PHY 342 HW Ch.5

Do problem 5.2 (use results of 5.1), 5.1* & 5.14*, plus the following (*=optional bonus).

q5.1
For a system of two spins, explicitly show that:
(a) $\vec{s}_1 \cdot \vec{s}_2 = s_{1z}s_{2z} + \frac{1}{2}(s_{1+}s_{2-} + s_{1-}s_{2+})$ where $s_{1\pm}$ and $s_{2\pm}$ are the raising and lowering spin operators for particle 1 and 2, respectively;
(b) the three members of the triplet are eigenfunctions of $\hat{S}^2 = \hat{s}_1^2 + \hat{s}_2^2 + 2s_{1z}s_{2z} + s_{1+}s_{2-} + s_{1-}s_{2+}$ with the eigenvalue $2\hbar^2$ and, of S_z with the eigenvalues $1\hbar, 0\hbar, -1\hbar$, respectively.

q5.2
Consider the addition of two spins, $s_1 = \frac{1}{2}$ and $s_2 = 1$. (a) List all possible total spin S and M. (b) The coupled state $|SM\rangle$ may be expressed in terms of the product states, $|s_1 m_1\rangle|s_2 m_2\rangle$, as $|SM\rangle = \sum_{m_1,m_2} c(m_1, m_2)|s_1 m_1\rangle|s_2 m_2\rangle$, where $M = m_1 + m_2$ and $c(m_1, m_2)$ is the Clebsch-Gordan coefficient. Show that $|\frac{3}{2}\,\frac{1}{2}\rangle = c_1|\frac{1}{2}\,\frac{1}{2}\rangle|1\,0\rangle + c_2|\frac{1}{2}, -\frac{1}{2}\rangle|1\,1\rangle$ with the coefficients $c_1 = \sqrt{2/3}$ and $c_2 = \sqrt{1/3}$. Hint: consider $S_\pm = s_{1\pm} + s_{2\pm}$.

q5.3
Let the wave function of a system of two identical particles be

$$\psi_\pm(x_1, x_2) = A[\phi_m(x_1)\phi_n(x_2) \pm \phi_n(x_1)\phi_m(x_2)].$$

(a) Verify that $\psi_\pm(x_1, x_2) = \pm\psi_\pm(x_2, x_1)$.
(b) Given ϕ_n forming an orthonormal basis set, calculate the normalization constant A for $m \neq n$ and for $m = n$. The latter is possible only for symmetric wave functions (bosons), of course.

q5.4
Experimentally, the energy required to eject two electrons from a helium atom is 79 eV. Find the energy to eject only one electron (single-ionization potential), and express it in eV and in a.u.

q5.5
Consider a system of two particles and three single-particle states $\psi_\alpha(x), \psi_\beta(x), \psi_\gamma(x)$. Make a table listing possible states of the system if the particles are (a) distinguishable; (b) identical bosons; and (c) identical fermions.
Do it yourself first, then use ChatGPT to generate a response; compare the two and give your critique.

q5.6
A certain metal has an atomic weight of 64 g/mole and a density of 9.0 g/cm^3.
(a) Assuming one free electron per atom, find the Fermi energy of the metal in eV.
(b) Find the electron velocity at this Fermi energy, and express it as a fraction of c, the speed of light.
(c) If each atom contributes two free electrons, what is the new Fermi energy? You needn't repeat the calculations (aka forbidden) in part (a).
`https://jwang.blogs.umassd.edu/p342/`