

PHY 341 HW Ch.2d

Do problems 2.29, 2.34 (a,b), 2.39*; plus the following (* = optional bonus):

q2-13 The momentum wave function for a square wave is

$$\phi(k) = \sqrt{\frac{a}{\pi}} \frac{\sin(ka)}{ka},$$

with $p = \hbar k$ as usual. Show that $\phi(k)$ is normalized in k -space. Use computer algebra or look up from a table of integrals.

q2-14 Let the energy of a particle be $E = \sqrt{E_0^2 + p^2 c^2}$. Using $p = \hbar k$, find the “dispersion” relation $\omega(k)$, and obtain the group and phase velocities. What happens when $E_0 = 0$? Explain.

q2-15

(a) Numerically compute all the even state energies for an electron in a finite potential well of width $2a$ and depth $-V_0$. Solve (or graph) the equation for allowed energies

$$g(E) = \sqrt{E + V_0} \tan \left[\sqrt{2(E + V_0)} a \right] - \sqrt{-E} = 0 \quad (\text{even}).$$

In the above equation, atomic units are used, so E and V_0 are in units of 27.2 eV, and a is in units of 0.529 Å.

Assume $a = 4$ and $V_0 = 1$. Give energies accurate to three digits. Follow the sample finite square well program at the course website below.

(b) Repeat above for the odd state energies. Use the results from 2.29,

$$g(E) = \sqrt{E + V_0} \cot \left[\sqrt{2(E + V_0)} a \right] + \sqrt{-E} = 0 \quad (\text{odd}).$$

(c) [bonus] Graph the unnormalized wave functions of the first two excited states (one odd, one even).

q2-16

Investigate the number of bound states in the finite square well as a function of widths and depths. Use the program on “visualizing bound states” on the course website. Again use atomic units.

(a) First set $a = 2$ and $V_0 = 2$, find the number of bound states by varying the energy from the program. Include both even and odd states.

(b) Next, double a but keep V_0 the same; find the number of bound states. Then, revert a to the original value but double V_0 ; record the number of bound states.

(c) To increase the number of bound states, what is more effective, increase a or V_0 ? Briefly discuss your results.

<https://jwang.sites.umassd.edu/p341/>