Mathematical Problem Solving by Pre-School Children
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Source: *Educational Studies in Mathematics*, Vol. 23, No. 4 (Aug., 1992), pp. 397-415
Published by: Springer
Stable URL: https://www.jstor.org/stable/3482991
Accessed: 17-02-2019 04:24 UTC

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# MATHEMATICAL PROBLEM SOLVING BY PRE-SCHOOL CHILDREN

ABSTRACT. In this article we provide new evidence for mathematical problem-solving abilities of pre-school children. These problem-solving behaviours occurred in a study of sharing of discrete items by dealing, in which we examined the abilities of three categories of counters to solve a discrete re-distribution problem. We detail the problem solving strategies used in the context of sharing by dealing as a common action scheme of pre-school children in clinical interviews.

## 1. ASPECTS OF THE DEALING SCHEME

Dealing, or distributive counting, is an action scheme for apportioning equal shares of a collection of discrete items to a finite set of recipients, whenever such an allocation is possible. Specifically by *dealing* we mean a cyclic distribution of discrete objects, regarded as identical, with the same number distributed to each place on each round of the cycle, until there are no more cycles possible, Davis and Pitkethly (1990). This procedure is also known as distributive counting: see Miller (1984), for example.

In the simplest form of dealing a cycle occurs when one object is given to each place. One cycle in this simple form is an instance of one-to-one correspondence – one object for each place. This one-to-one correspondence is then repeated over and over until all objects are used or it is not possible to proceed further. In a slightly more complex form of dealing the number of objects placed at each spot is fixed within a cycle but may vary from cycle to cycle. For example, a child might distribute 12 crackers evenly between 3 dolls by dealing out 1 cracker to each of the dolls, then immediately follow that by dealing out 2 to each doll, and then finish by dealing 1 to each doll. The *order* in which the items are dealt may vary from cycle to cycle. A child might first begin with the left hand doll, complete a cycle, and then start again from the right hand doll, or, as sometimes happens with 3 dolls, from the middle doll.

Dealing is primarily an *action scheme*. We explain what we mean by this in more detail in sections 1.4 and 2.1 below. The central idea is that dealing is composed of repeatable units of action that are performed over until the sharing task is completed. It is an important action scheme because it is an action basis for repeated one-one correspondences: a basic concept that is

*Educational Studies in Mathematics* **23**: 397–415, 1992.

mathematically anterior to the concept of number. As a perceptive seven year old said to us: "Dealing is sort of like counting. One there, one there, one there, one there".

Dealing has been used, by Streefland (1987) in particular, to help children build a knowledge of fractions and ratio. He has developed instructional material that takes the distribution of fair shares as a starting point. Kieren (1983) and Vergnaud (1983) have also identified sharing as an important strategy for establishing basic fraction knowledge.

### 1.1. *How Common Is Dealing and What Are its Social Origins*?

Dealing has been reported by a number of authors as a general activity in interviews concerned with sharing: Clements and Lean (1988), Hunting and Sharpley (1988), Hunting (1991), and Miller (1984). These similar reports, from three different cultures suggest that dealing is an action scheme that is common amongst children in many, if not all, cultures.

It is important to discover the origins, as distinct from the use, of the dealing scheme. This is because there is evidence that it is a very common action scheme across different cultures, and because it could potentially be used at an early age to help children build knowledge of fractions, ratios, and proportions. Some obvious questions are: Is it rooted in a child's early experiences with parents or siblings? Does it arise spontaneously without training? Are there sharing activities in early childhood that strengthen a child's capacity to share by dealing, or is this capacity largely unaffected by the presence or absence of such activities? Do first children have the same capacity to deal as later children in a family? These and similar questions were dealt with in an excellent and novel study by Hunting (1991). It transpires from Hunting's careful analysis that, as yet, we have no firm evidence of one or more factors that might be significant in a child's developing ability to share discrete items by dealing.

### 1.2. *How Conscious Are Children of the Effectiveness of Dealing*?

We have seen that by the age of 4 or 5, and even as early as 3 years of age, children commonly deal in response to an adult interviewer's request to share a collection of discrete items. To what extent do children perform the dealing strategy without being aware of its significance as an action scheme that of and by itself ensures equality of shares among recipients?

This question was addressed by Davis and Pitkethly (1990). Their study involved video clips of pre-school children who were engaged in sharing crackers to 2 or 3 dolls. Video segments of three of these pre-school children were shown to grade 2 children. The grade 2 children were asked for their views on whether there is a need to check, by counting or measurement, for equal shares after dealing. They were also asked why the pre-school children performed counting and measuring checks. Of the 17 children in that study 16 indicated that it was an essential part of establishing an even share to count after dealing.

### 1.3. *Is Sharing by Dealing a Spontaneous Activity*?

By this we mean: do children use dealing to form equal shares in their own social activities, when they are not under the direction of an adult? The limited evidence we have suggests that dealing is not a spontaneous activity in this sense. Davis and Hunting (1990) addressed the question of how pre-school children, in groups of 2 or 3, would share a collection of jelly babies, without any instructions from an adult that they were to share. These children were given a foil counting task and told they could have the jelly babies when they had finished. Despite there being occasions on which children expressed a need to establish fair shares, no child resorted to dealing to resolve the issue. Further evidence for an apparent lack of observed dealing comes from the work of Mulligan (1988). There were numerous instances in her study in which young children could have resolved a sharing issue by dealing but did not.

### 1.4. *One Cycle in Dealing as a Unit Item*

The evidence we have so far does not seem to tell us where dealing comes from in a child's development, and suggests that the dealing scheme is developmentally anterior to more abstract forms of counting – even counting of figural or motor unit items. An observation we have made on a number of different occasions, with different children, is that good sharers of discrete items seem to "track" what they are doing within a dealing cycle and between cycles. It is as if a tension develops as a child places an item in front of one spot (a doll, say) and then decides how to approach the remaining spots: that is, the child appears to be establishing a preferred *order* in which to complete a dealing cycle. Furthermore children can, and do, vary the order they create from one dealing cycle to another. This hypothesized tension appears to be relieved when a given dealing cycle is completed. It is in this sense that a dealing cycle appears to be a "marked off" identifiable piece of experience. When a cycle is regarded as an entity in itself it appears as a "unit" because it, or a variant of it, is repeated over and over until the task of sharing is complete. It is not a unit that is *counted*, but a unit that is *repeated* as an action scheme, until no further action of that type is possible.

## 2. CHILDREN'S STRATEGIES IN A SHARING PROBLEM

### 2.1. *A Sharing Problem*

There is a marked difference in childrens' ability to:

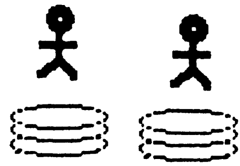
(a) share a number of discrete items between two dolls, and
(b) re-adjust the shares to accommodate a third doll.

A perceptive and articulate 8 year old child, Tom, whose responses to dealing and sharing tasks were detailed in Davis and Pitkethly (1990) and Davis (1990), found it less than straightforward to describe how a pre-schooler would re-adjust the shares of 12 crackers to two dolls to accommodate a third doll, Joey. The part of the interview with Tom that shows this goes as follows:

*Interviewer*: "How will he get those biscuits for Joey, do you think?
*Tom*: "I think, ah, ya put the two piles next to each other and then, ah, ya put the two piles next to each other. You ah, and ah, you, you get the piles and you put um one, one, what one for one doll from that pile na, one for another doll and then you do that until that pile is finished. Then with this one uh, you do it from this pile that, that, uh, that doll, that doll and then Joey, then that doll, that doll and then Joey. When that pile is finished you do the same with this one, and then you count them to make sure."
*Interviewer*: "So you don't dismantle the piles but you share one pile out first and then you share the other one out."
*Tom*: "Yeh, yeh, but you share it with three so they both . . . until it is fair."

This answer was expressed with considerable hesitation and not with Tom's usual clarity: we infer that even he saw this as a more difficult task. Admittedly he was describing, rather than doing, the task but it is just his descriptions of other childrens' actions that were so forceful and imaginative that lead us to believe that he might be able to describe this task easily.

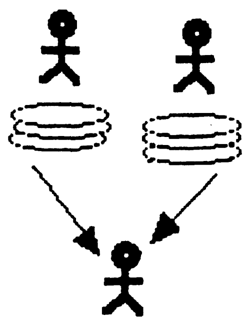
Let us look at the simplest sharing task between 2 dolls that can be re-adjusted for a fair share when a third doll arrives. This is the sharing of 6 items (crackers, say) between 2 dolls:



6 crackers shared evenly between 2 dolls.

Fig. 1.

The arrival of a third doll who must get an even share, before any crackers are eaten, creates a problem:



Then a third doll arrives

Fig. 2.

Whereas the request to share 6 crackers between 2 dolls could be handled by recourse to an *action scheme*, without further reflection, even this simple situation leads to a problem that requires cogitation. Of course a favourite dictum of mathematics – namely, reduce the problem to a previous case – applies here. It suffices to pick up the crackers from the 2 dolls and deal them out again, this time to 3 dolls. This is the way that Tom, cited above, suggests he would solve the problem for 12 crackers. In the case of 12, rather than 6, crackers shared between 2 dolls we have seen this strategy used relatively rarely: approximately only 5% of children we have observed did so. One might imagine the reason to be because the children do not see their dealing actions as providing them with a solution to a problem. In other words, dealing is something they can do in order to establish even shares so for them this is not a problem situation. The introduction of the third doll does however create a definite problem. It would seem therefore that in this very simple situation of sharing discrete items to dolls, and then re-adjusting the shares because of the arrival or removal of one or more dolls, lies some of the very first steps that children can make in mathematical problem solving.

## 2.2. *Good, Developing, and Poor Counters*

Since dealing, as a primitive action scheme to establish one-one correspondences, is logically anterior to counting, we might expect good counters to be already good at sharing by dealing, but not vice versa. That is, we might expect almost all children who are good at counting to be good at dealing, but conversely with young children we might expect a significant proportion who are good at dealing but not yet good counters. The counting categorizations we use are based on Steffe and Cobb's (1988) types of counters of unit items:

- "Poor" counters are counters of perceptual unit items only;
- "Developing" counters are counters of figural unit items;
- "Good" counters are the rest.

Recall, briefly, Steffe and Cobb's (1988) counting type classifications:

- Counters of perceptual unit items require actual perceptual items in order to establish units that can be counted. "They know how to count but need a collection of marbles, beads, fingers, etc., in order to carry out the activity." (p. 4)
- "One of the first manifestations of independence from immediate perception occurs when a collection of items is counted, even though it is not within the child's range of immediate perception or action. In this case the child might attempt to count the items of a screened collection by coordinating the sequential production of number words with the sequential production of visualized images of perceptual items. The child is then said to count figural unit items." (p. 4)
- The remaining counters are counters of motor unit items, verbal unit items, and abstract counters (Steffe and Cobb, 1988, pp. 4–6).

There is evidence (Pepper, 1990) that *all* good counters of age 4–5 years can perform the redistribution problem with 12 crackers to 3 dolls, but conversely many children of this age who can perform the re-distribution problem are still only counters of perceptual unit items. This suggests that the dealing re-adjustment tasks are genuinely pre-numeric problem solving tasks: they are capable of being solved by children who are generally not good counters.

### 2.3. *The Re-distribution Task*

We will call the task of re-distributing 12 crackers to 3 dolls when one has previously distributed the 12 crackers to just 2 dolls, the *distribution task*, or re-distribution problem. The question we have examined is this: do pre-school children who are placed into different categories of counters – poor, developing, or good – perform equally well at the distribution task? Our hypothesis was that the good counters would, by virtue of their ability to count flexibly, be significantly better at the re-distribution task. We gave the distribution task to 74 children from 3 different pre-school classes. The ages of the children ranged from 4 years 5 months to 5 years 8 months, with an average age of 4 years 11 months. The children were first placed into categories of poor, developing, or good counters. This was done by using a series of clinical interviews of approximately 15 minutes duration. These clinical interviews presented the children with a model farmyard on a table: the farmyard contained model animals and buildings. The children were asked a number of questions about the farmyard. These questions were designed to allow us to categorize the children as poor, developing, or good counters. The questions were as follows:

- *I would like to show you my play farm. Here are some brown sheep and some white sheep. Can you tell me the names of the other animals on the farm?*
- *There are five brown sheep in this field* (indicate) *and four white sheep in this field* (indicate). *How many sheep are there altogether?*
- *Over here there are two cows in the shed* (lift shed and briefly show the cows). *How many cows are there altogether?* (There were six visible.)
- *There are seven piglets hiding in the pig house. How many piglets are there altogether?* (Five piglets were visible.)
- *Here are eight geese* (indicate). *There are some more in the goose shed. There are twelve geese altogether. How many geese are in the shed?*
- *In this house there are six people* (indicate the white building), *and in this house there are five people* (indicate the red building). *How many people are in the two houses altogether?*

Children did not necessarily have to solve all the problems to be classified as good counters. However all children classified as good counters used strategies that we inferred would lead to a solution were they developed further. For example, good counters generally tackled the pig problem by counting the pigs inside the shed first either by counting to seven or by counting on from seven. Developing counters could solve the first two farm problems. However they had difficulty with finding a strategy to begin the pigs problem, and they could not solve the geese or people problems. Poor counters were unable to solve any problems where perceptual material was not readily available to them.

## 2.4. *Children's Strategies*

61% of the children were able to solve the re-distribution problem. There were 22 poor counters, 14 developing counters, and 9 good counters, who solved this problem. This was 49% of the poor counters, 61% of the developing counters, and 100% of the good counters. There were 26 different ways that these 45 children went about successfully solving this problem. Many of these solutions to the re-distribution problem are structurally similar, in that they branch off from a common sub-strategy. For example:

Give 1 cracker from doll 1 to doll 3, and 1 cracker from doll 2 to doll 3. Repeat these actions. Then:

(a) The task is completed (2 poor counters, 1 good counter).
(b) Repeat them again and then give 1 cracker from doll 3 to doll 1, and 1 cracker from doll 3 to doll 2 (1 developing counter).
(c) Give 1 cracker from doll 1 to doll 3, 1 cracker from doll 3 to doll 1 1 cracker from doll 2 to doll 1, and 1 cracker from doll 1 to doll 2 (1 poor counter).

(The beginning doll in any strategy is always labelled "doll 1", as a matter of convention.)

## 2.5. *Steps in a Strategy*

Each strategy carried out by the children can be regarded as being made up from a number of identifiable *steps*. For example, the three strategies described above can be detailed through the steps in Fig. 3.

Often children carried out a dealing action as part of solving the re-distribution problem. As we described in 1.4, above, we regard one cycle in a dealing procedure as a unit that is repeated, or potentially can be repeated. Consequently, we take a single cycle in dealing to be one step in

| | Start | Step 1 | Step 2 | Step 3 | Step 4 |
|---|---|---|---|---|---|
| Doll 1 | 6 | 5 | 5 | 4 | 4 |
| Doll 2 | 6 | 6 | 5 | 5 | 4 |
| Doll 3 | 0 | 1 | 2 | 3 | 4 |

| | Start | Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 | Step 7 | Step 8 |
|---|---|---|---|---|---|---|---|---|---|
| Doll 1 | 6 | 5 | 5 | 4 | 4 | 3 | 3 | 4 | 4 |
| Doll 2 | 6 | 6 | 5 | 5 | 4 | 4 | 3 | 3 | 4 |
| Doll 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 5 | 4 |

| | Start | Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 | Step 7 | Step 8 |
|---|---|---|---|---|---|---|---|---|---|
| Doll 1 | 6 | 5 | 5 | 4 | 4 | 3 | 4 | 5 | 4 |
| Doll 2 | 6 | 6 | 5 | 5 | 4 | 4 | 4 | 3 | 4 |
| Doll 3 | 0 | 1 | 2 | 3 | 4 | 5 | 4 | 4 | 4 |

Fig. 3.

solving the re-distribution task. We do this in the sense that we believe a single cycle in dealing is regarded by a child as a single repeatable unit of action, even though it consists of component actions of giving one (or more) crackers to each doll. Examples of this appear in the next pair of strategies, which have the first two steps as a common base:

Give 3 crackers from doll 1 to doll 3. Then take 3 crackers from doll 2, and then:

(a) place them in a pile in the middle. Then deal out the pile to the three dolls (2 poor counters, 1 developing counter, 1 good counter).
(b) give them to doll 3. Take 3 crackers from doll 3 and place them in a pile in the middle. Then deal from the pile to the three dolls (1 developing counter).

In terms of steps we can represent these strategies as follows:

| | Start | Step 1 | Step 2 | Deal | Step 3 |
|---|---|---|---|---|---|
| Doll 1 | 6 | 3 | 3 | one | 4 |
| Doll 2 | 6 | 6 | 3 | by | 4 |
| Doll 3 | 0 | 3 | 3 | one .. | 4 |

| | Start | Step 1 | Step 2 | Step 3 | Deal | Step 4 |
|---|---|---|---|---|---|---|
| Doll 1 | 6 | 3 | 3 | 3 | one | 4 |
| Doll 2 | 6 | 6 | 3 | 3 | by | 4 |
| Doll 3 | 0 | 3 | 6 | 3 | one .. | 4 |

Fig. 4.

## 2.6. *Number of Steps to a Solution*

The number of steps, in the above sense, that a child takes either to obtain an even distribution of all of the crackers, or to *recognize* that they have obtained such a distribution, is a measure of the ease with which they knowingly solved the re-distribution problem. The number of children in each of the 3 categories of counters who took a given number of steps to recognize that they had successfully solved the re-distribution problem is shown in the graphs below (2 children who crunched all crackers together and gave out piles of crumbs are not included in this count):

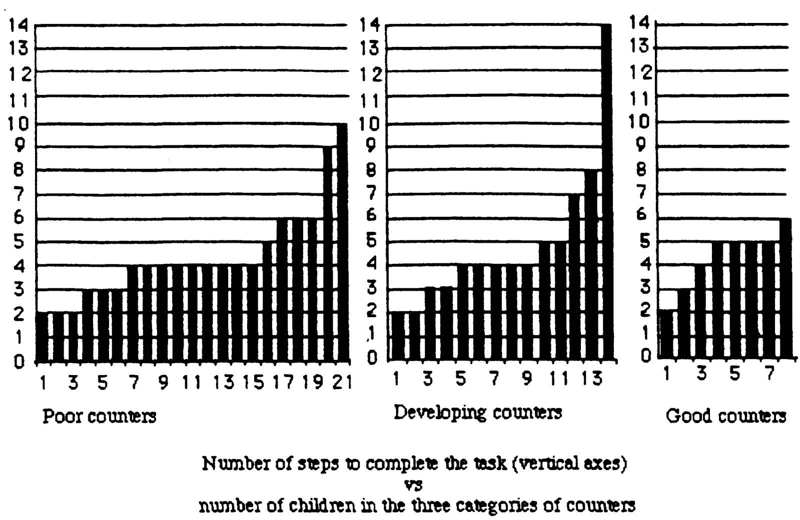



Fig. 5.

The descriptive statistics for the three categories of counters and the overall successful group are given below:

TABLE I

Descriptive statistics of the number of steps required to realize a solution to the re-distribution problem

| Type | Mean | Std. dev. | Std. error | Variance | Coef. var. | Count |
|---|---|---|---|---|---|---|
| Poor | 4.429 | 2.063 | .45 | 4.257 | 46.59 | 21 |
| Devel | 4.929 | 3.1 | .829 | 9.61 | 62.898 | 14 |
| Good | 4.375 | 1.302 | .46 | 1.696 | 29.771 | 8 |
| All | 4.581 | 2.312 | .353 | 5.344 | 50.461 | 43 |

The mean number of steps required to complete the re-distribution task is close to the number of steps required if the crackers are collected from the two dolls and then dealt one at a time to the three dolls, namely 5 steps. The relation between the proportion of all children who successfully completed the re-distribution problem and the number of steps taken, is indicated in the graph below in which the cumulative proportion of children who were successful at the re-distribution task, and completed it in $N$ or fewer steps is plotted below as a function of $N$:

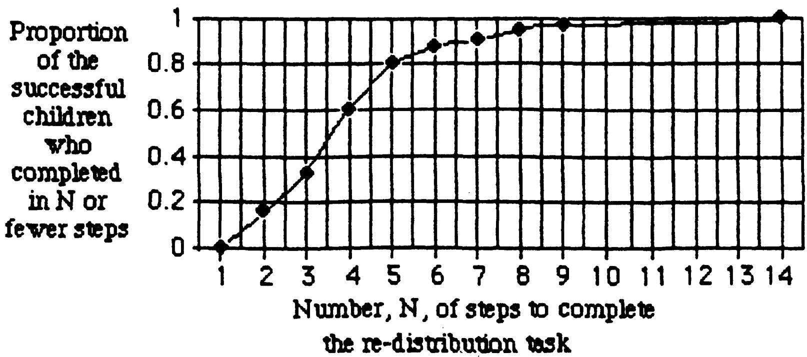



Fig. 6.

At the mean number of steps (4.5), over 65% of the children who were successful completed the task in fewer than that many steps. The linearly interpolated curve through the points in Fig. 6 can be viewed as an item characteristic curve (Anastasi, 1990, pp. 219–220) in which the number of steps taken is a test "score" $N$ and the number on the vertical axis is the probability that a child chosen at random from the group took $N$ or fewer steps to complete the task. In this sense the number of steps taken to complete the task is an *inverse* measure of ability on this particular task. With this interpretation the 50% threshold is approximately 3.5 steps. Were we able to take this collection of students as a random sample, representative of children in the 4–6 years age range, then we would expect approximately 60% of children of this age who completed the task to do so in 4 or fewer steps. As we have said in 2.4, 61% of our sample of pre-school children did solve the re-distribution problem so, if this sample were representative of this age group, we could expect approximately $\frac{1}{3}$ of children of this age to solve the problem in 4 steps or less.

## 2.7. *Re-distribution by Dealing*

Interestingly, 3 of the 9 good counters solved the re-distribution problem by reducing it to a known problem. Take the 6 crackers from doll 1 and from doll 2, and then deal out the 12 crackers by 1's. Only 1 other child – a developing counter – adopted this strategy. One could imagine that good counters could use their skills to solve the problem by counting, but there is no evidence that they did.

## 2.8. *The Most Common Strategies*

The strategies adopted by 3 or more children were:

- Give 2 crackers from doll 1 to doll 3, and then 2 from doll 2 to doll 3 (3 poor counters, 3 developing counters, 1 good counter, and 1 unknown due to a missing interview).
- Take the 6 crackers from doll 1 and from doll 2. Deal out the 12 crackers by 1's (1 developing counter, 3 good counters).
- Give 3 crackers from doll 1 to doll 3. Then take 3 crackers from doll 2 and
  - (a) give them to doll 3. Then give 1 cracker from doll 3 to doll 1, and 1 cracker from doll 3 to doll 2 (2 poor counters, 2 developing counters)
  - (b) place them in a pile in the middle. Then deal out the pile to the three dolls (2 poor counters, 1 developing counter, 1 good counter).
- Give 1 cracker from doll 1 to doll 3, and 1 cracker from doll 2 to doll 3. Then repeat this action (2 poor counters, 1 good counter).

These 5 strategies were adopted by a total of 45% of the children who were successful at solving the re-distribution problem. Apart from 2 other strategies (one of which was to crunch the crackers together – a strategy that we reluctantly agreed did solve the problem) the remaining were adopted by only one child, giving a total of 19 strategies adopted by a single individual. This evident and considerable *diversity* of response to the re-distribution task indicates to us that it was indeed a problematic situation for the children involved, and that most, if not all, of the children who successfully solved the problem were using untutored methods.

## 2.9. *Counting in the Re-distribution Task*

Although we have not indicated it in the description of the strategies there were 13 instances of counting during, or immediately after, successful completion of the re-distribution problem. Only 4 of these instances of counting appeared during the problem solution, and the other 9 instances appeared at the end of the solution. Notably, however, *every one* of the 4 children who used the strategy – "*Take the 6 crackers from doll 1 and from doll 2. Deal out the 12 crackers by 1's*" – counted each doll's allocation upon completion of the task. The children who counted upon completion of, but not during, their successful solution of the re-distribution problem seemed to be acting in accord with the point of view expressed by the 8 year old child Tom cited above, (Davis, 1990), who said:

*Interviewer*: "Do you have to count, to know if they are fair shares?"
*Tom*: "Well if you know what you are doing you probably don't, but if ah, ya sh . . . I always count them after, just to make sure, before I put the answer."

For the most common strategy – "*Give 2 crackers from doll 1 to doll 3, and then 2 from doll 2 to doll 3,*" – adopted by 8 children, only 2 counted upon completion of the task.

## 2.10. *The Shortest Strategies*

There were two shortest strategies adopted by the children in solving the re-distribution problem. The first was to give 2 crackers from doll 1 to doll 3, and then 2 from doll 2 to doll 3:

TABLE II

| | Start | Step 1 | Step 2 |
|---|---|---|---|
| Doll 1 | 6 | 4 | 4 |
| Doll 2 | 6 | 6 | 4 |
| Doll 3 | 0 | 2 | 4 |

The second strategy was to crunch all the 12 crackers into crumbs and place crumb piles in front of each doll. As we mentioned in 2.8 above, we reluctantly agreed that this strategy did solve the re-distribution problem, even though the unit nature of the individual crackers was destroyed. We did not, however, include it in the information listed in Figs. 5 and 6. This strategy was used by one poor and one good counter.

A third strategy on the face of it appeared to involve only two steps. This strategy was: "*Spread out all 12 biscuits and, without overt counting or* *dealing, give 4 to each doll.*" This could be interpreted as dealing by 4's and so be summarized as in the table below:

TABLE III

| | Start | Step 1 | Dealing | Step 2 |
|---|---|---|---|---|
| Doll 1 | 6 | 0 | by | 4 |
| Doll 2 | 6 | 0 | fours | 4 |
| Doll 3 | 0 | 0 | | 4 |

However the video record of the one child (a developing counter) who used this strategy indicates that we should not take her actions as parts of a single dealing cycle because she appeared to remove the 6 crackers from each of dolls 1 and 2 in two coordinated sequences of movements, and then place crackers in piles of 4 in coordinated movements. So we see this strategy as consisting of 5 steps, summarized as follows:

TABLE IV

| | Start | Step 1 | Step 2 | Step 3 | Step 4 | Step 5 |
|---|---|---|---|---|---|---|
| Doll 1 | 6 | 0 | 0 | 4 | 4 | 4 |
| Doll 2 | 6 | 6 | 0 | 0 | 4 | 4 |
| Doll 3 | 0 | 0 | 0 | 0 | 0 | 4 |

## 2.11. *Apportioning by Ratios*

The children who solved the re-distribution problem in the following way: "*Give 2 crackers from doll 1 to doll 3, and then 2 from doll 2 to doll 3,*" appeared, to us, to be able to mentally split a pile of 6 discrete objects in the ratio 2:1. If we include in this group the child (a poor counter) who adopted this strategy but also took a cracker from doll 1, counted, and replaced the cracker, we then have 9 children, or 20% of the entire group of successful solvers of the re-distribution task, who used this strategy. Interesting features of this strategy are;

- it was the strategy most commonly adopted by the successful solvers of the re-distribution problem;
- it was one of the three shortest solutions to this problem;

only 1 good counter was observed to use this strategy. The other children who did were 3 poor counters, 3 developing counters, 1 unknown (due to a missing interview), and the 1 poor counter, mentioned above, who used this as a base strategy.

The fact that 4 poor counters and 3 developing counters used this strategy whilst only 1 good counter was observed to do so, suggests that an ability to operate on patterns of objects such as crackers might be a developmental mode that is an alternative, or complementary, to counting. Only one of these children overtly counted during the solution process: this is rather remarkable because there is little evidence cited elsewhere in the literature that suggests children of this age are capable of spontaneously apportioning discrete items in the ratio 2:1, especially without recourse to overt counting.

### 2.12. *Strategies of the Good Counters*

The 8 good counters used a variety of strategies to solve the re-distribution problem. What is notable about the strategies used by the good counters however is that they were, with a single exception, *convergent strategies*. That is, the good counters used strategies which, at each step, moved closer to a solution. The one exception was the following strategy in which a solution is reached at step 4 through a process that diverges at step 3, and in which the child makes two extra moves (one the reverse of the other):

TABLE V

| | Start | Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
|---|---|---|---|---|---|---|---|
| Doll 1 | 6 | 5 | 5 | 3 | 4 | 5 | 4 |
| Doll 2 | 6 | 6 | 4 | 4 | 4 | 4 | 4 |
| Doll 3 | 0 | 1 | 3 | 5 | 4 | 3 | 4 |

### 2.13. *Deviation from a Solution*

There were 5 instances where we observed that children had reached an even distribution of 4 crackers to each doll and then continued their actions until they once again obtained such a distribution, and then stopped. An example is given above in 2.12. It appears therefore that these children did not recognize when they had first reached an even distribution of all the crackers. If we take the adult observer's idea of when a child had reached a solution then the mean number of steps to reach a solution is as follows:

TABLE VI

Mean number of steps required to complete the re-distribution problem

| Poor counters | Developing counters | Good counters | Overall |
|---|---|---|---|
| 4.0 | 4.5 | 3.9 | 4.1 |

The figure for the developing counters is inflated by a strategy that used 14 steps, namely: "*Pick up all the biscuits and re-deal to get 3, 5, and 4 crackers respectively. Compare heights, collect all 12 crackers again and re-deal to get 3, 4, and 5 crackers respectively. Collect all the crackers again and re-deal to get even shares.*" If this strategy is ignored the mean for the remaining 13 developing counters is 3.8.

### 2.14. *Strategies of the Poor Counters*

The poor counters who solved the re-distribution problem were remarkably successful: 15 out of 21 children (71%) classified as poor counters used 4 or fewer steps to knowingly solve the re-distribution problem. When we take into account those who re-adjusted an already correct solution, 18 out of 21 (86%) solved the problem in 4 or fewer steps. This compares with 10 out of 14 (71%, re-adjusted for first step correct) for the developing counters, and 4 out of 8 (50%, re-adjusted for first step correct) for the good counters. When we take into account the percentage of poor, developing, and good counters who did solve the re-distribution problem we get the following figures (after re-adjustment for the first step correct):

TABLE VII

Percentage of children, by counting category, who attempted the re-distribution problem and completed it in 4 or fewer steps (after adjustment for first correct step)

| Poor counters | Developing counters | Good counters |
|---|---|---|
| 42% | 43% | 50% |

This puts the poor counters in a good light compared with the developing and good counters, and suggests to us that the ability to efficiently re-distribute 12 crackers to 3 dolls after an initial distribution of 12 crackers to 2 dolls, is a pre-numeric ability that develops in parallel with the ability to count.

## 3. DISCUSSION

We began this study by asking whether children with demonstrable counting competence are more able to solve a discrete re-distribution problem. Our guarded answer is that they are, in the sense that all good counters could solve the problem. However, both developing and poor counters who can solve the problem often do so as efficiently as, or sometimes more efficiently than, the good counters. When we take into account the overall population in the study – not just those children who solved the problem – and also take into account the first time an adult observer noticed that a child had a solution to the problem, then little difference is evidenced between poor, developing, or good counters who could solve the problem in 4 or fewer steps.

We found a considerable diversity of solution strategies – more than one might expect in more advanced mathematical problem solving – and this probably points to the un-learned nature of the children's responses: in other words to the fact that the re-distribution task was a genuine non-routine problem for them. Already, even with only 3 classes of pre-school children we can see with such a simple task how their minds appear to be working in problem solving activities. On the other hand the most common strategy adopted by these children suggests to us that many children at this age are capable of apportioning discrete items in the ratio 2:1.

As we remarked in section 2.2 above, there is evidence that *all* good counters of age 4–5 years can perform the re-distribution problem with 12 crackers to 3 dolls, yet many children of this age who can perform the re-distribution problem are still only counters of perceptual unit items. It seems to us therefore that the dealing re-adjustment tasks are genuinely *pre-numeric* problem solving tasks in that they are capable of being solved efficiently by children who are generally not good counters.

The relation between counting ability and an ability to solve discrete re-distribution problems needs to be detailed carefully, as does the ability of such young children to establish mental images of stacks of discrete items in a given ratio. We still do not know the salient factors in the development of the dealing scheme for individual children. Given its role as an action basis for the establishment of one-one correspondences, this is an important issue to clarify. It does not seem to us that a larger scale statistical study of pre-school children's strategies for solving re-distribution problems will provide a great deal of extra information. Rather, we feel, it is important – certainly as important as statistical studies – to follow a few children over a year or so and track the development of their sharing and counting competencies. It would be particularly interesting, we believe, if the ability to solve discrete re-distribution tasks developed alongside a child's increasing ability to count. The idea that children are capable of measuring parts of a stack of discrete items in the ratio 2:1, particularly children who have difficulty finding the total of – for example – five animals and four animals, needs to be investigated more carefully in a variety of settings.

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