Which one of the following equations correctly represents the uncertainty principle between two operators $\hat{A}$ and $\hat{B}$?

A. ${\sigma_A}^2{\sigma_B}^2 \geq \left(\frac{1}{2}\langle[\hat{A},\hat{B}]\rangle\right)^2$
B. ${\sigma_A}^2{\sigma_B}^2 \geq \left(\frac{1}{2}[\hat{A},\hat{B}]\right)^2$
C. ${\sigma_A}^2{\sigma_B}^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$
D. ${\sigma_A}^2{\sigma_B}^2 \geq \left(\frac{1}{2i}[\hat{A},\hat{B}]\right)^2$
E. None of the above

1

For the wavefunction $\psi(x) = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}} e^{-ax^2}$, choose all of the following statements that are correct. (Hint: The standard form of a Gaussian function peaked about $x = x_0$ with a standard deviation σ is given by $f(x) = Ae^{-(x-x_0)^2/2\sigma^2}$.)

I. As a increases, σ_x for $\psi(x)$ increases and σ_p for $\varphi(p)$ decreases.
II. As a increases, σ_x for $\psi(x)$ decreases and σ_p for $\varphi(p)$ increases.
III. The product of the standard deviations (uncertainties) of $\psi(x)$ and $\varphi(p)$ is the same for all Gaussian functions of the type $\left(\frac{2a}{\pi}\right)^{\frac{1}{4}} e^{-ax^2}$ regardless of the value of a.

A. I only B. II only C. III only D. I and III only E. II and III only

2

A wavefunction which is highly localized in space can be approximated as a Dirac delta function $\delta(x)$. Which of the following is the correct Fourier transform $G(k)$ of a delta function $\delta(x)$ localized at $x = 0$?

A. $\frac{\delta(x)e^{+ikx}}{\sqrt{2\pi}}$
B. $\frac{e^{-ikx}}{\sqrt{2\pi}}$
C. $\frac{e^{-ik}}{\sqrt{2\pi}}$
D. $\frac{1}{\sqrt{2\pi}}$
E. None of the above

3

Which one of the following is correct about the delta function $\delta(x)$? (Hint: perform an inverse Fourier transform of $G(k) = \frac{1}{\sqrt{2\pi}}$.)

A. $\delta(x) = e^{ikx}$
B. $\delta(x) = \int_{-\infty}^{\infty} e^{ikx} dx$
C. $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} dk$
D. $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} dx$
E. None of the above

4

Suppose that at time $t = 0$, the wave packet of a quantum mechanical particle is highly peaked and can be approximately described as a delta function $\delta(x)$. Choose all of the following statements that are correct about the wave function of the particle at time $t = 0$. Ignore normalization issues. Something is well-defined when it has a probability distribution highly peaked about its mean value.

I. The momentum of the particle is well defined.
II. The position of the particle is well defined.
III. A delta function is a linear combination of infinitely many momentum eigenstates, so momentum of the particle is not at all well defined.

A. I only B. II only C. III only D. I and II only E. II and III only

5

Choose all of the following statements that are correct.

I. A particle with wavefunction $\Psi_1(x,0) = \sqrt{\frac{1}{a}}$ for $0 < x < a$ *and zero elsewhere* and another with $\Psi_2(x,0) = \sqrt{\frac{1}{a}}$ for $a < x < 2a$ *and zero elsewhere* have the same uncertainty in momentum at time $t = 0$.

II. A particle with wavefunction $\Psi_1(x,0) = \sqrt{\frac{1}{a}}$ for $0 < x < a$ *and zero elsewhere* and another with $\Psi_2(x,0) = \sqrt{\frac{1}{2a}}$ for $0 < x < 2a$ *and zero elsewhere* have the same uncertainty in momentum at time $t = 0$.

III. The uncertainty in the position of a particle with wavefunction $\Psi(x,0) = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}} e^{-ax^2}$ is $\sigma_x = \pm\sqrt{\frac{1}{2a}}$ at time $t = 0$.

A. 1 only B. I and II C. I and III only D. II and III only
E. all of the above

6

Suppose at time $t = 0$, the momentum space wavefunction $\varphi(p)$ is given as a function of p explicitly. Choose all of the following statements that are correct.

I. It is possible to determine the uncertainty in the position of the particle at time $t = 0$ without knowing the Hamiltonian of the system.
II. It is possible to determine the uncertainty in the position of the particle at time $t > 0$ without knowing the Hamiltonian of the system.
III. At a given time t, if the momentum space wavefunction is given explicitly, then the position space wavefunction can be determined by a Fourier Transform.

A. I only B. II only C. III only D. I and III only E. all of the above

7

For a three-dimensional free particle, choose all of the following pairs of observables which can be measured simultaneously in a given quantum state.

I. x and p_y
II. x and p_x
III. L_x and L^2
IV. S_x and S_y

A. I only B. I and II only C. I and III only D. II, III, and IV
E. All of the above

8

Hermitian operators $\hat{A}$ and $\hat{B}$ are compatible when the commutator $[\hat{A}, \hat{B}] = 0$ and incompatible when $[\hat{A}, \hat{B}] \neq 0$. Choose all of the following statements that are correct.

I. If $\hat{A}$ and $\hat{B}$ are <u>incompatible</u> operators with non-degenerate eigenstates, it is impossible to find a complete set of simultaneous eigenstates for $\hat{A}$ and $\hat{B}$.
II. If $\hat{A}$ and $\hat{B}$ are <u>compatible</u> operators with non-degenerate eigenstates, it is impossible to find a complete set of simultaneous eigenstates for $\hat{A}$ and $\hat{B}$.
III. If $\hat{A}$ and $\hat{B}$ are <u>incompatible</u> operators with non-degenerate eigenstates, it is possible to infer the value of the observable B after the measurement of the observable A returns a particle value for A.

A. I only B. II only C. III only D. I and III only E. II and III only

9

The generalized uncertainty principle for $\hat{S}_x$ and $\hat{S}_y$ is $\sigma_{S_x}{}^2\sigma_{S_y}{}^2 \geq \left(\frac{1}{2i}\langle[\hat{S}_x, \hat{S}_y]\rangle\right)^2$. Choose all of the following statements that are correct.

I. If the initial state of a spin-1/2 particle is $|\uparrow_y\rangle$, we have $\langle[\hat{S}_x, \hat{S}_y]\rangle = \langle\uparrow_y|[\hat{S}_x, \hat{S}_y]|\uparrow_y\rangle = 0$.
II. If the initial state of a spin-1/2 particle is $|\uparrow_y\rangle$, we can measure $\hat{S}_x$ and $\hat{S}_y$ simultaneously with 100% certainty.
III. If the initial state of a spin-1/2 particle is $|\uparrow_x\rangle$, we can measure $\hat{S}_x$ and $\hat{S}_y$ simultaneously with 100% certainty.

A. I only B. I and II only C. I and III only D. II and III only
E. All of the above

10

In a finite-dimensional vector space, two <u>compatible</u> operators $\hat{A}$ and $\hat{B}$ corresponding to physical observables are such that the eigenvalue spectrum of each has no degeneracy. The eigenvalue equation for $\hat{B}$ with eigenvalue β_i is $\hat{B}|\beta_i\rangle = \beta_i|\beta_i\rangle$ and the eigenvalue equation for $\hat{A}$ with eigenvalue α_i is $\hat{A}|\alpha_i\rangle = \alpha_i|\alpha_i\rangle$. Choose all of the following statements that are correct about measurements in a generic state $|\Psi\rangle$.

I. If you measure observable A first and then measure observable B, the joint probability (w.r.t. state $|\Psi\rangle$) of obtaining a particular eigenvalue β_i is $|\langle\beta_i|\Psi\rangle|^2$.
II. If you measure observable A first and then measure observable B, the joint probability (w.r.t. state $|\Psi\rangle$) of obtaining a particular eigenvalue β_i depends on the eigenstate of observable A that the wave function $|\Psi\rangle$ collapses to after the measurement of observable A.
III. If you measure observable B without measuring A first, the probability of obtaining a particular eigenvalue β_i is $|\langle\beta_i|\Psi\rangle|^2$.

A. I only B. II only C. III only D. I and III only E. II and III only

11

Suppose the orbital angular momentum state of an electron in a hydrogen atom is $|\Psi\rangle = \frac{1}{\sqrt{3}}(|0\ 0\rangle + |1\ 0\rangle + |1\ 1\rangle)$ in which the first quantum number in each $|l\ m\rangle$ for this superposition state refers to the total orbital angular momentum, L^2, and the second quantum number refers to quantum number for the z component of orbital angular momentum, L_z. Choose all of the following statements that are correct regarding the measurement of L^2 in state $|\Psi\rangle$:

I. If you measure L^2, the probability of obtaining 0 is 1/3.
II. If you measure L^2, the probability of obtaining $2\hbar^2$ is 2/3.
III. If you measure L^2 after you measure L_z, the joint probability (w.r.t. state $|\Psi\rangle$) of obtaining one of the eigenvalues of L^2 is different than if you had measured L^2 directly in state $|\Psi\rangle$.

A. I only B. I and II only C. I and III only D. II and III only
E. all of the above

12

Definition of joint probability:

In state $|\Psi\rangle$, when observable A is measured first, the joint probability for measuring β_j for observable B immediately following the measurement of A is defined as

$$P_{\beta_j}^{joint} = \sum_{i=1}^{N} \left|\langle\beta_j|\alpha_i\rangle\right|^2 |\langle\alpha_i|\Psi\rangle|^2$$

13

Suppose the orbital angular momentum state of an electron in a hydrogen atom is $|\Psi\rangle = \frac{1}{\sqrt{3}}(|0\ 0\rangle + |1\ 0\rangle + |1\ 1\rangle)$ in which the first quantum number in each $|l\ m\rangle$ for this superposition state refers to the quantum number for total orbital angular momentum, L^2, and the second quantum number refers to quantum number for the z component of orbital angular momentum, L_z. Choose all of the following statements that are correct if you first measure L_z and then measure L^2 in state $|\Psi\rangle$.

I. The probability of obtaining $\hbar$ for L_z is 1/3.
II. If you obtain the value 0 for L_z, the state collapses to $\frac{1}{\sqrt{2}}(|0\ 0\rangle + |1\ 0\rangle)$ and measurement of L^2 following L_z in this case will either yield $2\hbar^2$ with probability 1/3 or 0 with probability 1/3.
III. If you first measure L_z and *then* measure L^2, the joint probability (w.r.t. state $|\Psi\rangle$) of obtaining 0 for L^2 is 1/3 and the joint probability (w.r.t. the state $|\Psi\rangle$) of obtaining $2\hbar^2$ for L^2 is 2/3 (the same probabilities as if you had measured L^2 directly in the state $|\Psi\rangle$ without measuring L_z).

A. I only B. I and II only C. I and III only D. II and III only
E. all of the above

14

In a finite-dimensional vector space, two incompatible operators $\hat{A}$ and $\hat{B}$ corresponding to physical observables are such that the eigenvalue spectrum of each has no degeneracy. The eigenvalue equation for $\hat{B}$ with eigenvalue β_i is $\hat{B}|\beta_i\rangle = \beta_i|\beta_i\rangle$ and the eigenvalue equation for $\hat{A}$ with eigenvalue α_i is $\hat{A}|\alpha_i\rangle = \alpha_i|\alpha_i\rangle$. Choose all of the following statements that are correct about measurements in a generic state $|\Psi\rangle$.

I. If you measure observable A first and then measure observable B, the joint probability (w.r.t. state $|\Psi\rangle$) of obtaining a particular eigenvalue β_i is $|\langle\beta_i|\Psi\rangle|^2$.
II. If you measure observable A first and then measure observable B, the joint probability (w.r.t. state $|\Psi\rangle$) of obtaining a particular eigenvalue β_i depends on the eigenstate of observable A that the wave function $|\Psi\rangle$ collapses to after the measurement of observable A.
III. If you measure observable B without measuring A first, the probability of obtaining a particular eigenvalue β_i is $|\langle\beta_i|\Psi\rangle|^2$.

A. I only B. II only C. III only D. I and III only E. II and III only

15