QM2 Concept Test 17.1

The ground state wavefunction of the electron in a hydrogen atom is $\Psi_{100}(\vec{r})$. Choose all of the following functions that are reasonable guesses for trial wavefunctions for the electron in a hydrogen molecule ion ${H_2}^+$ for the variational upper bound on ground state energy. $\vec{r}_1$ and $\vec{r}_2$ are the coordinates of the electron from the two protons. A is a normalization constant.

1) $A[\Psi_{100}(\vec{r}_1) + \Psi_{100}(\vec{r}_2)]$
2) $A[\Psi_{100}(\vec{r}_1) - \Psi_{100}(\vec{r}_2)]$
3) $A[\Psi_{100}(\vec{r}_1) \cdot \Psi_{100}(\vec{r}_2)]$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. All of the above.

QM2 Concept Test 17.2

The ground state energy of a hydrogen atom is -13.6eV. Choose all of the following statements that are correct about the hydrogen molecule ion $H_2{}^+$.

1) If $H_2{}^+$ has a bound state, its ground state energy must be lower than that of a neutral hydrogen atom plus a free proton.
2) If $H_2{}^+$ is in a bound state, the electron will have a higher probability of being found between the protons compared to the state in which it is not bound.
3) For any normalized trial wavefunction Ψ, $\langle\Psi|\hat{H}|\Psi\rangle_{min}$ (minimized with respect to variational parameters) must be less than the energy of a neutral hydrogen atom plus a free proton.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above.

QM2 Concept Test 17.3

Sally uses the WKB approximation to evaluate the approximate wavefunction and energy of a particle (in the nth stationary state with energy E_n) interacting with a potential energy well defined by $V(x)$. Choose all of the following statements that are correct about the WKB approximation.

1) It is a semi-classical approximation.
2) It works well when the potential energy changes slowly on the length scale of the wavelength of the particle.
3) It works well for high n stationary states.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. All of the above.

QM2 Concept Test 17.4

A particle in a bound state with energy E is interacting with potential energy well given by $V(x)$. If $V(x)$ is not constant but varies slowly in comparison to the wavelength λ of the particle, choose all of the following statements that are correct.

1) Over a region containing many full wavelengths, the potential energy is essentially constant.
2) The wavefunction $\Psi(x)$ of the particle remains practically sinusoidal in classically allowed regions.
3) Near the classical turning points ($E \approx V$), the wavelength of the particle goes to zero.

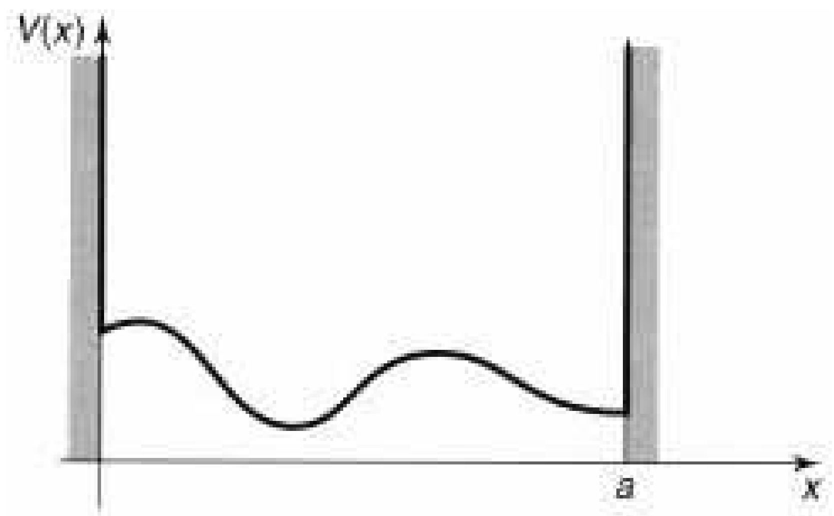
A. 1 only B. 2 only C. 1 and 2 only D. 2 and 3 only E. All of the above

QM2 Concept Test 17.5

A particle interacts with an infinite square well where the potential energy $V(x)$ has a bumpy bottom between $0 < x < a$ as shown in the figure below. $V(x)$ changes slowly compared to the relevant wavelength of the particle. Choose all of the following statements that are correct for large *energy states (n>>1)*.

1) Inside the well, the wavefunction can be approximated as $\Psi(x) \cong \frac{1}{\sqrt{p(x)}}[C_1 \sin\phi(x) + C_2 \cos\phi(x)]$.
2) The phase at $x = a$ is $\phi(a) = n\pi$
3) $\int_0^a p(x)dx = n\pi\hbar$.

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. All of the above.

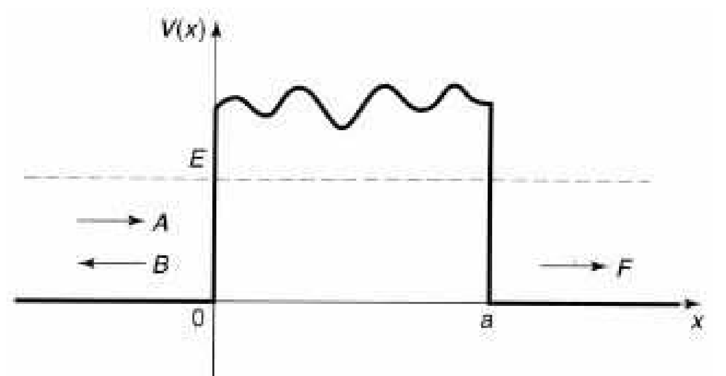


QM2 Concept Test 17.6

A particle interacts with a potential energy barrier where the potential energy $V(x)$ has a bumpy top between $0 < x < a$ as shown in the figure below. $V(x)$ changes slowly compared to the relevant wavelength of the particle. Choose all of the following statements that are correct for the particle with the energy shown.

1) The region between $x = 0$ and $x = a$ is classically forbidden.
2) In the tunneling region, the approximate stationary state is of the form $\Psi(x) \cong \frac{C}{\sqrt{|p(x)|}} e^{\frac{1}{\hbar}\int_0^x |p(x')|dx'} + \frac{D}{\sqrt{|p(x)|}} e^{-\frac{1}{\hbar}\int_0^x |p(x')|dx'}$.
3) If the barrier is very wide and/or high, then in the tunneling region the wavefunction is very close to $\Psi(x) \cong \frac{C}{\sqrt{|p(x)|}} e^{\frac{1}{\hbar}\int_0^x |p(x')|dx'}$.

A. 1 only
B. B. 2 only
C. 1 and 2 only
D. 2 and 3 only
E. All of the above



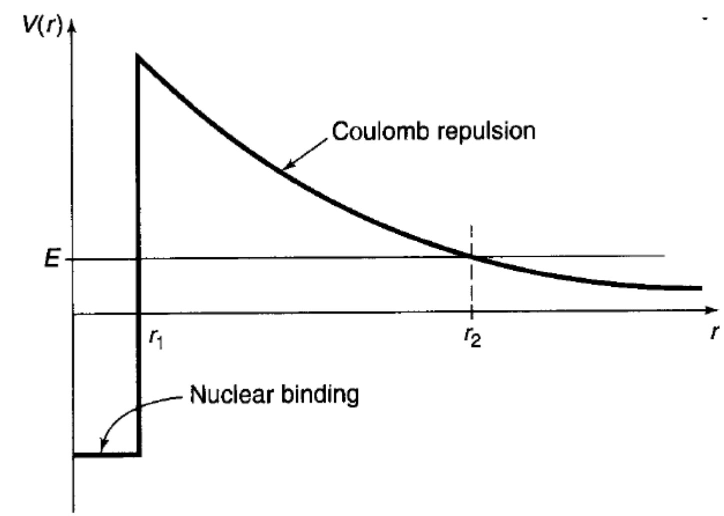
QM2 Concept Test 17.7

Using the WKB approximation, the probability of a particle tunneling through a wide potential energy barrier of width a is $T \cong e^{-2\gamma}$, where $\gamma = \frac{1}{\hbar}\int_0^a |p(x)|dx = \frac{1}{\hbar}\int_0^a \left|\sqrt{2m[E - V(x)]}\right|dx$. In alpha decay of uranium, assume that the alpha particle with energy E interacts with a potential energy curve as shown below. If the average speed of the alpha particle is v, choose all of the following statements that are correct.

1) The average time between the "collisions" of the alpha particle with the "wall" is $\frac{2r_1}{v}$.

2) The probability of the escape of an alpha particle at each collision is $e^{-2\gamma}$.

1) The lifetime of the uranium nucleus is $\tau = \frac{2r_1}{v} e^{2\gamma}$.

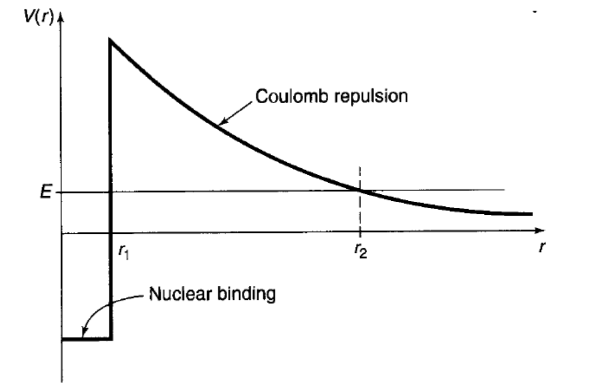


A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. All of the above.

QM2 Concept Test 17.8

Using the WKB approximation, the probability of a particle tunneling through a wide potential energy barrier of width a is $T \cong e^{-2\gamma}$, where $\gamma = \frac{1}{\hbar}\int_0^a |p(x)|dx = \frac{1}{\hbar}\int_0^a \left|\sqrt{2m[E - V(x)]}\right|dx$. In alpha decay of uranium, the alpha particle with energy E interacts with a potential energy curve as shown below. The lifetime of the uranium nucleus is $\tau = \frac{2r_1}{v}e^{2\gamma}$. Choose all of the following statements that are correct.

1) As the width and/or height of the Coulomb repulsion barrier increases, tunneling probability T decreases.
2) As the width and/or height of the Coulomb repulsion barrier increases, lifetime of alpha particle τ decreases.
3) The alpha particle is classically forbidden in the region between $r_1 < r < r_2$.



A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. All of the above.

QM2 Concept Test 17.9

Using the WKB approximation, the probability of a particle tunneling through a wide potential energy barrier of width a is $T \cong e^{-2\gamma}$, where $\gamma = \frac{1}{\hbar}\int_0^a |p(x)|dx = \frac{1}{\hbar}\int_0^a \left|\sqrt{2m[E - V(x)]}\right|dx$. The lifetime of the parent nucleus is $\tau = \frac{2r_1}{v}e^{2\gamma}$. Thus, $\ln \tau = 2\gamma = \frac{C_1 Z}{\sqrt{E}} - C_2\sqrt{Z}A^{1/6}$, where Z is the number of protons and A is the number of neutrons plus protons in the leftover parent nucleus. Choose all of the following statements that are correct if we plot $\ln \tau$ vs. $\frac{1}{\sqrt{E}}$.

1) We obtain a linear graph with a positive slope.
2) The slope of the graph of Thorium (90 protons) is greater than the slope of the graph of Uranium (92 protons).
3) The negative intercept (on $\ln \tau$ axis) of the graph of Thorium (90 protons) is greater in magnitude than that for Uranium (92 protons).

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. All of the above.

QM2 Concept Test 17.10

Using the WKB approximation, the probability of a particle tunneling through a wide potential energy barrier of width a is $T \cong e^{-2\gamma}$, where $\gamma = \frac{1}{\hbar}\int_0^a |p(x)|dx = \frac{1}{\hbar}\int_0^a \left|\sqrt{2m[E - V(x)]}\right| dx$. The lifetime of the parent nucleus is $\tau = \frac{2r_1}{v} e^{2\gamma}$. Thus, $\ln \tau = 2\gamma = \frac{C_1 Z}{\sqrt{E}} - C_2\sqrt{Z}A^{1/6}$, where Z is the number of protons and A is the number of neutrons plus protons in the leftover parent nucleus. Choose all of the following statements that are correct.

1) For a given element, as the energy of the alpha particle increases,life time τ of the parent nucleus decreases.
2) For a given element, as the energy of the alpha particle increases, the probability of the alpha particle tunneling through the potential barrier increases.
3) At a given energy, as the number of protons in the parent nucleus increases, the probability of the alpha particle tunneling through the potential barrier increases.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. All of the above.