Making Sense of Odds and Odds Ratios

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Odds and odds ratios are hard for many clinicians to understand. Odds are the probability of an event occurring divided by the probability of the event not occurring. An odds ratio is the odds of the event in one group, for example, those exposed to a drug, divided by the odds in another group not exposed. Odds ratios always exaggerate the true relative risk to some degree. When the probability of the disease is low (for example, less than 10%), the odds ratio approximates the true relative risk. As the event becomes more common, the exaggeration grows, and the odds ratio no longer is a useful proxy for the relative risk. Although the odds ratio is always a valid measure of association, it is not always a good substitute for the relative risk. Because of the difficulty in understanding odds ratios, their use should probably be limited to case-control studies and logistic regression, for which odds ratios are the proper measures of association.
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Despite their growing use in the medical literature,[1,2] odds ratios remain poorly understood by clinicians (and by some researchers, as well).[2] Because of confusion about odds and odds ratios, we will provide a brief overview of these terms. Our primary goal is to help busy clinicians interpret research reports that use these unfamiliar terms. Relying on simple examples from weather forecasts, tubal sterilization, and a deck of playing cards, we will explain odds and odds ratios and show their relationship to the better-understood probability and relative risk.

For most clinicians, odds ratios will remain . . . well, odd.

BASIC TERMS

Probability

Probability is the proportion (ie, percentage) of times an event would occur if an observation were repeated many times. Probabilities range from 0.0 to 1.0. For example, the weather forecast today, a typical day, might call for a probability of rain of 10%, or 0.10. A medical example would be that the cumulative 10-year probability of pregnancy after tubal sterilization (for all methods combined) in the U.S. Collaborative Review of Sterilization study was 19 per 1,000 women or 0.019.[3]

Odds

In contrast to probability, odds are not intuitive—except perhaps to gamblers. Odds are simply a different expression of the probability: the probability of an event divided by the probability of the event not happening. Thus, the odds of an event would be probability/(1–probability). Because this is a ratio, its values range from zero to infinity.

In the weather example above, the probability of rain today is 0.10. Hence, the odds of rain today would be 0.10/(1–0.10) or 0.10/0.90=0.11. Here, the odds of rain are 1/9 (one chance "yes" to 9 chances "no"). When the chance of rain is low, the probability (0.10) and odds (0.11) of rain are similar, and one could reasonably use these terms interchangeably.

But what if a nor'easter is approaching? In this situation, probability and odds of rain diverge. Assume that, with a major storm blowing in, the weather forecast calls for a 90% probability of rain (ie, 0.90). The odds of rain would be 0.90/(1–0.90) or 0.90/0.10=9.0. With the storm approaching, the odds of rain are 9/1 (nine chances "yes" to one chance "no"). The probability of rain is 0.90, while the odds of rain are 9.0, a tenfold difference. (Fortunately, the conversion between probability and odds is simple [see the Box, "Converting Probability and Odds"]).

Relative Risk

Both clinicians and patients readily understand relative risk. It is simply a

> **Converting Probability and Odds**
>
> **Probability to Odds**
>
> The formula for converting from probability to odds is
>
> *probability/(1–probability)= odds*
>
> **Example**
>
> With a probability of 0.30, the odds would be calculated as
>
> 0.30/(1–0.30)=0.30/0.70=0.43
>
> **Odds to Probability**
>
> In the reverse direction, the formula for converting odds to probability is
>
> *odds/(odds+ 1)=probability*
>
> **Example**
>
> With an odds of 4, the probability would be calculated as
>
> 4/(4+1)=4/5=0.80

ratio of probabilities. For example, in a cohort study, the relative risk would be the probability of the outcome in those exposed divided by the probability of the outcome in those not exposed. Because this expression is also a ratio, its values range from zero to infinity.

The tubal sterilization study above provides some examples. The probability of pregnancy after sterilization with the Hulka spring clip was 37 per 1,000 women, or 0.037. The comparable figure for postpartum partial salpingectomy was 7.5 per 1,000 women, or 0.0075.[3] Hence, the relative risk of pregnancy with the Hulka clip compared with postpartum salpingectomy was (0.037 per 1,000)/(0.0075 per 1,000)=4.9. The risk of pregnancy was nearly five times higher with this clip than with the postpartum operations. Relative risks always relate to some benchmark (sometimes called the "referent"), here, postpartum partial salpingectomy. Relative risks will vary, depending on the referent group used.

Odds Ratio

Odds ratios are more difficult to understand than are relative risks. As the name implies, an odds ratio is the odds of the outcome in one group divided by the odds of the outcome in the other group (analogous to relative risk). As a ratio, it ranges from zero to infinity. The odds ratios formula is more cumbersome, so a few abbreviations may help. Here, p_1 refers to the probability of the outcome in group 1, and p_2 is the probability of the outcome in group 2.

$$\text{Odds ratio}=\frac{p_1/(1-p_1)}{p_2/(1-p_2)}$$

In the sterilization example, the relative risk of pregnancy with the Hulka spring clip was 4.9 compared with postpartum partial salpingectomy. The odds ratio of pregnancy with the Hulka clip compared with postpartum partial salpingectomy would be 5.1, similar to the relative risk. Because the frequency of pregnancy was low with both methods (the "rare disease assumption"), the odds ratio and relative risk are close, and the two can be used interchangeably. The odds ratio and relative risk are similar when the outcome is uncommon; the odds ratio exaggerates the relative risk when the outcome is common.

In the weather example above, the odds of rain with a nor'easter approaching are 9.0, whereas the odds on an typical day are 0.11. Thus, the odds ratio of rain with an approaching nor'easter compared with a usual day is 9.0/0.11=82. A weather forecaster might excitedly proclaim on the evening news that rain is 82 times more likely with the impending storm compared with an average day! Not so: 82 times a 10% chance of rain (820%) is nonsense,[4] since the probability of rain cannot exceed 100%. The relative risk of rain in this situation is lower: probability with nor'easter/probability on average day=0.90/0.10=9.0. Thus, the likelihood of rain with a storm approaching is nine times higher than on an average day (relative risk), not 82 times higher (the odds ratio). Since the chance of rain is high, the odds ratio greatly exaggerates the relative risk; hence, the two terms cannot be used interchangeably.

This same exaggeration occurs when the odds ratio is inappropriately used as a proxy for the relative risk in clinical research.[5] For example, a study of the familial recurrence of dystocia reported (based on an odds ratio of 24) that "the risk is increased more than 20-fold . . . ." Since the baseline risk was 11%, a 20-fold increase in 11% is impossible because the probability of dystocia cannot exceed 100%.[2]

EXAMPLES WITH CARDS

We will next illustrate these four terms in sequence, using a deck of 52 playing cards (jokers removed).[6]

Probability

What is the probability of drawing a diamond card from a randomly shuffled deck? Since the deck contains 13 diamond cards out of a total of 52, the probability is 13/52=¼=0.25. Thus, with many repetitions (and with replacement of the drawn card each time), the probability of drawing a diamond will be 0.25.

Odds

What are the odds of drawing a diamond card from the same shuffled deck? The odds would be the probability of drawing a diamond card divided by the probability of drawing a card of another suit. Here, this would be 13/(52–13)=13/39=1/3=0.33. In other words, one can expect to draw one diamond for every three cards of another suit. The odds (0.33) exaggerate the probability (0.25) of drawing a diamond because the event is not uncommon (the probability of diamonds is 25%).

Relative Risk

What is the relative risk of drawing a diamond card compared with drawing the ace of spades? The probability of a diamond is 0.25; the probability of drawing the ace of spades is 1/52, or 0.019. Hence, the relative risk of drawing a diamond compared with drawing the ace of spades is 0.25/0.019=13. Thus, a diamond card is 13 times more likely than the ace of spades (the referent).

Odds Ratio

What would be the odds ratio of drawing a diamond compared with drawing the ace of spades? As calculated above, the odds of drawing a diamond are 0.33. The odds of drawing the ace of spades are 1/(52–1)=1/51=0.020. So the odds ratio would be 0.33/0.020=17. Once again, the odds ratio of drawing a diamond compared with the ace of spades (17) is higher than the relative risk (13), so the two terms cannot be used interchangeably.

PITFALLS OF ODDS RATIOS

Odds ratios have important drawbacks.[7] First, clinicians think in probabilities, not odds. Second, as noted above, odds ratios exaggerate the effect size compared with a relative risk, especially for common outcomes. If the odds ratio is greater than 1.0, it is larger than the relative risk. Conversely, if the odds ratio is less than 1.0, it is smaller than the relative risk.[8,9] This discrepancy becomes clinically important only when the baseline probabilities of the outcome exceed 0.10 to 0.20.

However, authors sometimes use the odds ratio as a proxy for the relative risk even when the disease is not rare, which is inappropriate and can lead to misinterpretation. A study of odds ratio use in two obstetrics and gynecology journals revealed that, in 44% of 107 articles, the odds ratio exaggerated the relative risk by more than 20%.[2] Other investigators have mistaken the odds ratio for the relative risk, leading to the claim that a twofold difference in cesarean delivery frequency was a threefold difference.[10]

WHY BOTHER WITH ODDS RATIOS?

Given these inherent problems with odds ratios, why use them at all? Odds ratios have several key roles to play. Although they are always a valid measure of association, they are not always a good substitute for relative risk. First, odds ratios are the appropriate measure of relative effect in case-control studies.[11] In these studies, researchers have a numerator (cases) but no denominator, so rates and relative risks cannot be determined. Instead, investigators compare the frequency of exposure among cases with the frequency of exposure among the controls. In case-control studies, the odds ratio is a good proxy for the true relative risk when the "rare disease assumption" is met. Examples of rare conditions include ovarian cancer, systemic lupus erythematosus, and uterine rupture; these have all been investigated with case-control studies.

Second, odds ratios are commonly used in meta-analysis, which aggregates research studies to increase the power to find differences. With frequent outcomes, relative risks are constrained.[12] For example, assume that baseline fertility is 0.85. What would be the relative risk and odds ratio if a new treatment boosted fertility to 0.99? The relative risk of fertility with the new treatment compared with baseline would be 0.99/0.85, or 1.2. Thus, the new treatment increased the fertility rate by only 20%. The highest possible increase in fertility rate can lead to only a small increase in the relative risk because the baseline rate (the referent) is so high.[12]

In epidemiology, relative risks in the range of 1.2 to 1.5 ("weak associations") are difficult to interpret; bias can easily account for them.[13] In contrast, the odds ratio of the fertility treatment in this example would be 99/5.7, or 17. The relative risk of 1.2 is not much different from 1.0 (no effect), in contrast to the odds ratio of 17. Here, the wide range of possible odds ratios is more informative than the narrow possible range of relative risks.

Third, odds ratios are the output of logistic regression, a technique often used to control for confounding bias in research analysis.[14] Adjustment of odds ratios in logistic regression analysis is widely available through software packages and is easily understood. Properly addressing confounding in research usually trumps concerns regarding odds ratios.

SUMMING UP

Odds ratios are hard for many clinicians to understand and susceptible to misinterpretation.[2] Under the "rare disease assumption," when the probability of an outcome is less than 0.10 or 0.20, the odds ratio and relative risk can be used interchangeably. This is not true when the probability is higher.

Methods are available to approximate the relative risk from an adjusted odds ratio.[9,15] This may provide more reader-friendly articles.[2] The formula proposed by

Zhang and Yu[15] is useful:

$$RR = \frac{OR}{[(1 - P_0) + (P_0 \times OR)]}$$

where P_0 is the proportion of those unexposed who develop the outcome, OR is the odds ratio, and RR is the relative risk estimated from the odds ratio. As the background rate of the outcome gets low (ie, P_0 approaches zero in the "rare disease assumption"), the denominator in brackets approaches 1.0, and the relative risk approaches the odds ratio.

The safest strategy, however, may be to limit the use of odds ratios to case-control studies and logistic regression (Deeks J. When can odds ratios mislead? [letter] BMJ 1998; 317:1155). We suggest that, in the interest of better understanding, researchers should report results as probabilities and relative risks (with confidence intervals)[16] whenever possible.[6] For most clinicians, odds ratios will remain . . . well, odd.[2]

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