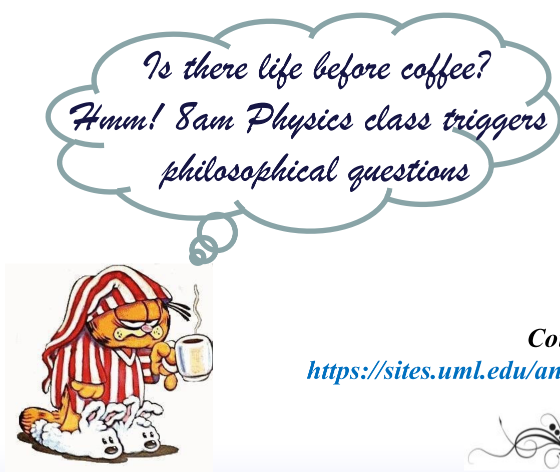
Lecture 11

Physics II

Chapter 26

Capacitors in Series and Parallel



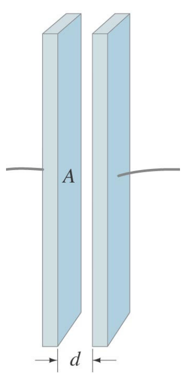
Course website:
https://sites.uml.edu/andriy-danylov/teaching/physics-ii/

Today we are going to discuss:

Chapter 26:

- *Section 26.5*

Parallel-plate capacitor

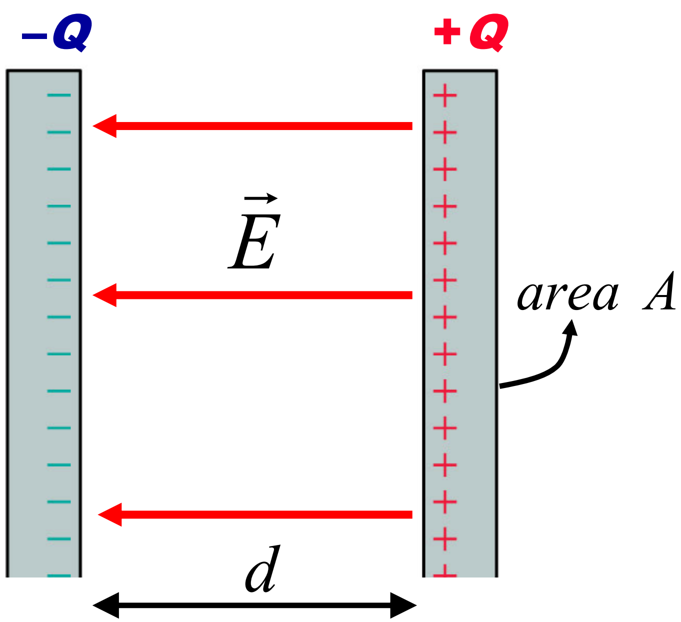


In its simplest form, a capacitor consists of a pair of parallel metal plates separated by air/insulating material.

Parallel-plate capacitor

Let's find capacitance of a parallel-plate capacitor

$$C \stackrel{\text{def}}{=} \frac{Q}{\Delta V_C}$$



We need to find Q and ΔV:

The electric field between the plates is

$$E = \frac{\eta}{\varepsilon_0} = \left\|\eta = \frac{Q}{A}\right\| = \frac{Q}{\varepsilon_0 A}$$

η - surface charge density

$$Q = \varepsilon_0 A E$$

The potential difference between plates:

$$\Delta V_C = Ed \quad \text{(Eq.25.26)}$$

This gives the capacitance: $C \stackrel{\text{def}}{=} \frac{Q}{\Delta V_C} = \frac{\varepsilon_0 AE}{Ed}$ ⇒ $C = \frac{\varepsilon_0 A}{d}$

Capacitance is a purely geometric property of two electrodes because it depends only on their surface area and spacing.

ConcepTest *Varying Capacitance*

A parallel-plate capacitor initially has a voltage of 400 V and stays connected to the battery. If the plate spacing is now doubled, what happens?

A) the voltage decreases

B) the voltage increases

C) the charge decreases

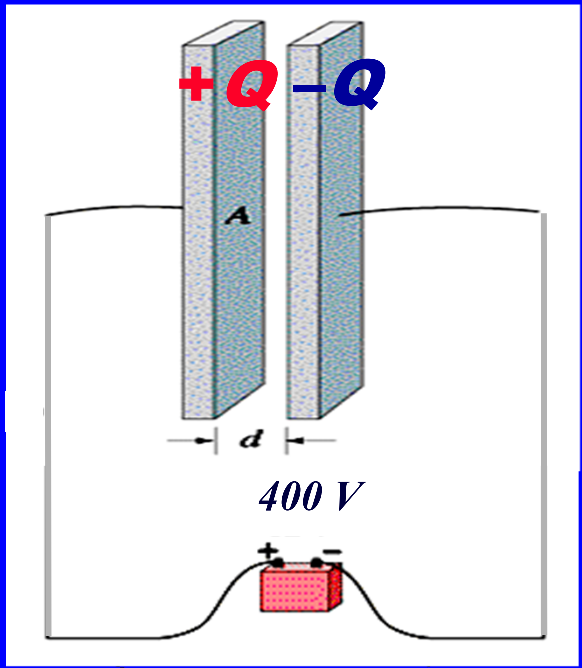
D) the charge increases

E) both voltage and charge change

Since the battery stays connected, the potential difference must remain constant! (A, B and E are out)

Since $C = \frac{\varepsilon_0 A}{d}$, when the spacing d is doubled, the capacitance C is halved.

And since $C \stackrel{\text{def}}{=} \frac{Q}{\Delta V_C} \Rightarrow Q = C\Delta V$, that means the charge must decrease.

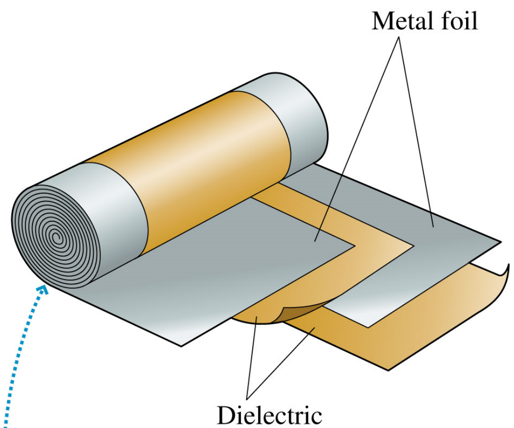


Follow-up: How do you increase the charge?

Parallel-plate capacitor

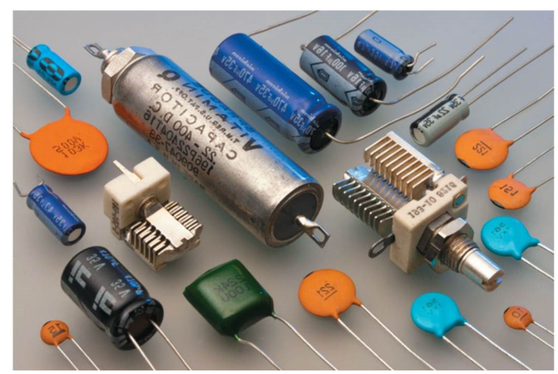
$$C = \frac{\varepsilon_0 A}{d}$$

We can increase capacitance by increasing area A by making "a roll of metal and insulator"

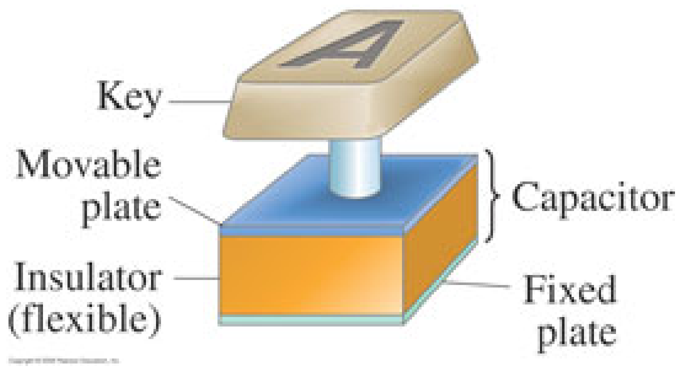


Many real capacitors are a rolled-up sandwich of metal foils and thin, insulating dielectrics.

© 2013 Pearson Education, Inc.



Parallel-plate capacitor/keyboard



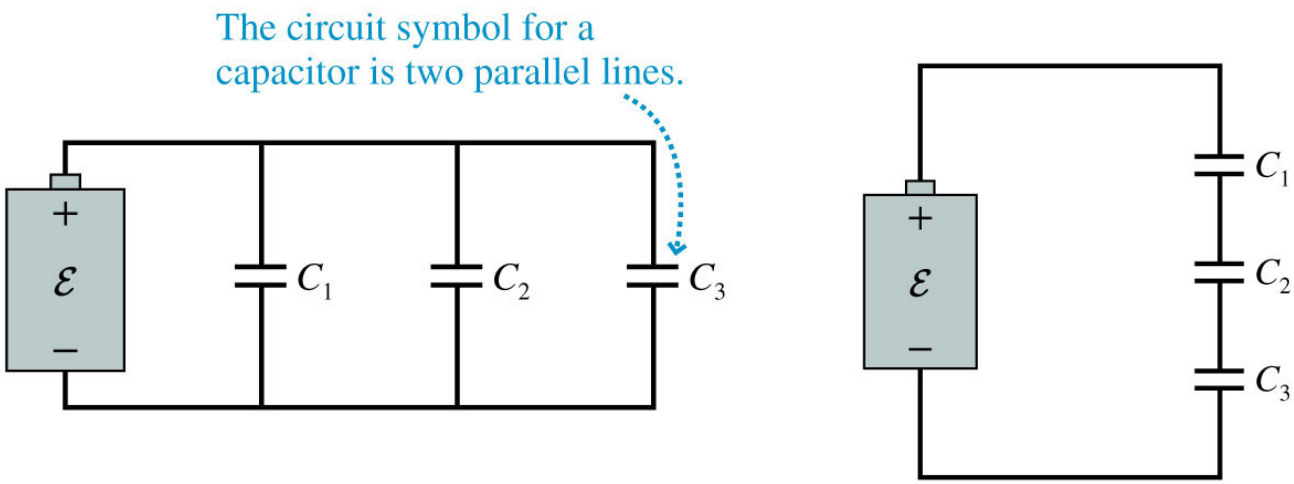
The keys on most computer keyboards are capacitor switches. Pressing the key pushes two capacitor plates closer together, increasing their capacitance.

Combinations of Capacitors

In practice, two or more capacitors are sometimes connected together.

The circuit diagrams below illustrate two basic combinations:

parallel capacitors and series capacitors.



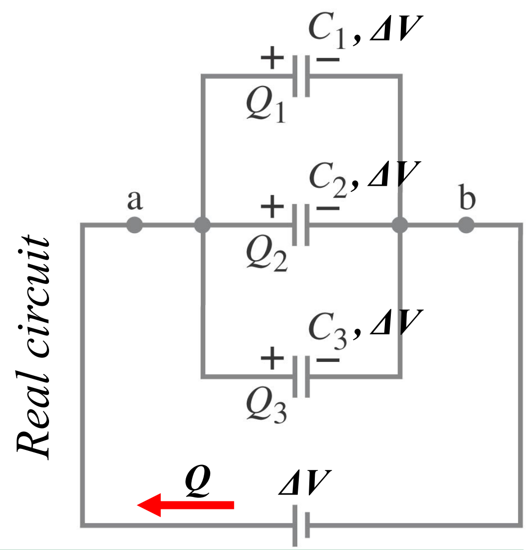
A method of an equivalent capacitor

The equivalent capacitance is the capacitance of the single capacitor that can replace a set of connected capacitors

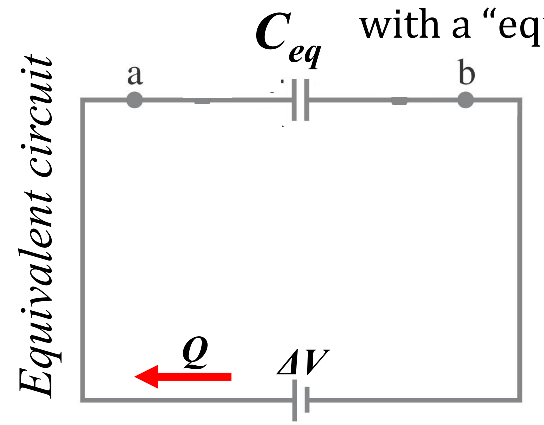
<u>without changing the operation of the circuit</u>

Capacitors in Parallel

Consider three capacitors connected in parallel.



We have replaced 3 capacitors with a "equivalent" capacitor.



Capacitors in parallel have the same potential difference, ΔV

$$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V_{eq}$$

C_{eq} is inserted <u>without changing the operation of the circuit,</u> so Q and ΔV are same as in the real circuit

$$Q = C_{eq}\Delta V$$

Q is a total charge drawn from the battery

Conservation of charge $Q = Q_1 + Q_2 + Q_3$ ⇨ $C_{eq}\Delta V = C_1\Delta V + C_2\Delta V + C_3\Delta V$

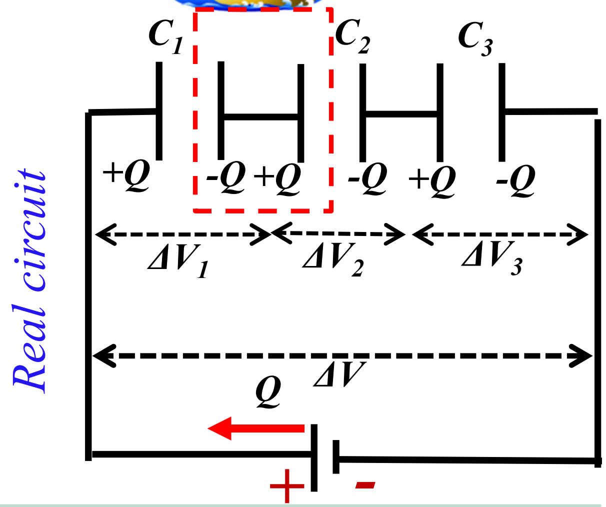
Since $C \stackrel{\text{def}}{=} \dfrac{Q}{\Delta V}$ ⇨ $Q_1 = C_1\Delta V;$ $Q_2 = C_2\Delta V;$ $Q_3 = C_3\Delta V;$

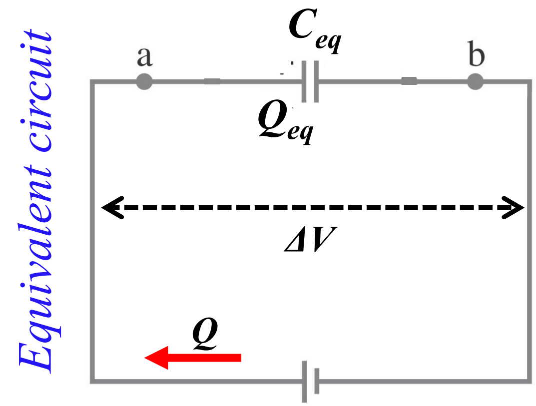
$$C = C_1 + C_2 + C_3$$

Equivalent capacitance of capacitors in parallel.

Capacitors in Series

Consider three capacitors connected in series.





Capacitors in series have the same charge, Q.

$$Q = Q_1 = Q_2 = Q_3 = Q_{eq}$$

C_{eq} is inserted without changing the operation of the circuit, so Q and ΔV are same as in the real circuit

$$\Delta V = Q/C_{eq}$$

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

$$\text{Since } C \overset{\text{def}}{=} \frac{Q}{\Delta V} \Rightarrow \quad \Delta V_1 = Q/C_1 \quad \Delta V_2 = Q/C_2 \quad \Delta V_3 = Q/C_3$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Equivalent capacitance of capacitors in series.

ConcepTest — *Varying Capacitance*

A parallel-plate capacitor initially has a potential difference of 400 V and is then disconnected from the charging battery. If the plate spacing is now doubled, what is the new value of the voltage?

A) 100 V

B) 200 V

C) 400 V

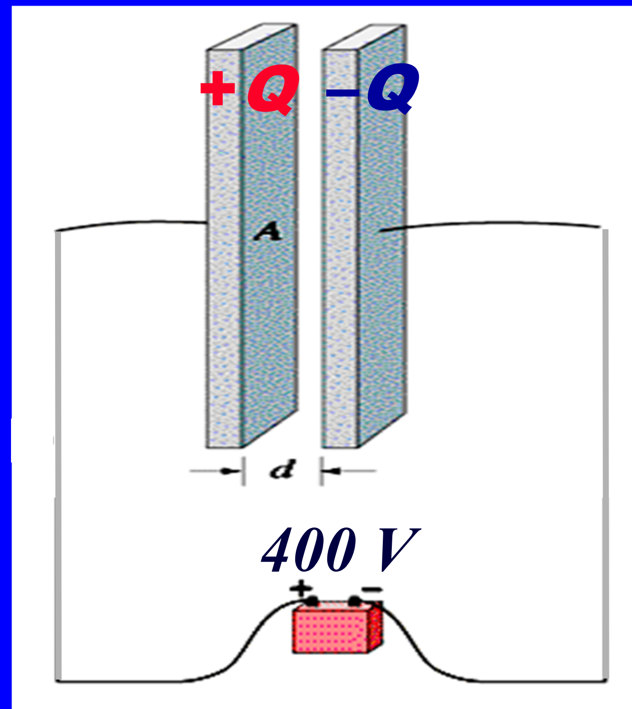
D) 800 V

E) 1600 V

Once the battery is disconnected, Q has to remain constant, since no charge can flow either to or from the battery.

Since $C = \dfrac{\varepsilon_0 A}{d}$, when the spacing d is doubled, the capacitance C is halved.

And since $C \stackrel{\text{def}}{=} \dfrac{Q}{\Delta V_C} \Rightarrow Q = C\Delta V$, that means the voltage must double.



End of Class

Example: Equivalent capacitance

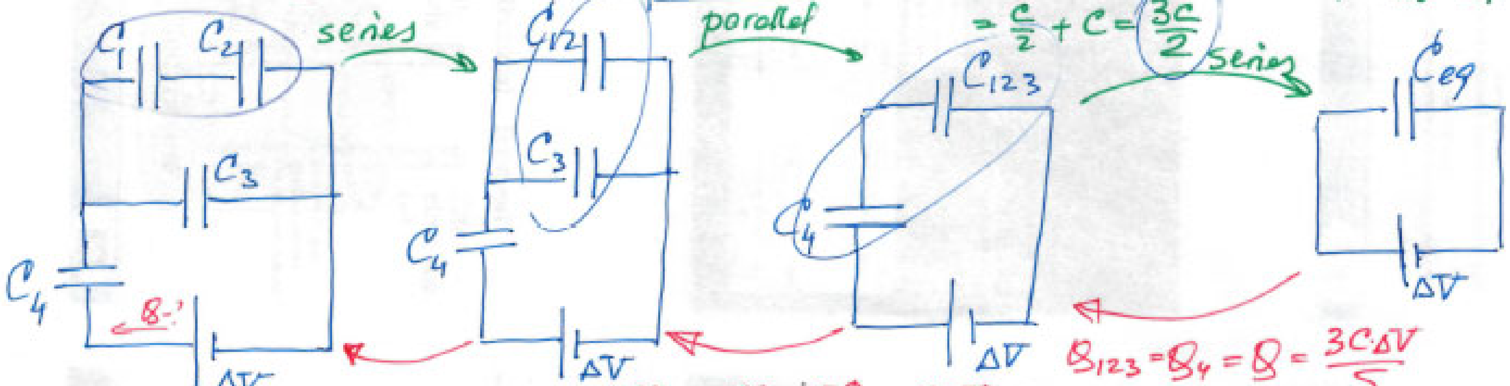
$C_1 = C_2 = C_3 = C_4 = C$, ΔV } given; Find: Analyze the circuit

useful: $C = \dfrac{Q}{\Delta V}$

$\dfrac{1}{C_{12}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{C} + \dfrac{1}{C} \Rightarrow C_{12} = \dfrac{C}{2}$

$C_{123} = C_{12} + C_3 = \dfrac{C}{2} + C = \dfrac{3C}{2}$

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_{123}} + \dfrac{1}{C_4} \Rightarrow C_{eq} = \dfrac{3C}{5}$



$Q = C_{eq} \cdot \Delta V = \dfrac{3C\Delta V}{5}$

– the goal of this simplification to find this total charge taken from the battery.

$Q_{123} = Q_4 = Q = \dfrac{3C\Delta V}{5}$

$\Delta V_4 = \dfrac{Q_4}{C_4} = \dfrac{3C\Delta V/5}{C} = \dfrac{3\Delta V}{5}$

$\Delta V_{123} = \dfrac{Q_{123}}{C_{123}} = \dfrac{3C\Delta V/5}{3C/2} = \dfrac{2\Delta V}{5}$

$\Delta V_4 + \Delta V_{123} = \Delta V$ as it should be.

$\Delta V_{12} = \Delta V_3 = \Delta V_{123} = \dfrac{2\Delta V}{5}$

$Q_3 = C_3 \cdot \Delta V_3 = C \cdot \dfrac{2\Delta V}{5} = \dfrac{2}{5} C\Delta V$

$Q_{12} = C_{12} \cdot \Delta V_{12} = \dfrac{C}{2} \cdot \dfrac{2\Delta V}{5} = \dfrac{C\Delta V}{5}$

$Q_1 = Q_2 = Q_{12} = \dfrac{C\Delta V}{5}$

$\Delta V_1 = \dfrac{Q_1}{C_1} = \dfrac{C\Delta V/5}{C} = \dfrac{\Delta V}{5}$

$\Delta V_2 = \dfrac{Q_2}{C_2} = \dfrac{C\Delta V/5}{C} = \dfrac{\Delta V}{5}$

Done. We found all Q's and ΔV's


Thank you
Bye Bye For Now