PHY 341 HW Ch.3b

Do problems 3.7(a), 3.17, plus the following.

q3-6

(a) Let $\hat{Q} = -\frac{d^2}{d\phi^2}$ where ϕ is the azimuthal angle between 0 and 2π. Is $\hat{Q}$ Hermitian? If so, find its eigenfunctions and eigenvalues. If no, why?
(b) Do the same if $\hat{Q} = i\frac{d^2}{d\phi^2}$.

q3-7

Consider the complete basis set $|n\rangle$ representing the nth eigenstate of the SHO, with $n = 0, 1, \cdots$. Let $|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|2\rangle)$ and $|\Psi_2\rangle = \frac{1}{\sqrt{6}}(|0\rangle - 2|1\rangle + i|2\rangle)$.
(a) Find the projections $a_n = \langle n|\Psi_1\rangle$ and $b_n = \langle n|\Psi_2\rangle$. (b) Write down $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as column vectors and $\langle\Psi_1|$ and $\langle\Psi_2|$ as row vectors. (c) Find $\langle\Psi_m|\Psi_n\rangle$ where $m(n) = 1, 2$. Is $\langle\Psi_1|\Psi_2\rangle$ zero? Why or why not? (d) Predict, without explicit calculation, whether $\langle x\rangle = \langle\Psi_1|x|\Psi_1\rangle$ and $\langle p\rangle = \langle\Psi_1|p|\Psi_1\rangle$ should be zero. Write down your predictions. (e) Calculate $\langle x\rangle$ and $\langle p\rangle$. You do not have to complete the calculations fully, and can stop as soon as whether a null result can be ascertained. Compare your results with the predictions, and discuss discrepancies, if any.

q3-8

Consider a two-state basis set consisting of the ground and first excited states of the SHO, $|1\rangle \equiv |\psi_0\rangle$ and $|2\rangle \equiv |\psi_1\rangle$, respectively. Assume an operator $U = \alpha x$. (a) Construct the matrix representation of $U_{mn} = \langle m|U|n\rangle$ with $m(n) = 1, 2$. Use existing results as much as possible (e.g. from the previous problem). (b) Find the eigenvalues and eigenvectors of U. Confirm that the eigenvectors are orthogonal.

q3-9

In a calculation such as $\langle x\rangle = \langle\Psi|x|\Psi\rangle$, position space is the natural choice, but it does not have to be so. (a) Show that in a complete basis set $|n\rangle$, $\langle x\rangle = \sum_{m,n} c_m^* c_n x_{mn}$, where $c_n = \langle n|\Psi\rangle$ and $x_{mn} = \langle m|x|n\rangle$. (b) [bonus] Derive an analogous formula for $\langle x\rangle$ in momentum space in the form of a double integral.