



The expectation value of the x-component of the spin operator, denoted as $\langle S_x \rangle$, for a spin-1/2 particle (e.g., an electron) described by the given wave function ψ can be calculated using the formula:

$$\langle S_x \rangle = \langle \psi | S_x | \psi \rangle$$

where S_x is the x-component of the spin operator, and ψ is the given wave function.

The spin operator S_x in terms of the Pauli matrices is given by:

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Given the wave function $\psi = A^* [2i, -1]$, and $A = 1/\sqrt{5}$ (as previously determined for normalization), we can calculate $\langle S_x \rangle$:

First, normalize the wave function:

$$\psi = \frac{1}{\sqrt{5}} \begin{pmatrix} 2i \\ -1 \end{pmatrix}$$

Now, calculate $\langle S_x \rangle$:

$$\begin{aligned} \langle S_x \rangle &= \langle \psi | S_x | \psi \rangle \\ &= \frac{1}{5} \begin{pmatrix} -2i & 1 \end{pmatrix} \left(\frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) \begin{pmatrix} 2i \\ -1 \end{pmatrix} \\ &= \frac{\hbar}{5} \begin{pmatrix} -2i & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -2i \end{pmatrix} \\ &= \frac{\hbar}{5} (2i) \\ &= \frac{2i\hbar}{5} \end{aligned}$$

So, the expectation value of the x-component of the spin operator is:

$$\langle S_x \rangle = \frac{2i\hbar}{5}$$