Physics II
Department of Physics and Applied Physics
University of Massachusetts Lowell

Mapping of the Magnetic Field from Helmholtz Coils

The magnetic field B, like force fields g and E, fills three-dimensional space. A pair of Helmholtz coils provide case of some importance for studying the spatial dependence of a field.

I. Introduction

The magnetic field from a coil made up of N turns of wire closely packed falls off with distance along its axis. To provide a region of essentially constant magnetic field, a second identical coil is placed along the same axis to compensate for this fall off. The distance between these two coils will be varied to find the most uniform field condition. At this optimum spacing, the two coils are called Helmholtz coils.

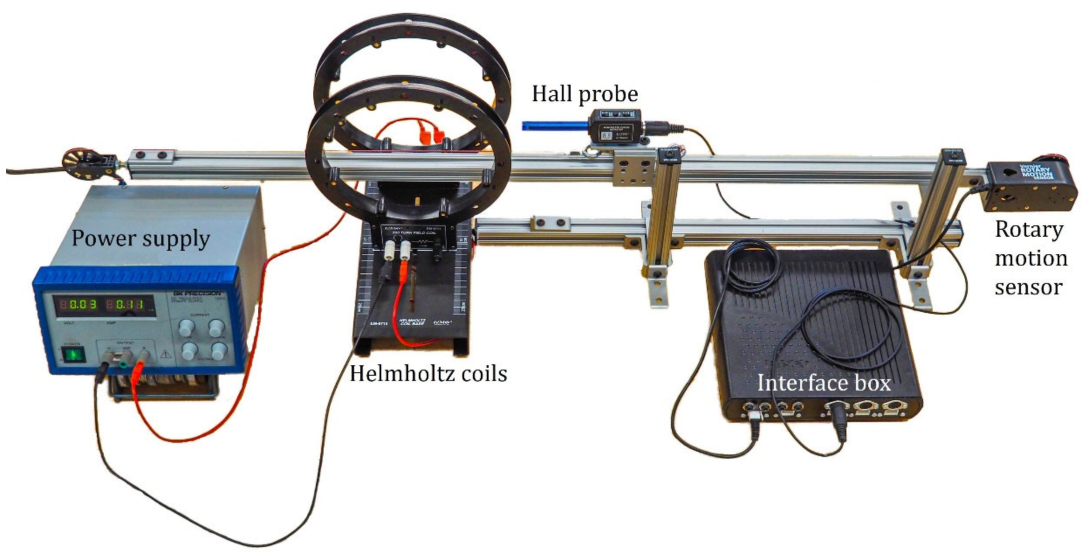
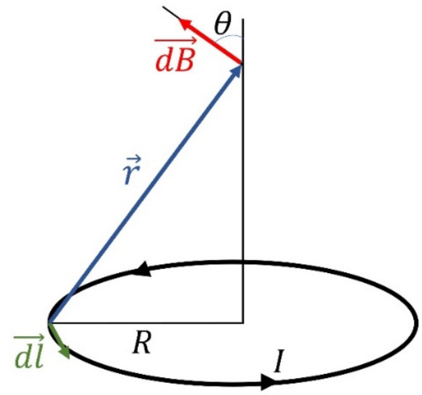


Fig. 1. Setup for radial scan of the magnetic field in the midplane.

The magnetic field intensity is measured as a function of distance along the axis for three different coil spacings. At each spacing the theoretical magnetic field is calculated and superimposed on the measured magnetic field for comparison. The spacing that provides the most uniform field region is sought in this study. In addition, the magnetic field is measured off axis at the optimum spacing setting again to see how constant the field remains this time in the radial direction.

The measurement of the magnetic field uses a Hall probe. This is a solid-state device through which a current is passed. In a magnetic field, moving charges experience a force perpendicular to both the magnetic field and the velocity. This causes the moving free charges of a particular sign to displace laterally and provide a small voltage which can be calibrated in terms of the magnetic field.



Equipment: Optics bench, linear translator, rotary motion sensor, Hall probe, Helmholtz coils, high current power supply.

Theory

Consider the magnetic field along the axis of a single current loop of radius R:

The law of Biot-Savart gives the relation between magnetic field and current:

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} I \frac{\overrightarrow{dl} \times \vec{r}}{r^3}, \quad \text{where } \mu_0 = 4\pi \cdot 10^{-7} \text{ Tm/A}$$

Horizontal components will cancel in pairs, only the z-components are additive.

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} I \frac{dl Cos\theta \vec{k}}{R^2 + z^2} = \frac{\mu_0}{4\pi} I \frac{dl\, R\, \vec{k}}{(R^2 + z^2)^{3/2}}$$

This integrates to

$$\vec{B}_z = \frac{\mu_0}{4\pi} I \frac{2\pi R^2\, \vec{k}}{(R^2 + z^2)^{\frac{3}{2}}}, \qquad (1)$$

For two such coils N turns each separated by distance h, with one coil centered at the origin (i.e. z = 0):

$$\vec{B}_z = \frac{\mu_0 N I R^2\, \vec{k}}{2(R^2 + z^2)^{\frac{3}{2}}} + \frac{\mu_0 N I R^2\, \vec{k}}{2(R^2 + (h - z)^2)^{\frac{3}{2}}} \qquad (2)$$

To find what value of z will provide a maximum (or local minimum) in the magnetic field, set the derivative with respect to z equal to zero:

$$\frac{\overrightarrow{dB_z}}{dz} = 0$$

This gives as a solution the point on the z axis midway between the two coils,

$$z = {}^{h}/_{2} \qquad (3)$$

At this midpoint field, the h value must still be chosen to provide the most uniform field. With $\overrightarrow{B_z}$, evaluated at z = h/2, set the derivative with respect to h equal to zero:

$$\frac{\overrightarrow{dB_z}(z = \frac{h}{2})}{dh} = 0$$

This gives as a solution the spacing of the coils which provide region of the most uniform field midway between the coils along the z direction:

$$h = R \qquad (4)$$

The magnetic field at the midpoint under this optimum spacing has h = R and z = R/2:

$$\vec{B}_z = \frac{8\mu_0 NI\,\vec{k}}{R(125)^{\frac{1}{2}}} \qquad (5)$$

The SI units on the B-field are teslas (T).

Moving charges in the Hall probe subjected to a magnetic field experience the Lorentz force:

$$\vec{F} = q\vec{v} \times \vec{B} \qquad (6)$$

This force is perpendicular to both the velocity vector and the B-field vector and usually produces a spiral path for a constant speed particle.

II. Experimental part.

Initial Setup:

a) Study Fig.1 on the title page to become familiar with the experimental setup.
b) Wires from the rotary motion sensor are plugged into digital channels 1 and 2 on interface box.
c) The Hall probe wire plugs into analog channel A on the interface box.
d) Coils are hooked up in series and then to the current supply.
e) Open the "E&M" folder.
f) Open the "HelmholtzCoil.ds" file.

Procedures and Analysis:

A. *The axial magnetic field will be measured for three different spacings of the coils and compared to theoretical calculations.*

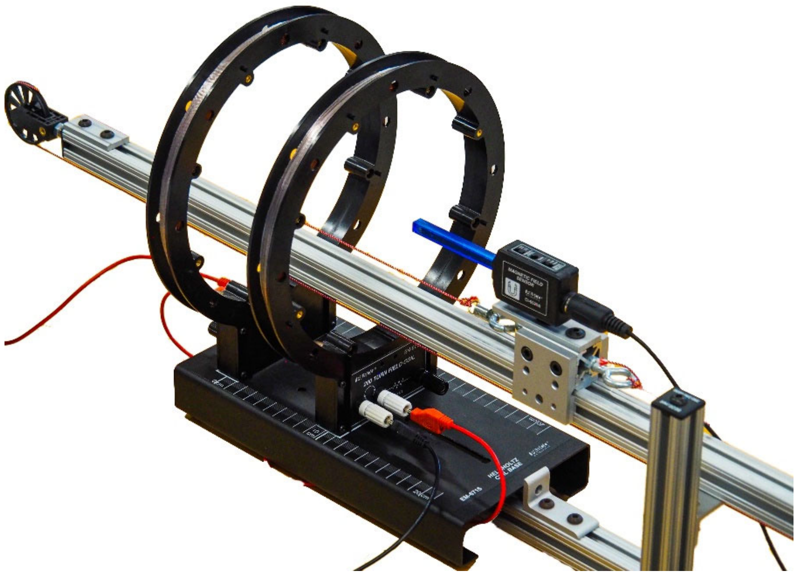


Fig. 2. Setup for the axial scan of the magnetic field.

a) The Fig.2. setup will be used in this procedure. Make sure that the Hall probe will traverse along the axis beyond both coils.
b) Measure the mean radius of the coils, R, and enter this value in data entry box below the graph.
c) Set the mean spacing (distance between centers of coils), h =1.5R, and enter this value in the data entry box.
d) Enter the number of turns in a single coil.
e) Before taking a data set, practice moving the Hall probe in a slow, smooth, and continuous fashion by rotating the pulley on the rotary motion sensor.
f) Check that the Hall probe switch is set to axial.
g) With the current off, click START and zero the Hall probe by pressing the Taar button. Then click STOP and pull down EXPERIMENT menu and DELETE the data run.

h) Set the current to 1 A and enter this value in the data box.
i) Position the Hall probe outside the nearest coil (i.e. move to one extreme).
j) Click START and slowly rotate the pulley moving the Hall probe along the axis to the other extreme and click STOP. Inspect the pattern. It should appear smooth and symmetric. If this is not the case, pull down EXPERIMENT menu and DELETE the present data. Then repeat the measurement procedure.

k) Using the Smart Tool, locate the position, Z_{center}, which will lie in the middle of the valley (called "saddle point"). Enter this value in the data entry box.
l) This will readjust the origin to place one coil at z=0 and the other at z= 1.5R.
m) The theoretical curve should now be superimposed. Print out the graph in Landscape and label the graph with h = 1.5R.
n) Print out all the input parameters, I, R, h.
o) Next repeat the procedure with h = 0.5 R and the current still at 1A.
p) Print out the graph and label with h =0.5 R.
q) Print out all the input parameters.
r) Finally repeat procedure and printouts for h = R and the current still at 1A.

B. With the spacing at h = R, the magnetic field intensity is measured in the radial direction in the plane midway between the two coils (Fig. 1).

a) Turn off the current to the Helmholtz coils.
b) The Helmholtz coils at their optimum h = R spacing are now repositioned so that the Hall probe will traverse the central plane parallel to the coils. Align the probe tip so its traverse takes it through the axis of the coils and covers the full diameter distance.
c) The switch on the Hall probe must be set to Radial.
d) With the current off, click START and zero the Hall probe by pressing the Taar button. Then click STOP and pull down EXPERIMENT menu and DELETE the data run.
e) Set the current to 1A.
f) Position the Hall probe at one extreme.
g) Click START and slowly rotate the pulley moving the Hall probe across the midplane to the other extreme and click STOP.
h) Inspect graph for smoothness and symmetry. With the Smart Tool, locate the center of the distribution, y_{center}. This should be the position of the axis.
i) Print out the graph in "landscape" and label "Midplane Traverse". Also write the y_{center} value on the graph.

Questions:

1. How uniform is the magnetic field around the midplane for h =R setting? Give answer in terms of % deviation for +l cm, +2cm, +3cm, etc. first in the axial direction, then in the radial direction starting from the center point.

2. What energy would an electron have which orbits at a radius of 3cm in the magnetic field that was measured at the h= R setting?