

PHY 342 HW Ch.5

Do problem 5.2 (use results of 5.1), 5.1* & 5.14*, plus the following (*=optional bonus).

q5.1

For a system of two spins, explicitly show that:

(a) $\vec{s}_1 \cdot \vec{s}_2 = s_{1z}s_{2z} + \frac{1}{2}(s_{1+}s_{2-} + s_{1-}s_{2+})$ where $s_{1\pm}$ and $s_{2\pm}$ are the raising and lowering spin operators for particle 1 and 2, respectively;

(b) the three members of the triplet are eigenfunctions of $\hat{S}^2 = \hat{s}_1^2 + \hat{s}_2^2 + 2s_{1z}s_{2z} + s_{1+}s_{2-} + s_{1-}s_{2+}$ with the eigenvalue $2\hbar^2$ and, of S_z with the eigenvalues $1\hbar, 0\hbar, -1\hbar$, respectively.

q5.2

Let the wave function of a system of two identical particles be

$$\psi_{\pm}(x_1, x_2) = A[\phi_m(x_1)\phi_n(x_2) \pm \phi_n(x_1)\phi_m(x_2)].$$

(a) Verify that $\psi_{\pm}(x_1, x_2) = \pm\psi_{\pm}(x_2, x_1)$.

(b) Given ϕ_n forming an orthonormal basis set, calculate the normalization constant A for $m \neq n$ and for $m = n$. The latter is possible only for symmetric wave functions (bosons), of course.

q5.3

Experimentally, the energy required to eject two electrons from a helium atom is 79 eV. Find the energy to eject only one electron (single-ionization potential), and express it in eV and in a.u.

q5.4

A certain metal has an atomic weight of 64 g/mole and a density of 9.0 g/cm³.

(a) Assuming one free electron per atom, find the Fermi energy of the metal in eV.

(b) Find the electron velocity at this Fermi energy, and express it as a fraction of c , the speed of light.

(c) If each atom contributes two free electrons, what is the new Fermi energy? You needn't repeat the calculations (aka forbidden) in part (a).

q5.5

Consider a system of two particles and three single-particle states $\psi_{\alpha}(x), \psi_{\beta}(x), \psi_{\gamma}(x)$.

Make a table listing possible states of the system if the particles are (a) distinguishable;

(b) identical bosons; and (c) identical fermions.

Do it yourself first, then use ChatGPT to generate a response; compare the two and give your critique.