PHY 342 HW Ch.4b

Do problem 4.11, plus the following ($*$=optional bonus).

q4.4
For $l \neq 0$ states in the 3D infinite spherical well:
(a) Sketch the centrifugal and the effective potentials, and indicate where bound states can be found. (b) The eigenenergies are determined by the zeros of $j_l(ka) = 0$ (Eq. 4.48), or $k_{nl} = x_{nl}/a$ with x_{nl} being the n-th zero of the spherical Bessel function $j_l(x)$. Find the first 5 zeros of $j_l(x)$ for $l = 1, 2, 3$. Then give the energies E_{nl} for each l in units of $\hbar^2\pi^2/2ma^2$. Sketch the energy levels, with n being the vertical axis and l the horizontal axis including $l = 0$.
You can use the sample program discussed and provided on the course website. Alternatively, try to get AI bots like ChatGPT to write a code to accomplish the same thing. In either case, include the code and results. If you do use AI bots, comment on the process and where it's helpful/unhelpful.
For extra challenge, search for "Scipy zeros spherical Bessel function", or follow this link
`https://docs.scipy.org/doc/scipy/reference/generated/scipy.special.spherical_jn.html`
to write your own program to obtain the zeros graphically (or numerically with root finding).

q4.5
Use dimensional analysis, find the powers i, j, k, l such that $\varepsilon_0^i e^j \hbar^k m^l$ has the dimension of length.

q4.6
Given the equation for v

$$\rho\frac{d^2v}{d\rho^2} + 2(\ell + 1 - \rho)\frac{dv}{d\rho} + (\rho_0 - 2\ell - 2)v = 0,$$

and the power series solution $v = \sum_{j=0}^{\infty} c_j \rho^j$, fill in the steps leading to

$$\sum_{j=0}^{\infty} \Big[(j+1)(j+2\ell+2)c_{j+1} - 2jc_j + (\rho_0 - 2\ell - 2)c_j\Big]\rho^j = 0.$$

q4.7
(a) Plot the radial wave functions R_{20} and R_{21} using a computer. Omit the $a^{-3/2}$ factor, and use Bohr radius as the units for r.
(b) Plot the radial probability distributions, i.e., $|R_{nl}|^2 r^2$, for the two cases. Restrict $0 \leq r/a \leq 10$.

q4.8
Derive the most probable value of r for finding the electron in the states R_{20} and R_{21}. Note this is different than the highest probability density. Compare and discuss your answers with the graphs above.