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Source: *Journal for Research in Mathematics Education*, Vol. 21, No. 2 (Mar., 1990), pp. 145-153

Published by: National Council of Teachers of Mathematics

Stable URL: <https://www.jstor.org/stable/749141>

Accessed: 16-02-2019 18:24 UTC

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COGNITIVE ASPECTS OF SHARING

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To probe certain logical aspects of preschoolers' dealing and counting we showed edited highlights of three preschool children engaged in sharing activities to 17 second-grade children in an interview situation. In the videotaped episodes, the preschoolers routinely counted or checked the heights of the stacks when asked how they knew there was a fair share; the second graders, with one notable, and another possible, exception, expressed the view that counting after dealing was essential to tell if there was a fair share. One second grader gave us articulate explanations about the procedures used by the preschoolers, and another apparently changed her mind during the course of the interview about the need to count after dealing out.

This is a report on a short series of interviews that was designed to probe logical and procedural features of young children's sharing activities. The procedure we focused on was that of systematic dealing (as in card games). By *dealing* we mean a cyclic distribution of discrete objects (regarded as identical), with the same number distributed to each place on each round of the cycle until there are none left. This procedure is also known as distributive counting (Miller, 1984).

In the simplest form of dealing, a cycle occurs when one object is given to each place. One cycle in this simple form is an instance of one-to-one correspondence—one object for each place. This one-to-one correspondence is then repeated over and over until all the objects are used or it is not possible to proceed further. In a slightly more involved form of dealing, the number of objects placed at each spot is fixed within a cycle but may vary from cycle to cycle. In this sense we regard dealing as a repeated application of one-to-one correspondences, although it is important to note that we do not claim that children see it this way.

Experimental results indicate that the most common sharing strategy for young children placed in a structured interview situation is dealing without overt counting (Miller, 1984; Hunting & Sharpley, 1988; Clements & Lean, 1988). However, as a result of a study with 75 preschool children that was designed to probe features of the children's understanding of fractional terms such as $1/2$, $1/4$, and $1/3$ and their prefractional sharing knowledge and strategies (Hunting & Davis, 1989), we found that features of the preschoolers' responses to questions such as "Is there a fair share?" when they had just apportioned a number of discrete items using a dealing strategy, seemed to require further investigation. These were, principally,

This research was partially funded by a grant from the Commonwealth Tertiary Education Commission, Australia. We are grateful to Peter Egeberg of the Northern Metropolitan Regional Office of the Victorian Ministry of Education for his assistance in locating a suitable research site, to Marjorie Home and Kath Richter of Westgarth Primary School for allowing us to be part of their grade 2 class, and to Geraldine Woolnough and Debbie Chesser of Crellin Crescent and Diamond Hills kindergartens for providing us with the facilities to produce the preschoolers' videotape.

whether the children in the study were aware that dealing was sufficient to ensure equality of shares and that counting after sharing is simply a checking procedure, or whether they indeed felt that counting or some other check was essential in determining a fair share. It was puzzling to us that the preschool children shared evenly by dealing but then resorted to a counting or measurement check. Our aim in this research was to look at the question of whether young children were aware that dealing without counting is sufficient to ensure equality of shares in a discrete sharing situation. The question we have addressed in this study is this: Do second-grade children, watching the sharing activities of preschoolers, express a view that dealing without counting is sufficient to establish the equality of shares?

METHOD

Videotape Episode

We compiled edited highlights of videotapes of preschoolers involved in sharing activities. The edited tape consisted of three five-year-old children—one boy and two girls—performing sharing tasks with 12 biscuits (crackers). Initially each child was presented with two dolls and asked to share all the biscuits evenly between the two dolls. Then a third doll joined the other two, just before they were supposed to eat the biscuits, and the child was asked to share all the biscuits evenly among the three dolls. The strategies used by the three children varied considerably.

Boy. The boy gave one biscuit to each doll and stopped. The interviewer indicated that all the biscuits should be shared, so he then continued to give out one to each (without changing the order) until all the biscuits were shared, with a stack in front of each doll. He counted the number of biscuits, using a knife as a pointer, and correctly named six in each stack. When the third doll (Joey) arrived, he picked up three biscuits at a time from each pile and stacked them in front of Joey. He then picked up two biscuits from Joey's stack and placed one on each of the other dolls' stacks. He correctly counted the number of biscuits in Joey's stack. He moved to another stack and continued counting on from four until he reached six. He then started again with Joey's stack and counted all stacks correctly to conclude that each doll had four biscuits.

First girl. The first girl asked if she had to break the biscuits. She then gave one to each doll and asked if she had to give out all the biscuits. She did so, one biscuit at a time to each doll in turn, ending with a stack in front of each doll. She checked that each doll had "the same" by comparing the heights of the stacks. (She commented at this time: "What about if we move them and see if they've got the right amount?") When Joey arrived she took one biscuit from a stack and put it in front of Joey. She began to take another biscuit from the same stack but instead took one from the other stack. She then took one from each stack and placed them on Joey's stack. At this point (even though the dolls had an even share) she touched the stacks and moved one more biscuit onto Joey's stack. She continued to touch the stacks

and move biscuits until the interviewer suggested counting the number in each stack. She counted each stack by restacking it into a new one and counted on from the first to the second stack before correcting herself. She correctly counted four in two of the stacks and then said that each of the three dolls had four biscuits.

Second girl. The second girl gave to the two dolls, in turn, two biscuits, two biscuits, one biscuit, and one biscuit. She did not place them in a stack. She did not overtly count the biscuits and insisted they both had “the same.” When Joey arrived, she gave one biscuit to Joey and picked up all but one of the biscuits from in front of the other two dolls. She placed the ones she picked up in the middle and dealt out one at a time to the three dolls. She correctly counted four biscuits for each of two dolls and said that all three dolls had a fair share.

Sample interview questions

The following is a sample of the questions we directed to the second-grade children about the boy’s procedure for creating fair shares:

1. Why does he stack the biscuits?
2. Will it be easier to tell if each doll has a fair share if he stacks the biscuits? (A reference to the boy placing the biscuits in a stack in front of each of two dolls)
3. Did he know the dolls had a fair share before he started counting? (How did he know?)
4. Why did he count the biscuits? (A reference to his counting the number of biscuits in each of the two stacks)
5. What will he do with the biscuits when Joey comes? (A reference to the third doll, Joey, who was just about to make an appearance)
6. Why did he count?
7. Why does he go back and count the first pile of biscuits again? (References to his counting the three piles of biscuits and apparently making a mistake)

Subjects

We selected a school that was geographically and socioeconomically similar to the preschool in the videotaped episodes, from a working lower middle class area in the northern suburbs of Melbourne, and questioned 17 second-grade children in small groups about aspects of the edited videotape. These children varied in age from 7 years 4 months to 8 years 3 months. There were six groups, consisting of four, three, two, two, three, and three children, respectively.

Procedures

We showed edited videotaped highlights of the preschoolers’ actions to the second graders and questioned them on their views concerning why the preschoolers counted or measured after dealing. A specific list of questions, of which those above are a sample, were asked of the children in each group. Each question was

asked after an inserted color-bar break in the videotape. The questions were asked of the group as a whole and then of each child who had not already answered. The grade 2 children in a given group were free to respond to the interviewer's questions in any way they chose. Often we picked up a point made by one child and asked the other children in the group whether they agreed with or could amplify that response.

The method of interviewing children in groups rather than individually requires some explanation. First, our experience with preschool children indicated that although interaction in a group interview could be construed as contamination of a single child's response, we were likely to elicit more verbal response from the grade 2 children if they were interviewed in groups rather than individually.

Second, it was our plan to stimulate a discussion session rather than an interview as such and then to examine the video record of these discussion sessions for salient points relating to the logical basis of dealing and the perceived necessity of the preschoolers to count. As it happened, however, there was little interaction between the children. They most often answered the interviewer directly as a result of a question about the video record or about a point made by one of the other children.

Although the interview was scripted by our list of questions, we did encourage the children to adopt a conversational style in answering. Throughout the interviews we often diverted from the script to follow through relevant statements with other questions or restatements to encourage a further response.

All the interviews with the second graders were videotaped. Children's key statements relating both to the logical basis of dealing and the necessity to count were transcribed.

RESULTS

Statements about counting assumed the greatest proportion of the children's comments. We have extracted statements about counting from the interviews and cited examples below. Some of the second-grade children saw or inferred counting being performed by the preschoolers during the stacking or dealing process. Most of them commented on counting after the individual sharing tasks, and here they made remarks on the essential or inessential nature of counting.

They commented not only on what counting they saw but also on the general nature of the role of counting in sharing. Most of them perceived counting to be an essential part of the sharing process (whether or not it was performed by the preschoolers). For them, it was one of the steps in creating a fair share, not simply a check.

The following comments are typical of those that relate to the essential nature of counting. Anne and Gary are the interviewers, and the children interviewed are Megan, Steven, Lena, Lindon, George, and Boban.

In the discussion of the video interview with the preschool boy, Megan and Steven clearly express a belief that even though the boy dealt out biscuits to two dolls one at a time, he did not know the dolls had a fair share until he counted.

Megan expressed the view that the boy could have counted covertly, and if he did not count, then he could measure the stacks of biscuits to determine if there was a fair share.

Excerpt 1 (referring to the boy in the video episode who counted after dealing the biscuits to two dolls).

Anne: Would he have known they had fair shares before he started counting?

Megan: No.

Anne: Why do you say that?

Megan: But if he did, he would just go one for him and one for him and one for her and one for him and counted while he was doing it.

Anne: Did he know they were a fair share before he even counted? Did he know they were the same equal piles?

Steven: No.

Megan: He just counted them.

Anne: So why did he count?

Megan: So he knew how much they had. But if he didn't count he wouldn't know how much they had in each of the piles.

Gary: But would he know if they had a fair share?

Megan: If he measured it.

In the next excerpt Lindon seems to understand that dealing out would lead to a fair share, but then he finishes off with "... and counting them."

Excerpt 2 (referring to the boy in the video episode who stacked the biscuits in a pile in front of each doll as he dealt them).

Anne: Did you see how he put the biscuits up in a pile? Why did he do that?

Lindon: Yeh. He was sort of sharing them out to see if he's got enough for both of them. Shared between two people.

Anne: And would he know he had enough for both of them if they were in a pile?

Lindon: Sort of...sharing them out and then counting them.

Lindon and Lena, in excerpt 3, express as strong a view as any of the children interviewed that counting is essential to the establishment of a fair share. This is followed, in excerpt 4, by a clear statement from Megan that counting is essential. In excerpt 5, Megan, who did not observe the second preschool girl count but did see her behave in an apparently self-confident manner, suggests that the girl is either magic or very smart, if she did not count the biscuits.

Excerpt 3 (referring to the boy in the video episode when he counted after dealing biscuits to two dolls).

Anne: And so he counted them. But did he know they were fair shares even before he counted them?

Lena and Lindon: No.

Anne: He didn't. He didn't know. Why didn't he know?

Lindon: Because he hadn't counted them.

Anne (to Lena): What do you think?

Lena (shaking head): He hasn't counted them.

Anne: Is that the only way you can tell if they were fair shares though?

Lena and *Lindon* both nod affirmatively.

Gary: You think he had to count them, you reckon?

Lindon: Yes.

Excerpt 4 (referring to the first girl in the video episode, after she has dealt biscuits to two dolls).

Anne: Does she know if the dolls have got the same?

Megan: No.

Anne: Why doesn't she know?

Megan: Because she probably didn't count them or she probably counted them in her mind. Because we don't know because we weren't there and she didn't tell us.

Excerpt 5 (referring to the second girl in the video episode who did not check after dealing biscuits to two dolls).

Anne: How did she know they were the same?

Megan: She probably would be magic. But I don't think she could know. She thought she was smart. She was so smart she could know even if she didn't count them.

There were other comments from the interviews indicating the extent to which the children felt that counting after dealing is necessary to establish a fair share. They include the following significant statements:

"You've got to take them apart, put them around and then count them."

"They weren't the same because she didn't count them when she was going along."

"She was probably counting while she was doing it. She could've gone one, one, two, two, and then she probably knew."

"She is probably wrong because they haven't counted them."

She counted them "to see if they were the same."

"If she didn't count them she wouldn't know how much they got."

Counting is "to see if they're right, a fair share."

"You have to count them to prove it was the right number."

One child, Tom, indicated by his responses that he had a clear indication of the sufficiency of dealing to establish fair shares.

Excerpt 6 (referring to the boy in the video episode who counted after dealing the biscuits to two dolls).

Anne: Do the dolls have the same? Tell me why.

Tom: The child put out the biscuits. They give one to one doll and then another to the other doll and they kept on doing it until there was no biscuits left. So that the dolls both had the same—like when you're playing cards and you want to get the equal number of cards, sometimes. So you say, one for me and one for you and so on until all the cards have filled up. And you sometimes put some of the cards out in the middle.

Anne: Did he know the dolls had a fair share even before he counted?

Tom: Yeh, I think so. I thought he was going to count them in the next thing.

Anne: Do you have to count, to know if they are fair shares?

Tom: Well, if you know what you are doing you probably don't, but if...uh...I always count them after, just to make sure, before I put the answer.

Anne (referring to the second girl in the video episode): Will she count them, do you think?

Tom: She may count them or do what the second girl did—just pile them and see if she got it right.

Anne: Is it important that she does either of those, to know if they are the same?

Tom: I think it is important to do, unless you are really good and she knows exactly what she is doing.

One of the second-grade students appeared to suddenly change her mind toward the end of the interview about the necessity to count after dealing. Previously, she had emphatically stated that it was essential to count to ensure equality. Finally, another of the children expressed some understanding of the logical connection between dealing and counting when he said, "Dealing is sort of like counting. One there, one there, one there, one there."

Two reasons for the preschoolers stacking the biscuits were suggested by the children. First, the procedure resulted in a neat pile—there was less confusion about which biscuits had been dealt to which doll, and it was easier to count. Second, it enabled a height comparison. By stacking they could tell if the piles were the "same size."

The following are statements from the interviews that relate to stacking:

"She just spreaded it all down and then do it like the other one, and it would be all messed up, and how could she count it?" (Megan)

"First, then he done the second one, first he goes higher and then he puts another one and then he goes the same again, then the other one goes higher, then he gives another one and it goes like the same." (Steven)

"Because they look the same height." (Lindon)

"If they're the same size, that is they're probably the same." (Lena)

DISCUSSION

The children in this study indicated by their statements that they understood dealing to mean giving to each doll in sequence until all the biscuits had been given out. Of the children we interviewed, only Tom saw this procedure as adequate in itself for establishing a fair share; another child apparently came to this belief by the end of the interview; and a third declared the method adequate for establishing a fair share, but when asked, along with others in his group, if one had to check, he stated "Yes."

With two exceptions, the children did not articulate an awareness that it is unnecessary to count the number of biscuits in each pile in order to ascertain equality of shares when the biscuits had just been dealt out. In only one of these two cases were we prepared to say that the child interviewed could articulate the situation clearly.

It appears that the second-grade children we interviewed saw counting, in the sense of Gelman and Gallistel's (1978) Cardinal Principle, as a necessary activity to establish one-to-one correspondence. It is as if these children have replaced the logically simple situation of a one-to-one correspondence between the biscuits belonging to each doll—a one-to-one correspondence established by dealing—with a more complicated sequence of one-to-one correspondences: between biscuits and number words at one time, between biscuits and number words at a later time, and between the final tag word each time.

Our results suggest that a high proportion of children of approximately 8 years of age have a firm conviction that counting after a discrete sharing task is not simply a check for accuracy but is really an essential part of the process—without it one would not know there was a fair share.

Our work with preschoolers (Hunting & Davis, 1989) and other work on sharing activities with preschoolers and young school children (Hunting & Sharpley, 1988; Miller, 1984) and also with older school children (Clements & Lean, 1988) lead us to believe that children manifest a capacity at an early age to share discrete objects by dealing. This means that it is important to understand why young children cannot assert the adequacy of such a widespread ability.

Whether these children can assert the correctness of the result of the sharing procedure, however, seems to depend on the strength of their grasp of numbers as a counting device or their awareness of other methods, such as height comparison of stacks. To a large degree it does not seem to depend on an awareness that the dealing procedure itself is sufficient to assert equality of shares, principally because children of this age do not seem to have such an awareness, despite dealing without overt counting being perhaps the most common sharing method for young children in interviews.

Of the 17 second-grade children we interviewed, only one (Tom) expressed a clear awareness of his and other people's appropriate and possible procedures for making fair shares. The other children either said little at all or described what they themselves would do in a sharing situation. Their remarks seemed to be action-based and did not express an awareness of the underlying logical aspects of dealing or other sharing procedures. It is as if the question "How do you know they have the same?" is an invitation to act rather than an invitation to reflect. Children spontaneously seem to want to do something to answer this question, rather than think about how they arrived at their result.

It has been suggested to us that questions such as "How do you know?" imply to children that their answers or procedures are incorrect, so they adopt some action-based checking procedure. In the context of the reported experiment we are inclined to doubt this interpretation. We cannot rule out the possibility that as a result of a question like "How do you know?" many children have a doubt that their procedure gave a correct result and that it may be this doubt that drives the action response to check. It may also be true that an implied doubt from an adult is a strong incentive to check. This reinforces our belief, however, that children's awareness of the adequacy of their own manifest dealing procedures is not strong.

The resolution of the children's doubt is more likely to come from checking and not from reflection.

There remains the possibility that the second-grade children we interviewed see counting as an essential check for fairness, because they believed that the preschoolers did forget or may have forgotten at which doll they started. We have no evidence to support this possibility, but we also have no clear evidence to rule it out.

Our observations and inferences suggest to us certain characteristics of dealing as a strategy. The major features are as follows:

(1) Dealing without counting is an implicit, widespread sharing strategy. By saying it is an *implicit* procedure, we mean that young children do not apparently have a conscious awareness that it is an adequate procedure and when questioned as to its validity will resort to other, checking, procedures.

(2) Counting is a stronger strategy for checking for fair shares. Although more explicit but not as widespread in preschool children, it predominates as a checking procedure. Similarly, height comparison is a stronger strategy than dealing.

(3) Checking procedures predominate over the intrinsic logical completeness of other procedures (in particular, over dealing to obtain a fair share).

Although we believe that in a structured interview situation, dealing is the most prevalent form of sharing for young children, we have no evidence that dealing is a common method of sharing by young children. Preliminary studies we have conducted on spontaneous sharing performed by preschool children indicate a marked absence of dealing in order to share when an adult is not present to direct questions.

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