PHY 342 HW Ch.7, 8

q7.1
Consider a particle of mass m in the linear potential $V = \alpha|x|$.

(a) Use dimensional analysis to estimate the order-of-magnitude of energy, and denote it by ϵ_0. This is the characteristic energy scale.

(b) Choose a Gaussian $\psi = \exp(-\beta x^2)$ as the trial wave function, and calculate the ground state energy with the variational principle. Determine the parameter β which minimizes the energy, and find E_{vp}. Express E_{vp} in units of ϵ_0, i.e., express $E_{vp} = f \times \epsilon_0$, and give the numerical value of the factor f. This is the upper bound of the true ground state energy, E_{gs}.

(c) The exact ground state energy may be obtained from solving the Schrödinger equation with the so-called Airy function. It is $E_{gs} = 0.80861\,\epsilon_0$. Compare E_{vp} with the exact result, and find the relative error between them. Comment on the result.

q7.2
Calculate the variational ground state energies of H^- and Li^+ using the one-parameter wave function similar to that for helium. Express the results in a.u. and in eV. Are they both stable against self-disintegration? Explain.

q7.3
Let $\psi_{100}(\vec{r})$ be the hydrogenic wave function with charge Z. Indicate true or false for each statement below with a brief explanation:

(a) $\Psi(\vec{r}_1, \vec{r}_2, S, m_s) = \psi_{100}(\vec{r}_1)\psi_{100}(\vec{r}_2) \times$ (singlet) is an exact wave function of helium ($Z = 2$) if electron-electron interaction is ignored.

(b) $\Psi(\vec{r}_1, \vec{r}_2, S, m_s) = \psi_{100}(\vec{r}_1)\psi_{100}(\vec{r}_2) \times$ (triplet) is an exact wave function of helium ($Z = 2$) if electron-electron interaction is ignored.

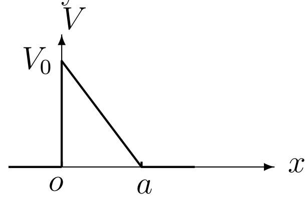
(c) In a variational-principle calculation, $\langle H \rangle = \langle T \rangle + \frac{e^2}{4\pi\varepsilon_0}\left(\langle \frac{1}{r_{12}} \rangle - Z^* \langle \frac{1}{r_1} + \frac{1}{r_2} \rangle\right)$, with Z^* as the effective screened charge.

(d) Depending on the trial wave function used, inclusion of the electron-electron interaction could either increase or decrease the variational energy.

Also do problem 8.3 plus the following.
q8.1
Use the WKB quantization rule without hard walls to calculate the eigenenergies in a linear potential $V = \alpha|x|$. Compare the ground state value with that from q7.1 using the variational principle, and with the exact result.

Also compare the WKB and the exact result for the first excited state. The latter is $E_1 = 2.33811\,\epsilon_0$ with the same ϵ_0 as in q7.1. Briefly discuss the comparison.

q8.2
A potential is zero everywhere but linear in $0 \le x \le a$ as shown below.



(a) Write out the piece-wise expression of the potential function $V(x)$. (b) Calculate the tunneling probability for an electron of energy $E < V_0$. Comment on the result, such as its dependence on V_0 and a.