QM1 Concept test 2.1

Suppose Q is an observable and its corresponding operator in the relevant Hilbert space is $\hat{Q}$. Choose all of the following statements that are correct.

(1) The expectation value of $\hat{Q}$ must be real.
(2) $\hat{Q}$ must be a hermitian operator.
(3) $\langle f|\hat{Q}g\rangle = \langle \hat{Q}f|g\rangle$ for all states $|f\rangle$ and $|g\rangle$ in the Hilbert space.

A. 1 only B. 2 only C. 3 only D. 2 and 3 only
E. all of the above

1

QM1 Concept test 2.2

A hermitian operator $\hat{Q}$ has a discrete spectrum. Choose all of the following statements that are correct.

(1) The eigenvalues of $\hat{Q}$ must be real.
(2) The eigenvectors belonging to different eigenvalues are orthogonal.
(3) If two eigenvectors have the same eigenvalue, they must be orthogonal.

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. all of the above

2

QM1 Concept test 2.3

Suppose $|\psi_i\rangle$ $(i = 1, 2, 3, \ldots N)$ form a complete set of orthonormal eigenstates of an operator $\hat{Q}$ with eigenvalues q_i. Choose all of the following statements that are correct.

(1) $\sum_i |\psi_i\rangle\langle\psi_i| = \hat{I}$
(2) $\sum_i q_i |\psi_i\rangle\langle\psi_i| = \hat{I}$
(3) $|\psi_i\rangle\langle\psi_i|$ is a hermitian operator that projects any state of the system along $|\psi_i\rangle$.

A. 1 only B. 2 only C. 3 only D. 1 and 3 only E. 2 and 3 only

3

QM1 Concept test 2.4

Suppose $|\psi_i\rangle$ $(i = 1, 2, 3, \ldots N)$ form a complete set of orthonormal eigenstates of an operator $\hat{Q}$ with eigenvalues q_i. The projection operator is $\hat{P}_i = |\psi_i\rangle\langle\psi_i|$. Choose all of the following statements that are correct.

(1) $\hat{P}_i|\psi_j\rangle = \delta_{ij}|\psi_j\rangle$
(2) The eigenvalues of $\hat{P}_i$ are 0 and 1.
(3) In a three dimensional Hilbert space (N=3), the eigenvalues of $\hat{P}_i$ are non-degenerate.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. all of the above

4

QM1 Concept test 2.5

Suppose $|\psi_i\rangle$ $(i = 1, 2, 3, \ldots N)$ form a complete set of orthonormal eigenstates of an operator $\hat{Q}$ with eigenvalues q_i and the projection operator is $\hat{P}_i = |\psi_i\rangle\langle\psi_i|$. Choose all of the following statements that are correct.

(I) $\langle\Psi|\hat{P}_i|\Psi\rangle$ is the probability of measuring q_i when the system is in a generic quantum state $|\Psi\rangle$
(II) $\hat{P}_i|\Psi\rangle = c_i|\psi_i\rangle$, where c_i is the component of $|\Psi\rangle$ along the basis vector $|\psi_i\rangle$.
(III) $\langle\Psi|\hat{Q}|\Psi\rangle = \sum_i q_i |\langle\psi_i|\Psi\rangle|^2$

A. (I) only B. (II) only C. (III) only D. (I) and (III) only
E. all of the above

5

QM1 Concept test 2.6

A quantum state in the Hilbert space is $|\psi\rangle$. Choose all of the following statements that are correct about the position space and momentum space wavefunctions for this quantum state.

(1) The position space wavefunction is $\psi(x) = \langle x|\psi\rangle$ where x is a continuous index.
(2) The momentum space wavefunction is $\psi(p) = \langle p|\psi\rangle$ where p is a continuous index.
(3) The momentum space wavefunction is $\psi(p) = \int -i\hbar\frac{\partial}{\partial x}\psi(x)dx$

A. 1 only B. 2 only C. 3 only D. 1 and 2 only
E. all of the above

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