Note: For the following concept tests about time-dependent perturbation theory,

The general state of a two-state system system at time t is

$$\Psi(t) = c_a(t)e^{\frac{-iE_a t}{\hbar}}\Psi_a + c_b(t)e^{\frac{-iE_b t}{\hbar}}\Psi_b.$$

$H'_{ij} = \langle\Psi_i|\hat{H}'|\Psi_j\rangle$ and if $H'_{aa} = H'_{bb} = 0$, then

$$\frac{d}{dt}c_a(t) = -\frac{i}{\hbar}H'_{ab}e^{-i\omega_0 t}c_b(t), \text{ and}$$

$$\frac{d}{dt}c_b(t) = -\frac{i}{\hbar}H'_{ba}e^{i\omega_0 t}c_a(t), \text{ where}$$

$$\omega_0 = \frac{E_b - E_a}{\hbar}.$$

QM2 Concept Test 12.1

Suppose an unperturbed two-state system with the Hamiltonian $\hat{H}_0$ has two non-degenerate stationary states Ψ_a and Ψ_b. The state of the system is

$$\Psi(t) = c_a(t)e^{\frac{-iE_a t}{\hbar}}\Psi_a + c_b(t)e^{\frac{-iE_b t}{\hbar}}\Psi_b$$

where $c_a(t=0) = 1, c_b(t=0) = 0$. If a time-dependent perturbation $\hat{H}'(t)$ acts on this system, choose all of the following statements that are necessarily correct.

1) $\left(\hat{H}_0 + \hat{H}'(t)\right)\Psi(t) = i\hbar\frac{\partial\Psi(t)}{\partial t}$
2) $H'_{ji} = H'_{ij}$, $(H'_{ij} = \langle\Psi_i|\hat{H}'|\Psi_j\rangle)$
3) The perturbation can cause a transition from Ψ_a to Ψ_b if $H'_{ab} \neq 0$.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

QM2 concept Test 12.2

Suppose an unperturbed two-state system with the Hamiltonian $\hat{H}_0$ has two non-degenerate stationary states Ψ_a and Ψ_b. The state of the system is

$$\Psi(t) = c_a(t)e^{\frac{-iE_a t}{\hbar}}\Psi_a + c_b(t)e^{\frac{-iE_b t}{\hbar}}\Psi_b$$ where

$c_a(t=0) = 1, c_b(t=0) = 0$. If a time-dependent perturbation $\hat{H}'(t)$ acts on this system and $H'_{aa} = H'_{bb} = 0$, choose all of the following statements that are correct about the coefficients $c_a{}^1(t)$ and $c_b{}^1(t)$ to first order (including zeroth order term + first order correction).

1) $\frac{d}{dt}c_b{}^1(t) = 0$
2) $c_a{}^1(t) = 1$
3) $c_b{}^1(t) = 0$

A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only
E. None of the above.

QM2 Concept Test 12.3

Suppose an unperturbed two-state system with Hamiltonian $\hat{H}_0$ has two non-degenerate stationary states Ψ_a and Ψ_b. The state of the system is $\Psi(t) = c_a(t)e^{\frac{-iE_a t}{\hbar}}\Psi_a + c_b(t)e^{\frac{-iE_b t}{\hbar}}\Psi_b$ where $c_a(t=0) = 1, c_b(t=0) = 0$. Choose all of the following statements that are correct.

1) The coefficients to zeroth order must satisfy $|{c_a}^0(t)|^2 + |{c_b}^0(t)|^2 = 1$.
2) The coefficients to first order must satisfy $|{c_a}^1(t)|^2 + |{c_b}^1(t)|^2 = 1$.
3) The exact coefficients (including corrections to all orders) must satisfy $|c_a(t)|^2 + |c_b(t)|^2 = 1$.

A. 1 only B. 3 only C. 1 and 3 only D. 2 and 3 only

E. All of the above

QM2 Concept Test 12.4

Suppose an unperturbed two-state system with Hamiltonian $\hat{H}_0$ has two non-degenerate stationary states Ψ_a and Ψ_b. The general state of the system at time *t* is $\Psi(t) = c_a(t)e^{\frac{-iE_at}{\hbar}}\Psi_a + c_b(t)e^{\frac{-iE_bt}{\hbar}}\Psi_b$. Suppose the initial state of the system is $\Psi(t=0) = \Psi_b$, i.e., $c_a(t=0) = 0$, $c_b(t=0) = 1$. If a sinusoidal time-dependent perturbation (with driving frequency ω) is applied starting at time $t = 0$, choose all of the following statements that are correct.

1) $c_b{}^1(t) = 1$
2) $c_a{}^1(t) = 0$
3) The transition probability from state Ψ_b to Ψ_a is equal to the transition probability from state Ψ_a to Ψ_b when ω is close to the transition frequency ω_0.

A. 1 only B. 1 and 2 only C. 1 and 3 only D 2 and 3 only
E. All of the above

QM2 Concept Test 12.5

In the sinusoidal time-dependent perturbation of a two-level system, the transition probability from Ψ_a to Ψ_b is $|c_b(t)|^2 = \frac{|V_{ab}|^2}{\hbar^2}\frac{sin^2(\omega_0-\omega)t/2}{(\omega_0-\omega)^2}$ at time t, where $V_{ab} = \langle\Psi_a|V|\Psi_b\rangle$ and the driving frequency ω is close to the transition frequency ω_0. Choose all of the following statements that are correct.

1) If we measure the energy of the system at time $t = \frac{2\pi}{|\omega_0-\omega|}$, we will find the particle in state Ψ_b with 100% probability.
2) If we measure the energy of the system at time $t = \frac{\pi}{|\omega_0-\omega|}$, we will find the particle in state Ψ_b with 100% probability.
3) The longer we wait before measuring the energy of the system, the higher the probability of inducing a transition.

A. 1 only B. 2 only C. 3 only D. 2 and 3 only

E. None of the above

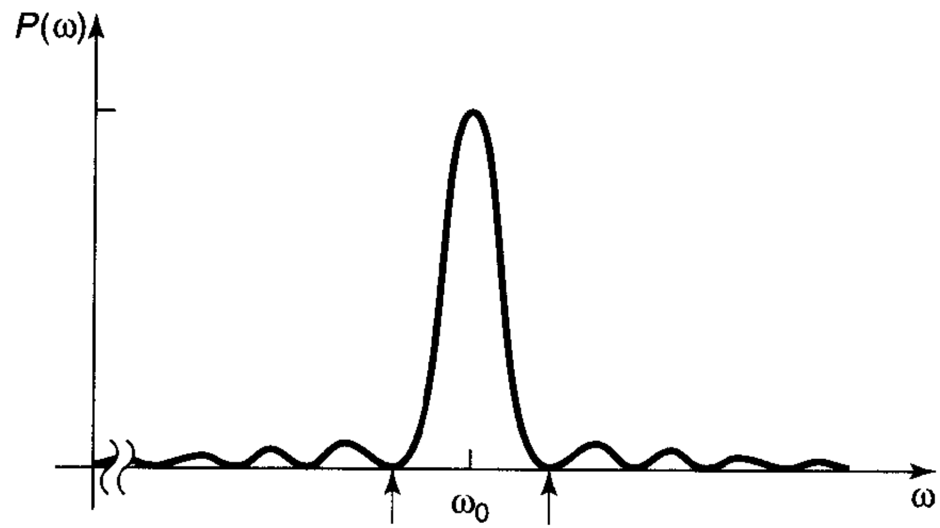
QM2 Concept Test 12.6

In the sinusoidal time-dependent perturbation of a two-level system, the transition probability from Ψ_a to Ψ_b is $|c_b(t)|^2 = \frac{|V_{ab}|^2}{\hbar^2}\frac{sin^2(\omega_0-\omega)t/2}{(\omega_0-\omega)^2}$, when $\omega \approx \omega_0$. $P_{a\to b}(\omega)$ *vs.* ω is plotted below.

Choose all of the following statements that are correct.

1) The transition probability is greatest when the driving frequency ω is close to the transition frequency ω_0.
2) The peak of the transition probability is a time-independent constant.
3) The first zero points (see arrows in the figure below) around the peak of the transition probability are at $\omega = \omega_0 \pm \frac{2\pi}{t}$.

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. All of the above.



QM2 Concept Test 12.7

Choose all of the following statements that are correct about the transition probability $P_{a\to b}(t) = \frac{|V_{ab}|^2}{\hbar^2} \frac{\sin^2\frac{(\omega_0-\omega)t}{2}}{(\omega_0-\omega)^2}$ $(\omega \approx \omega_0)$ for a two-level system if a sinusoidal time-dependent perturbation is applied at time $t = 0$.

1) The transition probability $P_{a\to b} \to \infty$ when time $t \to +\infty$.
2) The transition probability will be one (100% probability of transition) when $t \to +\infty$.
3) We must include the higher order corrections in the transition amplitude $c_b{}^n(t), (n \gg 1)$ when $t \to +\infty$.

A. 1 only B. 2 only C. 3 only D. 2 and 3 only
E. None of the above