

REFLECTIONS ON DEALING:  
AN ANALYSIS OF ONE CHILD'S INTERPRETATIONS

Gary Davis, The Institute of Mathematics Education,  
La Trobe University, Melbourne

*We detail the responses of a grade 2 child who was shown video episodes of preschool children engaged in sharing activities, and was asked why the preschoolers counted or made height checks after dealing. The grade 2 child's responses indicate a keen awareness of the logical completeness of dealing to establish fair shares, and an acute sense of the need to check. We discuss this child's responses in the context of language as a vehicle for heightened awareness of mathematical procedures.*

The procedure of dealing out a collection of discrete objects in order to share them evenly is interesting, because although a majority of pre-school and primary school children can, and do, use it in response to a sharing question (Clements and Lean (1988), Hunting and Sharpley (1988), Miller (1984)) it appears that few of them express a belief that dealing is actually sufficient to establish a fair share. Instead, young children of this age mainly refer to counting or height or length checking of piles of discrete items to check for equality of shares (Davis and Pitkethly, 1990). Children between the ages of 5 and 8 who can express the sufficiency of dealing to establish fair shares seem to be relatively rare. In a study of preschoolers, Hunting and Davis (1989) found only one child in 85 who expressed the sufficiency of dealing to establish fair shares, and in a study of 17 grade 2 children, Davis and Pitkethly (1990), observed only one such child.

In the Davis and Pitkethly (1990) study, edited highlights of videotapes of preschoolers involved in sharing activities were compiled. The edited tape consisted of three five-year old children, one boy and two girls, performing sharing tasks with 12 biscuits. Each child was presented with two dolls and asked to share all the biscuits evenly between the two dolls. Then the interviewer presented a third doll just before the other dolls were supposed to eat the biscuits, and the child was asked to share all the biscuits evenly between the three dolls. These video-taped episodes were shown to 17 grade 2 children in small groups, and the grade 2 children were questioned for their views why the preschoolers counted or measured after dealing. There was a specific list of questions that were asked of the children in each group. The questions were asked of the group as a whole and then of each child who had not already answered. The major conclusion of that study was that the grade 2 children interviewed, with a notable exception, expressed almost no awareness that dealing by itself is sufficient to establish fair shares. Rather they saw it as necessary to count, or to do height or length comparisons of shared items.

This article is about the responses of a particular eight year old child, Tom, who gave a clear statement of the sufficiency of dealing in the Davis and Piethly (1990) study. I have selected Tom and analysed his responses to questions in detail because, in the context of the interviews, he demonstrated an unusual depth of access to procedural matters and to the applicability of different procedures. The purpose of the analysis of Tom's responses is to indicate the depth to which a particular eight year old child was capable of expressing the logic inherent in the dealing strategy, and to examine the reasons why he could attain such depth of expression.

Tom thought about and reported on the procedures used by the younger children. He was reporting not only what they did but what he thought they were thinking when they did it. In this sense he was reflecting on the cognitive activity of other children, but at the same time providing evidence of his cognitive activity.

Tom's responses are important not simply because he appears to be an exceptional child. They are important because he expresses a clear awareness of his and others procedures without the use of explicit symbolism. Steffe (1988) says that "Children's operations seem to be primarily outside their awareness, and, without the use of symbols, they have little chance of becoming aware of them nor can they elaborate those operations beyond their primitive forms." I think that Tom, and the methodology we used with him, provides us with another way in which awareness of operations and procedures can be made explicit, and can potentially lead to an elaboration of those operations and procedures.

Tom's answers seemed to reflect his thought processes: it is as if one could see things clicking over in his head as he digested what was asked of him. His hesitations and restatements of the questions indicate an attempt on his part to understand fully what it was the interviewer was asking. It seems that he was able to put himself in another person's position and to imagine what they were thinking; indeed what they were capable of thinking. Furthermore Tom himself had a predominantly pictorial way of thinking: it seemed most times that he was simply reporting on something that he could literally see in his mind. He communicated in a very vivid way and he constantly used his hands to help communicate his thoughts.

Tom's responses were of a different character, both qualitatively and quantitatively, to those of the other children in the study. There were 28 situations in which statements or questions were put to Tom. They elicited 57 responses from him that were judged to be related in some way to aspects of the dealing procedure or more general procedures. This number was more than

twice as many responses per question as that given by the next most responsive child, Davis and Pitkethly (1990).

Tom seemed to have a clear understanding of the sufficiency of dealing to establish fair shares: his opening statement below that uses a card analogy establishes this, and he did not express a contrary point of view throughout the interview. He is aware of human error and the need to check because he himself sometimes makes mistakes: the 9 statements he made concerning possible or actual errors in the application of procedures supports this conclusion. What is also interesting is the number of statements of Tom's that are inferences about thought processes, or logical aspects of procedures.

These responses indicate a high level of intellectual activity on Tom's part. He appears to be observant, aware of human error, and has conceptual models of how people think. Some of Tom's statements about logical aspects of procedures actually refer to definitions. It is remarkable that a child who was just 8 years of age was capable of expressing a definition succinctly so that it would be accepted by most reflective adults. Other statements of Tom's that related to logical aspects of procedures dealt clearly with the underlying reason that the procedure was carried out as it was. The other remarkable point was the number of inferences about the thought processes of the video-taped preschoolers: Tom made 11 statements of this nature.

More importantly however, Tom expressed an understanding of the logical aspects of the dealing procedure without using symbolism. Of course it is a debatable point whether pictures that one sees in one's head are symbols in this sense, but I will take it that they are not: a symbol indicates a written mark that is used consistently as a sign. Tom, and children like him, are counter examples to the notion that without the use of symbols they have little chance of becoming aware of their procedures and operations. We see in Tom's statements a clear understanding of the nature of dealing, of its different applications, its sufficiency to establish fair shares, and the desirability - but not necessity - of a check because of the possibility of error.

Whether, without symbolism, Tom could elaborate the dealing procedure beyond a primitive form into the concept of fractions, for example, is a moot point: there is no evidence either way.

What appeared to allow Tom to be so clear in his statements about dealing was a primarily vivid pictorial manner of thinking. The video tape record shows, and the transcript below indicates, a child whose speech was hesitant and who often backtracked, because he appeared to be scanning images in his mind. Tom used his hands in a vivid and suggestive way as he talked and appeared to be relating pictures of imagined events. This, of course, does not explain his considerable and unusual awareness of the facets of the dealing procedure, but it does allow

that something other than symbols allowed him to talk about these facets with considerable accuracy. In fact Tom used spoken language to express his awareness of the dealing procedure. He was an articulate child who was capable of searching for and successfully finding words to describe his thoughts, which appeared to have a large pictorial component.

In my interpretation of Tom's ability to have, and to express, an awareness of logical aspects of a particular procedure, there are implications for learning via communication in primary mathematics classrooms. One way to allow students to progress to an awareness of operations and procedures in mathematics is to enhance their ability to use written symbols. Another approach that is suggested by Tom's responses is to engage the children in dialogue in order to give them an opportunity to develop an awareness of the logic inherent in a given mathematics episode. This awareness will not normally develop in the course of a single interview episode (Davis and Pitkethly, 1990). On-going discussion and reflection would appear to be required to allow children in the early grades of school to obtain the clarity of vision and expression shown by Tom. What is important is that if Tom can use words to express an awareness of logical aspects of a procedure there seems to be no reason why other children, through extended and directed dialogue, could not attain a similar awareness of this and other procedures and operations.

It is easy in mathematics classes to overestimate the power of symbolism and to underestimate the power of visual images that are normally dealt with by oral or written language. It may be that for many children in their later years of mathematical experiences these two aspects of mathematical awareness and thought are in opposition, but there is no reason that they should be. Pre-school and the early grades of school are where the process of dialogue and discussion in mathematics can and should begin.

#### COMMENTARY ON A SELECTION OF TOM'S ANSWERS

The following is a transcription of a selection of Tom's remarks and the questions and comments which prompted them, together with commentary on the answers. Anne and Gary are the interviewers and Alexis is another grade 2 child - one of the three children interviewed at this time.

##### Excerpt 1

The preschool boy in the video episodes gave one biscuit to each doll and stopped. He then continued to give out one to each, until all biscuits were used, after the interviewer indicated that all the biscuits should be shared.

Anne : " Do the dolls have the same? Tell me why."

This question was asked of the three children present. Tom's answer to the question is revealing. It indicated that he understood the dealing procedure to be

sufficient to ensure equality of shares, and then related it to another familiar example. Alexis then suggested that one can deal by twos and Tom explained that there is no essential difference, just that it is faster.

Tom: " The child put out the biscuits. They give one to one doll and then another to the other doll and they kept on doing it until there was no biscuits left So that the dolls both had the same. Like when you're playing cards and you want to get the equal number of cards . Sometimes. So you say, one for me and one for you and so on and until all the cards have filled up. And ya sometimes put some of the cards out in the middle. "

Alexis: " You could do it by twos."

Tom: " Like you pick up two biscuits. You go two, two, two. Its just a bit faster way of doing it."

Tom's replies to the next three question show that he did not think it was necessary to count after dealing to establish fair shares, but that he was aware of human error, including his own.

### Excerpt 2

The preschool boy counted the number of biscuits and correctly stated that there were 6 in each stack.

Anne: " Do you have to count, to know if they are fair shares?"

Tom: " Well if you know what you are doing you probably don't, but if ah, ya sh... I always count them after, just to make sure, before I put the answer."

Anne: " Did he know the dolls had a fair share even before he counted?"

Tom: " Yeh, I think so. I thought he was going to count them in the next thing ."

Anne: " You thought it was logical that he would have actually counted them at some stage. Did he know they had fair shares before he even counted them, do you think?"

Tom: " Well, if he'd done it quite a lot he would probably know it was alright , but since he was only in Kindergarten he'd probably count them, just to make sure."

### Excerpt 3

The preschool boy in the video episodes picked up three biscuits at a time from each pile and stacked them in front of the corresponding doll. He then picked up two biscuits from Joey's stack and placed one on each of the other dolls' stacks. He correctly counted the number of biscuits in Joey's stack. He moved to another stack and continued counting on from 4 until he reached 6. He then started again with Joey's stack and counted all stacks correctly to conclude that each doll had 4 biscuits.

Tom is then asked why the boy counted. His answer indicates, as before, that counting is not a part of the dealing process but it is a sensible check:

Anne : " So he did count, didn't he, and you watched how he shared them out and then he counted them. Why did he count?"

Tom : " Ah, because he probably counted and ah, the same as he..the same reason he did last time, and also because it was a little bit harder. So he counted just to make sure that he'd put the right amount of biscuits in each pile."

Anne : " Did he do it the way you suggested?" (i.e. deal from one pile completely then the other).

Tom : " He did it kind of, he did it.. he did it the same except he didn't put all the piles together. He kind of shared them. He kind of looked at this..looked at both the piles and he ah..he saw how many was on that pile and how many was on that pile and he shared them around ah.. to Joey and then... then he looked at the piles again. If they weren't fair he kept sharing them until they were fair and then he checked them with a knife."

#### Excerpt 4

The first preschool girl in the video episodes gave one biscuit to each doll and asked if she had to give out all the biscuits. She did so, one biscuit at a time to each doll in turn, and ended with a stack in front of each doll. In this excerpt Tom has seen the girl deal out the 12 biscuits.

Anne : " Do the dolls have the same?"

Tom : " Well probably ... since she used the right method of counting, ah, sharing them out, she'd have probably got them right."

Tom's answer was somewhat curious. He had seen the girl deal the biscuits, and he had previously stated that dealing was sufficient to ensure even shares. Yet he said "she'd have probably got them right". My interpretation of Tom's remarks here is that he has placed himself in the position of the girl in the video episode, and he is reporting what he thought she was feeling. This interpretation is strengthened by Tom's answer to the next question:

Anne : " How do you think she knows the dolls have got the same? She hasn't counted them, has she? How could she tell?"

Tom : " Well she doesn't... probably she doesn't really know that, she doesn't, she's not absolutely sure yet. And she'll probably count them."

#### Excerpt 5

The first girl in the video episodes then checked the height of the two stacks of biscuits she had made. Tom was asked if that is reasonable, and he gave a wonderful image for his answer. In so doing he tells us in precise detail that a height check is indeed one way to check for fairness:

Anne : " She looked to see if they were the same height. Now, does that seem reasonable to you Tom?"

Tom : " Yes. It seems alright because since they were the same size biscuits and the same length, if she puts them together and they're, they're, one's higher and

one's lower, it wouldn't be fair and if one, if one's lower and one's higher then it wouldn't be fair. But if they're flat across the top, if you can ah... if you can walk... if you were very little you can walk across the top without walking up a step or anything then it will be fair."

Tom was then asked about the relative merits of height checks and counting, and whether a height check is something he himself would have done:

**Anne** : " So that's another, is that another way to tell if you've got fair shares?"

**Tom** : " Yes."

**Anne** : " Is it perhaps as reliable as counting?"

**Tom** : " Well probably about the same ."

**Anne** : " About the same."

**Tom** : " It's.. it.., counting would be a better idea if the things were.., counting would be a better idea if the things weren't the same shape and size but if they're all the same like the biscuits were, it's alright to put them together."

**Anne** : "But it wasn't an idea that came to you until you saw the little girl do it, was it? Was it something you would do yourself? Have you got a preference?"

**Tom** : " I ... if things were different shapes I would normally count them but if they're the same I would normally count them, or I would put them together "

This is a revealing answer. If we take Tom at his word then he would count the biscuits after dealing or else do a height comparison. This is despite the fact that he has stated that dealing is sufficient to establish fair shares. His previous statements about possible errors give us a clue that he is not satisfied until he has checked by counting or a height comparison. This may well be the same for children other than Tom: the difference that appeared in the Davis and Pitkethly (1990) study is that most children did not express an awareness that dealing is by itself sufficient to establish fair shares. To the contrary the other children in that study were adamant that counting or some other check was essential for the establishment of fair shares.

#### Excerpt 6

The second girl in the video episodes gave out biscuits to the two dolls, in turn: 2 biscuits, 2 biscuits, 1 biscuit, and 1 biscuit. She did not place them in a stack and she did not overtly/count the biscuits, and insisted both dolls had "the same".

Tom was asked why the girl did not stack the biscuits like the other two children. His answer indicated that she could count the biscuits, as did the others, but with the biscuits in a line rather than stacked. He was not referring to counting out the biscuits but rather to counting them after they had been shared:

**Anne** : " Why didn't she stack them? The others stacked them, she didn't. Is there some reason why she didn't stack them?"