PHY 342 HW Ch.6a

Do problem 6.2, 6.3, and 6.6* [bonus] plus the following.

q6.0
Using the density of states $\rho(k)$, derive the total energy E_{tot} of a Fermi gas (free electrons), expressed in terms of V and k_F.

q6.1
Consider the Dirac δ atom, i.e., the bound state to the potential $-\alpha\delta(x)$.
(a) Write down the unperturbed Hamiltonian. Look up its unperturbed bound state energy and wave function, and also write them down.
(b) Suppose a perturbation is introduced in the form of a square well, $H' = -V_0$ for $|x| \leq \frac{a}{2}$ and zero elsewhere. Calculate the first-order energy correction E^1 to the bound state.
(c) Let $V_0 = V_c$ be the condition when E^1 is equal to the unperturbed energy. If $V_0 \geq V_c$, the perturbation is said to be strong; if $V_0 \ll V_c$, it is said to be weak, and this is the regime first-order perturbation is considered valid. Find V_c, and comment on the result.

q6.2
Suppose the nuclear charge of a hydrogen-like atom is changed from Z to $Z+\delta$. Calculate the first-order energy correction of the ground state.
If you do it the eeeezy way, no computation of integrals should be necessary.

q6.3* [bonus]
First, familiarize yourself with the program on the class website computing the energy levels of a double-well potential, and make sure it runs on your machine. Everything there and here will be in a.u.
(a) Modify the program to compute the energy levels of a single well of width 1.5 and depth of 6. Compare the energies with those of the double well, and note any observations.
(b) Calculate the energy levels for the same single well using the method from the last semester (HW2d, q2-15), and compare your results with those above (both even and odd states). Briefly discuss the results.
(c) Now modify the program to compute the energy levels of an electron in a SHO with $V = \frac{1}{2}x^2$, all the way up to $E = 5$. Comment on your results.
(d) Add a small perturbation to (c), say a square well centered at origin of width 1 and depth 0.1. Repeat (c). How are the energy levels shifted?
(e) Calculate the first-order corrections to the ground and first excited states energies. Compare them with results from part (d). Briefly explain the results.