = 1/2$), we will definitely obtain zero if we measure the square of the magnitude of the angular momentum $\hat{L}^2$.
3) Including the fine structure correction, when the electron is in the state ($n = 2, j = 3/2$), we will definitely obtain $2\hbar^2$ if we measure the square of the magnitude of the angular momentum $\hat{L}^2$.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above

QM2 Concept Test 11.7

For hydrogen atom, the Zeeman term in the perturbation is given by $H'_Z = \frac{e}{2m}(\vec{L} + 2\vec{S}) \cdot \vec{B}_{ext}$. Choose all of the following statements that are true about the intermediate field Zeeman effect, where neither the Zeeman term H'_Z nor the fine structure term H'_{fs} dominates.

1) The "good" basis states for the perturbation are the coupled states $\left|n, l, s, j, m_j\right\rangle$.
2) The "good" basis states for the perturbation are the uncoupled states $|n, l, m_l, s, m_s\rangle$.
3) Both the coupled and uncoupled states are equally "good" states for the perturbation.

A. 1 only B. 2 only C. 3 only D. Not enough information
E. None of the above