

PHY 341 HW Ch.3a

Do problems 3.5; plus the following (* = optional bonus):

q3-1

For each of the following wave functions, sketch it and determine whether it is a Hilbert space wave function ($a > 0$, $-\infty < x < \infty$):

(a) $A \exp(-ax)$; (b) $A \sin(kx)$; (c) $A \exp(-ax^2)$; (d) $A/\sqrt{-x}$; (e) $A \exp(-a|x|^2)/x^{1/4}$.

q3-2

Which of the following operators are Hermitian? Briefly explain.

d/dx , d^2/dx^2 , ix , a_+ , a_- , a_+a_- , $i\partial/\partial t$.

q3-3

Consider a particle in the box with ψ_n as the stationary states.

(a) Determine whether ψ_n are eigenfunctions of the Hamiltonian H , momentum \hat{p} , position \hat{x} , and momentum squared \hat{p}^2 .

(b) Are these pairs of operators mutually compatible? 1) H & \hat{p}^2 ; 2) H & \hat{p} ; 3) \hat{p} & \hat{p}^2 .

(c) For each mutually compatible pair identified above, is ψ_n a common eigenfunction? Discuss, given your answers to part (a) above.

q3-4

Calculate the following inner product:

(a) $\langle a|b \rangle$ and $\langle b|a \rangle$, where $a = [1, 3, 2i, -2]^T$, $b = [-i, -1, i, 1]^T$.

(b) $\langle f|g \rangle$ where $f = e^{ikx-x} \sin 2x$, $g = e^{ikx-x}$, $0 \leq x < \infty$, and k is real.

q3-5* Earlier we proved $[f(x), \hat{p}] = i\hbar f'$ in **q2-4**. Now prove this explicitly in momentum space. Hint: Let $\Phi = \Phi(p)$ be a test wave function in momentum space, and $\Phi^{(n)} \equiv \frac{\partial^n \Phi(p)}{\partial p^n}$.

First show that $(p\Phi)^{(n)} = p\Phi^{(n)} + n\Phi^{(n-1)}$, then expand $f(x)$ as a Taylor series, such that $[f(x), p] = \sum_n c_n [x^n, p]$ with $c_n = d^n f/dx^n|_{x=0}/n!$. Noting $x = i\hbar\partial/\partial p$, calculate the commutator of the n -th term with p , namely $[x^n, p]\Phi$. Finally reverse the sum as the Taylor series of the desired result.