Concept test 15.1

Suppose at time $t = 0$, the initial wavefunction of a particle in a 1D infinite square well is $\Psi(x) = \frac{1}{\sqrt{3}}\Psi_1(x) + \sqrt{\frac{2}{3}}\Psi_2(x)$, where $\Psi_1(x)$ and $\Psi_2(x)$ are the ground state and first excited state wavefunctions. If you measure the energy at time t=0, which one of the following statements is correct?

A. The wavefunction will become either $\Psi_1(x)$ or $\Psi_2(x)$ immediately after the measurement and the system will remain in that energy eigenstate at future times.

B. The wavefunction will become either $\Psi_1(x)$ or $\Psi_2(x)$ immediately after the measurement and the system will go back to $\Psi(x) = \frac{1}{\sqrt{3}}\Psi_1(x) + \sqrt{\frac{2}{3}}\Psi_2(x)$ after a long time.

C. The wavefunction will become either $\Psi_1(x)$ or $\Psi_2(x)$ immediately after the measurement and the system will evolve into a state which is superposition of all the stationary states after a long time.

D. The wavefunction $\Psi(x) = \frac{1}{\sqrt{3}}\Psi_1(x) + \sqrt{\frac{2}{3}}\Psi_2(x)$ will become a delta function immediately after the energy measurement and the system will stay in that state for all future times.

E. The wavefunction will become a delta function immediately after the energy measurement and go back to $\Psi(x) = \frac{1}{\sqrt{3}}\Psi_1(x) + \sqrt{\frac{2}{3}}\Psi_2(x)$ after a long time.

1

Concept test 15.2

Suppose at time $t = 0$, the initial wavefunction of a particle in a 1D infinite square well is $\Psi(x) = \frac{1}{\sqrt{3}}\Psi_1(x) + \sqrt{\frac{2}{3}}\Psi_2(x)$, where $\Psi_1(x)$ and $\Psi_2(x)$ are the ground state and first excited state wavefunctions. Choose all of the following statements that are correct about the expectation value of the energy of the system at time t=0.

1. $\langle E\rangle = \frac{1}{\sqrt{3}}E_1 + \sqrt{\frac{2}{3}}E_2$
2. $\langle E\rangle = \frac{1}{3}E_1 + \frac{2}{3}E_2$
3. $\langle E\rangle = \int_{-\infty}^{\infty}\Psi^*(x)\hat{H}\Psi(x)dx$

A. 1 only B. 2 only C. 3 only D. 1 and 3 only E. 2 and 3 only

2

Concept test 15.3

Suppose $\Psi_1(x)$ and $\Psi_2(x)$ are the ground state and first excited state wavefunctions for a particle in a 1D infinite square well of width a. Choose all of the following functions that represent the same state as $\Psi(x) = \frac{1}{\sqrt{2}}\Psi_1(x) + \frac{1}{\sqrt{2}}\Psi_2(x)$.

1. $\Psi(x) = \frac{1}{\sqrt{2}}\Psi_1(x) - \frac{1}{\sqrt{2}}\Psi_2(x)$
2. $\Psi(x) = \frac{1}{\sqrt{2}}\Psi_1(x) + \frac{i}{\sqrt{2}}\Psi_2(x)$
3. $\Psi(x) = \frac{i}{\sqrt{2}}\Psi_1(x) + \frac{i}{\sqrt{2}}\Psi_2(x)$

A. 1 only B. 2 only C. 3 only D. 1 and 3 only E. none of the above

3

Concept Test 15.4

At time t=*0*, the wavefunction for a particle in an infinite square well of width a ($0 \le x \le a$) is $\Psi(x,0) = A\sin^2\left(\frac{\pi x}{a}\right)$ where A is a suitable normalization constant. Which one of the following statements is correct about the wavefunction $\Psi(x,t)$ at time $t > 0$?

A. $\Psi(x,t) = A\sin^2\left(\frac{\pi x}{a}\right)e^{\frac{iEt}{\hbar}}$ where E is the average energy of the system.

B. $\Psi(x,t) = A\sin^2\left(\frac{\pi x}{a}\right)e^{\frac{-iEt}{\hbar}}$ where E is the average energy.

C. $\Psi(x,t) = A\sin^2\left(\frac{\pi x}{a}\right)e^{\frac{-iE_2t}{\hbar}}$, where E_2 is the energy corresponding to the $\Psi_2(x)$ energy eigenstate.

D. $\Psi(x,t) = \sum_n c_n \sin\left(\frac{n\pi x}{a}\right)e^{\frac{-iE_nt}{\hbar}}$ (c_n are suitable expansion coefficients).

E. None of the above

4

Concept Test 15.5

Suppose at time $t = 0$, the initial wavefunction of a particle in a 1D infinite square well is $\Psi(x,0) = Ax(a-x)$ for $0 \le x \le a$, where A is a suitable normalization constant. Which one of the following is the probability density $|\Psi(x,t)|^2$, at time $t > 0$?

A. $|\Psi(x,t)|^2 = |A|^2 x^2 (a-x)^2 \cos^2\left(\frac{Et}{\hbar}\right)$, where E is the expectation value of energy.
B. $|\Psi(x,t)|^2 = |A|^2 x^2 (a-x)^2 e^{\frac{-2iEt}{\hbar}}$, where E is the expectation value of energy.
C. $|\Psi(x,t)|^2 = |A|^2 x^2 (a-x)^2 \sin^2\left(\frac{Et}{\hbar}\right)$, where E is the expectation value of energy.
D. $|\Psi(x,t)|^2 = |A|^2 x^2 (a-x)^2$, which is time-independent.
E. None of the above.

5

Concept Test 15.6

At time $t = 0$, the initial state of a particle in an infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$, where $\Psi_1(x)$ and $\Psi_2(x)$ are the ground state and first excited state wavefunctions. If we measure the energy of the particle after time t, what results can we obtain?

A. $\frac{E_1+E_2}{2}$

B. E_1 or E_2

C. $\frac{E_1 e^{\frac{-iE_1 t}{\hbar}} + E_2 e^{\frac{-iE_2 t}{\hbar}}}{2}$

D. $E_1 e^{\frac{-iE_1 t}{\hbar}}$ or $E_2 e^{\frac{-iE_2 t}{\hbar}}$

E. Any energy eigenvalue E_n, where $n = 1,2,3 \ldots \infty$

6

Concept Test 15.7

At time $t = 0$, the initial state of a particle in an infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$, where $\Psi_1(x)$ and $\Psi_2(x)$ are the ground state and first excited state wavefunctions. If we measure the energy of the particle after time t, what is the state of the particle right BEFORE the measurement of energy?

A. $\frac{1}{\sqrt{2}}\left(\Psi_1(x) + \Psi_2(x)\right)$

B. $\Psi_1(x)$ or $\Psi_2(x)$

C. $\frac{1}{\sqrt{2}}\left(e^{\frac{-iE_1 t}{\hbar}}\Psi_1(x) + e^{\frac{-iE_2 t}{\hbar}}\Psi_2(x)\right)$

D. $e^{\frac{-iE_1 t}{\hbar}}\Psi_1(x)$ or $e^{\frac{-iE_2 t}{\hbar}}\Psi_2(x)$

E. None of the above

7

Concept Test 15.8

At time $t = 0$, the initial state of a particle in an infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$, where $\Psi_1(x)$ and $\Psi_2(x)$ are the ground state and first excited state wavefunctions. If we measure the energy of the particle after time t, what is the normalized state of the particle right AFTER the measurement of energy?

A. $\frac{1}{\sqrt{2}}\left(\Psi_1(x) + \Psi_2(x)\right)$

B. $\Psi_1(x)$ or $\Psi_2(x)$

C. $\frac{1}{\sqrt{2}}\left(e^{\frac{-iE_1 t}{\hbar}}\Psi_1(x) + e^{\frac{-iE_2 t}{\hbar}}\Psi_2(x)\right)$

D. $\frac{\Psi_1(x)}{\sqrt{2}}$ or $\frac{\Psi_2(x)}{\sqrt{2}}$

E. Any energy eigenfunction $\Psi_n(x)$, where $n = 1,2,3 \ldots \infty$

8

Concept Test 15.9

At time $t = 0$, the initial state of a particle in a 1D infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$. You measure the energy of the particle at time $t = t_0$ and obtain E_1. Then you immediately measure the position of the particle at time $t = t_0$. What is the probability of finding the particle in the region between x_0 and $x_0 + dx$?

1. $|\langle x_0|\Psi_1\rangle|^2$
2. $|\Psi_1(x_0)|^2 dx$
3. $|\langle x_0|\Psi_1\rangle|^2 dx$

A. 1 Only B. 2 only C. 3 only D. 2 and 3 only E. None of the above

9

Concept Test 15.10

At time $t = 0$, the initial state of a particle in a 1D infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$. You measure the energy of the particle at time $t = t_0$. Then you immediately measure the position of the particle at time $t = t_0$. If you don't know the energy measurement outcome, choose all of the following expressions that correctly represent the probability density for finding the particle at the position $x = x_0$.

1. $|\langle x_0|\Psi\rangle|^2$
2. $\frac{1}{2}|\langle x_0|\Psi_1\rangle + \langle x_0|\Psi_2\rangle|^2$
3. $\frac{1}{2}|\langle x_0|\Psi_1\rangle|^2 + \frac{1}{2}|\langle x_0|\Psi_2\rangle|^2$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. None of the above

10

Concept Test 15.11

At time $t = 0$, the initial state of a particle in a 1D infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$. You measure the energy of the particle at time $t = t_0$ and obtain E_1. Then you measure the position of the particle at time $t = t_1$ (not immediately after the measurement of energy). What is the probability of finding the particle in the region between x_0 and $x_0 + dx$?

1. $|\Psi_1(x_0)|^2 dx$
2. $\left|e^{\frac{-iE_1t_1}{\hbar}}\Psi_1(x_0)\right|^2 dx$
3. $\left|\sum_n c_n e^{\frac{-iE_nt_1}{\hbar}}\Psi_n(x_0)\right|^2 dx$, where $c_n = \langle\Psi_n|\Psi\rangle$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. None of the above

11

Concept Test 15.12

At time $t = 0$, the initial state of a particle in a 1D infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$. You measure the position of the particle at time $t = t_0$ and obtain x_0. Immediately after the position measurement, you measure the energy. What possible results could you obtain for the energy measurement?

A. Either E_1 or E_2
B. $\frac{E_1+E_2}{2}$
C. In general, you could obtain one out of an infinite number of energy eigenvalues E_n, where n may equal 1,2,3 ... ∞.
D. You would measure E, which is the expectation value of the energy.
E. None of the above.

12

Concept Test 15.13

At time $t = 0$, the initial state of a particle in a 1D infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$. You measure the position of the particle at time $t = t_0$ and obtain x_0. Immediately after the position measurement, you measure the energy. Choose all of the following statements that are correct.

1. The particle will either be in a state $\Psi_1(x)$ or $\Psi_2(x)$ after the energy measurement.
2. The particle will either be in a state $\Psi_1(x)$ or $\Psi_2(x)$ <u>a long time after</u> the energy measurement.
3. The particle will be in the state $\frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$ <u>a long time after</u> the energy measurement.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. None of the above

13

Concept test 15.14

At time $t = 0$, the initial state of a particle in a 1D infinite square well is $\Psi(x,0) = \frac{1}{\sqrt{2}}(\Psi_1(x) + \Psi_2(x))$. Choose all of the following statements that are correct.

1. If there is <u>no</u> external perturbation, the shape of the probability density for measuring the position of the particle will change with time.
2. If we measure the position of the particle, the probability of finding the particle between x_0 and $x_0 + dx$ will change with time.
3. If we measure the energy of the particle, the probability of obtaining the energy E_1 will change with time.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. all of the above

14

Concept test 15.15

Suppose $\Psi_{x'}(x)$ is an eigenfunction of the position operator $\hat{x}$ with eigenvalue x'. Choose all of the following statements that are correct.

1. $\hat{x}\Psi_{x'}(x) = x'\Psi_{x'}(x)$
2. $\Psi_{x'}(x) = \delta(x - x')$
3. $\hat{H}\Psi_{x'}(x) = E\Psi_{x'}(x)$
4. $\hat{H}\Psi_{x'}(x) = x'\Psi_{x'}(x)$

A. 1 only B. 3 only C. 4 only D. 1 and 2 only
E. 1, 2, and 4 only

15

Concept test 15.16

Choose all of the following statements that are correct about an electron in an eigenstate of the position operator.

1. The energy of the electron is well defined in this state.
2. The momentum of the electron cannot be well defined in this state due to the uncertainty principle.
3. The measurement of position in this state will yield a definite value with 100% certainty.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. All of the above.

16

Concept Test 15.17

A particle is initially in the ground state of an infinite square well between $0 \le x \le a$. At time *t=0*, the width of the well suddenly increases to $0 \le x \le 2a$, so fast that the wavefunction does not have the time to change. Which one of the following statements is correct?

A. The wavefunction of the particle will stay in the ground state of the initial well and the probability density will not change with time even though the well has expanded.
B. The wavefunction of the particle will initially change with time but then go back to the ground state of the initial well an remain in the ground state of the initial well.
C. The wavefunction of the particle will initially evolve to the ground state wavefunction of the new well and then stay in that state.
D. The wavefunction of the particle will evolve to the first excited state of the new well and then stay in that state.
E. The wavefunction of the particle keeps changing with time and will not remain in the stationary state of the old or new well for times $t>0$.

17

QM1 Concept test 15.18

Choose all of the following statements that are correct about a particle interacting with a simple harmonic oscillator potential energy well:

1) When in the ground state, the particle has the highest probability of being found at the ends of the well classically, but it has the highest probability at the center of the well quantum mechanically.
2) The probability of finding the particle at the center of the well is zero for the first excited state.
3) The probability of finding the particle at $x \to \pm\infty$ is zero for all stationary states.

A. 1 only B. 2 only C. 1 and 3 only D. 2 and 3 only
E. all of the above

18

QM1 Concept test 15.19

If the initial wavefunction for a particle interacting with a simple harmonic oscillator (SHO) potential energy well $V = \frac{1}{2}kx^2$ is $\Psi(x, t = 0) = \Psi_2(x)$ (the second excited state of the SHO), choose all of the following expectation values that are zero at time *t=0.*

1) $\langle x \rangle$
2) $\langle p \rangle$
3) $\langle E \rangle$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. all of the above

19

QM1 Concept test 15.20

If the initial wavefunction for a particle interacting with a simple harmonic oscillator (SHO) potential energy well $V = \frac{1}{2}kx^2$ is $\Psi(x, t = 0) = \Psi_2(x)$ (the second excited state of the SHO), choose all of the statements that are correct.

1) The expectation value of the position of this particle depends on time *t*.
2) The expectation value of the momentum of this particle depends on time *t*.
3) The expectation value of the energy of this particle depends on time *t*.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. none of the above

20

QM1 Concept test 15.21

If the initial wavefunction for a particle interacting with a simple harmonic oscillator potential energy well $V = \frac{1}{2}kx^2$ is $\Psi(x, t = 0) = \frac{1}{\sqrt{2}}\Psi_1(x) + \frac{1}{\sqrt{2}}\Psi_2(x)$, choose all of the statements that are correct.

1) The expectation value of the <u>position</u> of this particle depends on time *t*.
2) The expectation value of the <u>momentum</u> of this particle depends on time *t*.
3) The expectation value of the <u>energy</u> of this particle depends on time *t*.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. all of the above

21

Concept Test 15.22

For a particle interacting with a simple harmonic potential energy well with $V = \frac{1}{2}kx^2$, $\Psi_n(x)$ is an energy eigenfunction with corresponding energy eigenvalue E_n. Choose all of the following statements that are correct.

1) $E_{n+1} - E_n \to 0$ as $n \to +\infty$.
2) $\frac{E_{n+1} - E_n}{E_n} \to 0$ as $n \to +\infty$.
3) The probability density $|\Psi_n(x)|^2$ for finding the particle approaches the classical distribution as $n \to +\infty$.

A. 2 only B. 3 only C. 1 and 2 only D. 2 and 3 only
E. all of the above

22

Concept Test 15.23

A particle is initially in the ground state of a simple harmonic oscillator (SHO) potential energy well with $V = \frac{1}{2}kx^2$ and frequency ω_0. If at time *t=0*, the SHO frequency suddenly changes to $2\omega_0$(presumably due to a change in *k*), choose all of the following statements that are correct.

1) The particle will remain in the ground state of the old SHO for future times because it is a stationary state.
2) At *t=0*, the particle stays momentarily in the ground state of the old SHO which is a superposition of the stationary states of the new SHO. Then, the state evolves according to the new H.
3) As time passes, the wave function will evolve into the ground state wave function of the new SHO.

A. 1 only B. 2 only C. 3 only D. 2 and 3
E. None of the above

23

Concept Test 15.24

e^{ikx} is an energy eigenfunction of a free particle. If the wavefunction of a particle is $e^{ikx} + e^{-ikx}$ (ignore the normalization issues), choose all of the following statements that are correct.

1) The particle is still in an energy eigenstate.
2) The expectation value of the momentum of the particle is zero.
3) The expectation value of the energy of the particle is zero.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 1 and 3 only

24

Concept Test 15.25

Choose all of the following statements that are correct about a quantum simple harmonic oscillator and a free particle.

1) Energy is discrete for the simple harmonic oscillator, but continuous for the free particle.
2) A **possible wavefunction** for the simple harmonic oscillator can be normalized, but NO **possible wavefunction** for a free particle can be normalized.
3) A particle interacting with a simple harmonic oscillator potential energy can be found in a stationary state, but a free particle cannot be found in a stationary state.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. 1 and 3 only

25

Concept Test 15.26

Although the scattering state wavefunctions for a free particle are not normalizable, we can build a normalized wave packet $\Psi(x,t=0)=\frac{1}{\sqrt{2\pi}}\int \phi(k)e^{ikx}dk$ at time *t=0*. Choose all of the following statements that are correct.

1) $\phi(k)$ is the Fourier transform of the position space wavefunction.
2) $\phi(k)$ is the momentum space wavefunction for the free particle.
3) $|\phi(k)|^2 dk$ gives the probability of measuring the momentum between $\hbar k$ and $\hbar(k+dk)$.

A. 1 only B. 1 and 2 only C. 2 and 3 only D. 1 and 3 only
E. all of the above

26

Concept Test 15.27

A normalized wave packet for a free particle is $\Psi(x,t=0)=\frac{1}{\sqrt{2\pi}}\int \phi(k)e^{ikx}dk$ at time *t=0*. Choose all of the following equations that correctly represent the wave packet at time *t>0*. E_k is the total energy for wave number *k*.

1) $\Psi(x,t)=\frac{1}{\sqrt{2\pi}}\int \phi(k)e^{ikx-\frac{iE_kt}{\hbar}}dk$

2) $\Psi(x,t)=\frac{1}{\sqrt{2\pi}}\int \phi(k)e^{ikx-\frac{i\hbar k^2t}{2m}}dk$

3) $\Psi(x,t)=\frac{1}{\sqrt{2\pi}}e^{\frac{-iE_kt}{\hbar}}\int \phi(k)e^{ikx}dk$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. All of the above

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