Georg Essl,* Stefania Serafin,†§
Perry R. Cook,‡ and Julius O. Smith§
*Computer and Information Science and
Engineering Department
University of Florida
Gainesville, Florida 32611 USA
gessl@cise.ufl.edu
†Department of Computer Science and
Engineering, Medialogy Group
Aalborg University Esbjerg
Niels Bohrs Vej 8
6700 Esbjerg, Denmark
sts@cs.aue.auc.dk
‡Computer Science and Music Departments
Princeton University
Princeton, New Jersey 08544 USA
prc@cs.princeton.edu
§Center for Computer Research in Music
and Acoustics
Department of Music
Stanford University
Stanford, California 94305 USA
jos@ccrma.stanford.edu

Musical Applications of Banded Waveguides

In a companion article found in this issue of *Computer Music Journal* (Essl et al. 2003), we introduced the theory of banded waveguides, showing the advantages of this synthesis technique that allows efficient simulation of highly inharmonic vibrating structures. In this article, we provide an overview of different musical instruments that have been modeled efficiently using banded waveguides. Additional detail can be found in Cook (2002); Essl and Cook (1999); Essl and Cook (2000); Essl (2002); Kapur et al. (2002); Serafin et al. (2002); and Serafin, Wilkerson, and Smith (2002). Links to software implementations of these models available online can be found in the conclusion of this article.

First, we discuss a banded-waveguide model of bar percussion instruments followed by a model of a musical saw. Next, we show how to use banded waveguides to model bowed glasses and bowls, and we conclude by presenting models of a Tabla and a bowed cymbal.

Computer Music Journal, 28:1, pp. 51–63, Spring 2004

Bar Percussion Instruments

In this section, we discuss the simulation of bar percussion instruments using banded waveguides. This type of instrument was the first to be modeled using this approach. In fact, banded waveguides were originally invented to model the case of bowed bar percussion instruments. The problem of efficiently modeling this instrument had not been solved, nor had it previously received experimental attention. Both the development of the synthesis method for this case and experimental measurements of bowed bars were reported in detail in Essl and Cook (2000). It was realized that the difficulties that vibrating solid bars pose can be overcome by modeling the resonant modes of bars as spectrally separated closed traveling waves, as described in detail in Essl et al. (2003).

In the past, struck bar percussion instruments have been modeled using resonant modal filters (Wawrzynek 1989; Cook 1997) or additive sinusoidal synthesis (Serra 1986; van den Doel and Pai 1998). Acoustical properties of bar percussion instruments have been studied using finite difference and element methods (Bork 1995; Chaigne and Doutaut 1997; Doutaut, Matignon, and Chaigne

Figure 1. A banded wave-guide structure as proposed in Essl and Cook (1999).

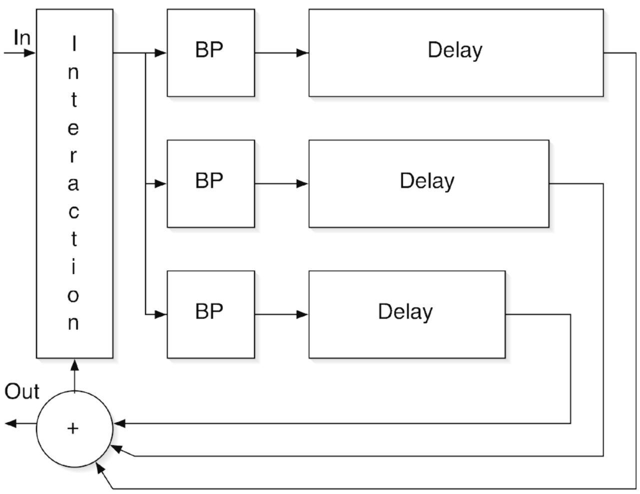


Figure 2. Sonogram of the simulated bowed bar. Note how many partials appear in the spectrum owing to the nonlinearity of the excitation mechanism.

1998; Orduña Bustamante 1991; Bretos, Santamaría, and Moral 1999; Bork et al. 1999), but these methods are too computationally expensive to run in real time and thus have not been used for interactive performances. Finite element methods have also been used to model combined visual and acoustic simulations of sounding objects, including bar percussion instruments (O'Brien, Cook, and Essl 2001). For summaries and reviews of the research on bar percussion instruments, see Moore (1970), Rossing (1976), and Fletcher and Rossing (1998). Some of the results described in this section have also been presented in Essl and Cook (1999), Essl and Cook (2000), and Essl (2002).

Modeling Bowed Bars

As explained in the companion article on theory (Essl et al. 2003), banded waveguides are filter structures that consist of a simple band-pass filter and delay line for each significant mode to be modeled. Banded waveguides can be constructed from physical dynamics or from modal measurements.

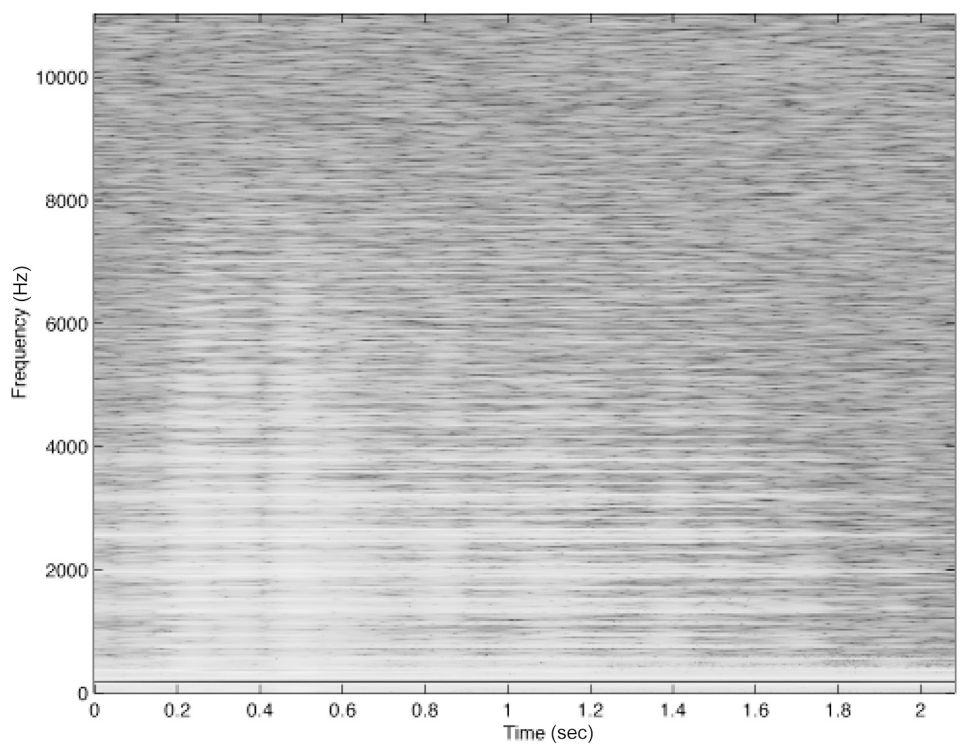


Figure 3. One of the authors playing a saw.

The original banded-waveguide structure as proposed in Essl and Cook (1999) is shown in Figure 1.

Here and in later applications, we use the modal measurement approach. For details on dynamical derivations and interpretations, refer to Essl (2002). The uniform bar measurement yields the well-known stretching of the inharmonic partials of a uniform bar (1:2.756:5.404:8.933 and so on) as heard from glockenspiels (Fletcher and Rossing 1998). Marimba, xylophone, and vibraphone bars are undercut, stretching the partials into harmonic ratios of either 1:4:10 or 1:3:6 (Moore 1970).

Using these frequencies, the length of the delays as well as the frequencies of the band-pass filters of all banded wave paths are tuned, forming the complete resonator model. When including interaction models as described in the companion article, the full banded-waveguide synthesis model is constructed. The resulting spectrum of a bowed bar simulation using the lowest four partials can be seen in Figure 2. Note that same structure models both the struck and the bowed bar without modification, and alternating or combined playing styles are easily possible without changing the model's parameters.

Musical Saw

As another application of one-dimensional banded waveguides, we propose a model of a musical saw (Serafin et al. 2002). When an ordinary handsaw is bent into an "S"-shape, an interesting acoustical effect can occur. Tapping the blade of the saw reveals that, beyond a certain critical degree of curvature, a very lightly damped vibration mode appears that is confined to the middle region of the "S." This confined mode can be excited by a violin bow to produce the characteristically pure sound of the musical saw. Scott and Woodhouse (1992) provide a detailed description of the vibrational behavior of an elastic strip with varying curvature.

The origins of the musical saw go back to the early 20th century, thanks in particular to Leon Weaver. Later on, June Weaver started playing the saw using a violin bow in a lap style, as shown in Figure 3.

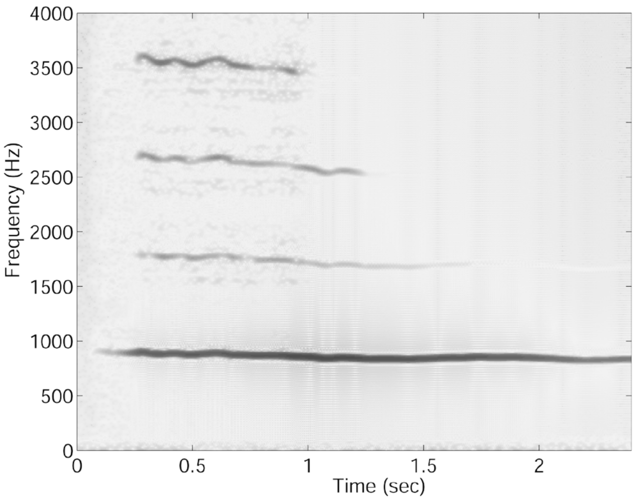


Figure 4. Sonogram of a bowed saw tone. The saw is bowed for about one sec and then left to resonate. Whereas the fundamental has a long decay time, the higher harmonics are quickly damped.

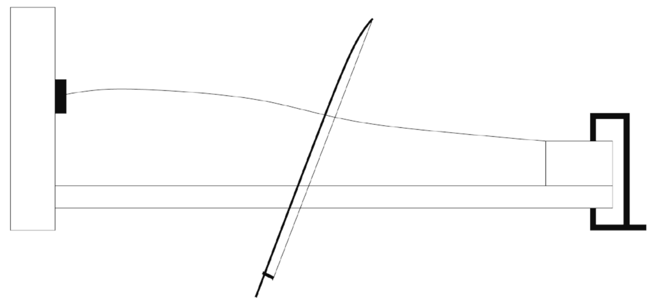


Figure 5. Configuration of a saw fixed at both ends.

Modeling a Musical Saw

Figure 4 shows the spectrogram of the sound obtained from a Stanley 26-inch crosscut saw bowed at the curvature. The saw was blocked on one side using a clamp and bent as shown in Figure 5. This S-shape allows certain modes to be confined to the vicinity of the inflection by a process of reflection from points of critical curvature. The microphone was placed in front of the player at about 20 cm away in the same horizontal plane as the curvature point.

The tone produced is almost sinusoidal, and the player controls the fundamental frequency by changing the curvature of the blade. Increasing the curvature produces higher fundamental frequency. The vibrato is obtained by slightly moving the extremity of the saw in the hand of the player. While the saw is bowed, many partials appear in the spectrum, but when the bow is released, the fundamental frequency resonates primarily.

Considering its relatively simple spectrum, the musical saw can be easily implemented using one banded waveguide excited by the same friction-driven mechanism explained in Essl et al. (2003). In this way, the model of the musical saw is a simplified version of the model of the bowed bar. The spectrum of the simulated saw is shown in Figure 6. Note how, as in the recordings of the real instrument shown in Figure 4, when the saw is sustained by the bow, a rich spectrum appears, but when it is released only the fundamental frequency appears.

To achieve vibrato, the digital waveguide uses third-order fractional delay interpolation (Laakso et al. 1996), so the length of the waveguide can change almost continuously.

Wine Glasses and Glass Harmonicas

Another instrument for which the theory of banded waveguides efficiently applies is the glass harmonica (Essl and Cook 2002; Serafin et al. 2002; Essl 2002; Cook 2002). Glass harmonicas can be found in two forms. The first, invented by Benjamin Franklin in 1757 and shown in Figure 7, adopts glass bowls turned on their horizontal axis on a common spindle so that one side of the bowl dips into a trough of water. The second one, which is the one we model, is a combination of wine glasses of different sizes, as shown in Figure 8.

Melodies can be played on a set of tuned glasses (filled with appropriate amounts of water or carefully selected by size) simply by rubbing the edge of the glass with a moist finger. The glasses can also be excited using a violin bow (Rossing 1994). Figure 9 shows the spectra of a wine glass hit with a hard mallet, bowed with a cello bow, and rubbed with a wet finger, respectively. Note how the modes's locations are consistent with the ones described in Rossing (1994).

Modeling a Glass Harmonica

A wine glass is a three-dimensional object, and disturbances travel along the object in all dimensions.

Figure 6. Sonogram of a synthetic bowed-saw tone. The saw is bowed for about 1.5 sec and then left to resonate. Note that, as in the real instrument, the fundamental has a long decay time while the higher harmonics are quickly damped.

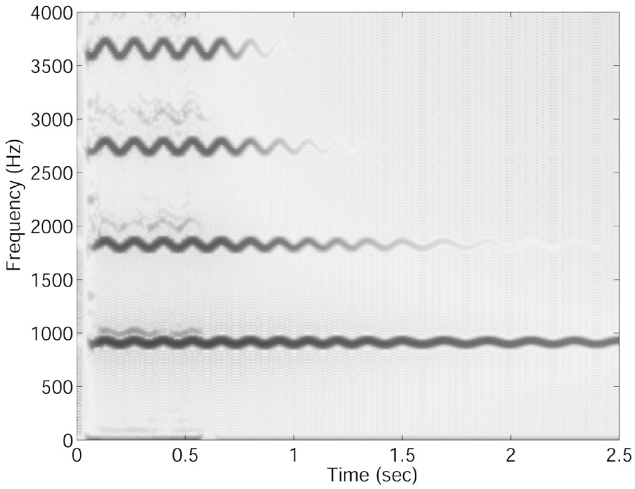


Figure 7. Benjamin Franklin's glass harmonica, which he called "armonica," as seen in the Franklin Institute Science Museum in Philadelphia. Picture courtesy of Ed Gaida.

The object is however axially symmetrical, and the dominant modes are essentially circular modes (Rossing 2000). Energy travels along the rim of the glass, creating a closed path as described in Essl et al. (2003). Essentially, the rim represents a bar bent into a circular shape, closing onto itself at both ends. Hence the path is quasi-one-dimensional.

In order to model waves propagating along the rim of a wine glass, we use a network of circular banded waveguides (CBW), each waveguide being tuned to the fundamental frequency of the corresponding mode. A CBW is a connection of two waveguides band-limited by a band-pass filter. The output of each waveguide is connected to the input of the other waveguide in a loop, as Figure 10 shows.

Figure 10 illustrates the situation in which only one mode is present. In the simulated instrument, many modes appear that are connected to the excitation model as described in Essl et al. (2003).

Tibetan Singing Bowl

The Tibetan singing bowl is another related instrument that can be modeled using banded waveguides (Essl and Cook 2002; Serafin et al. 2002; Serafin, Wilkerson, and Smith 2002; Essl 2002). The Tibetan bowl has lately seen a strong interest

Figure 8. Wine glasses being played by rubbing the edge of the glasses.

Figure 9. Spectrum of a small wine glass. Top: frequency response; center: bowing with a cello bow; bottom: rubbing with a wet finger.

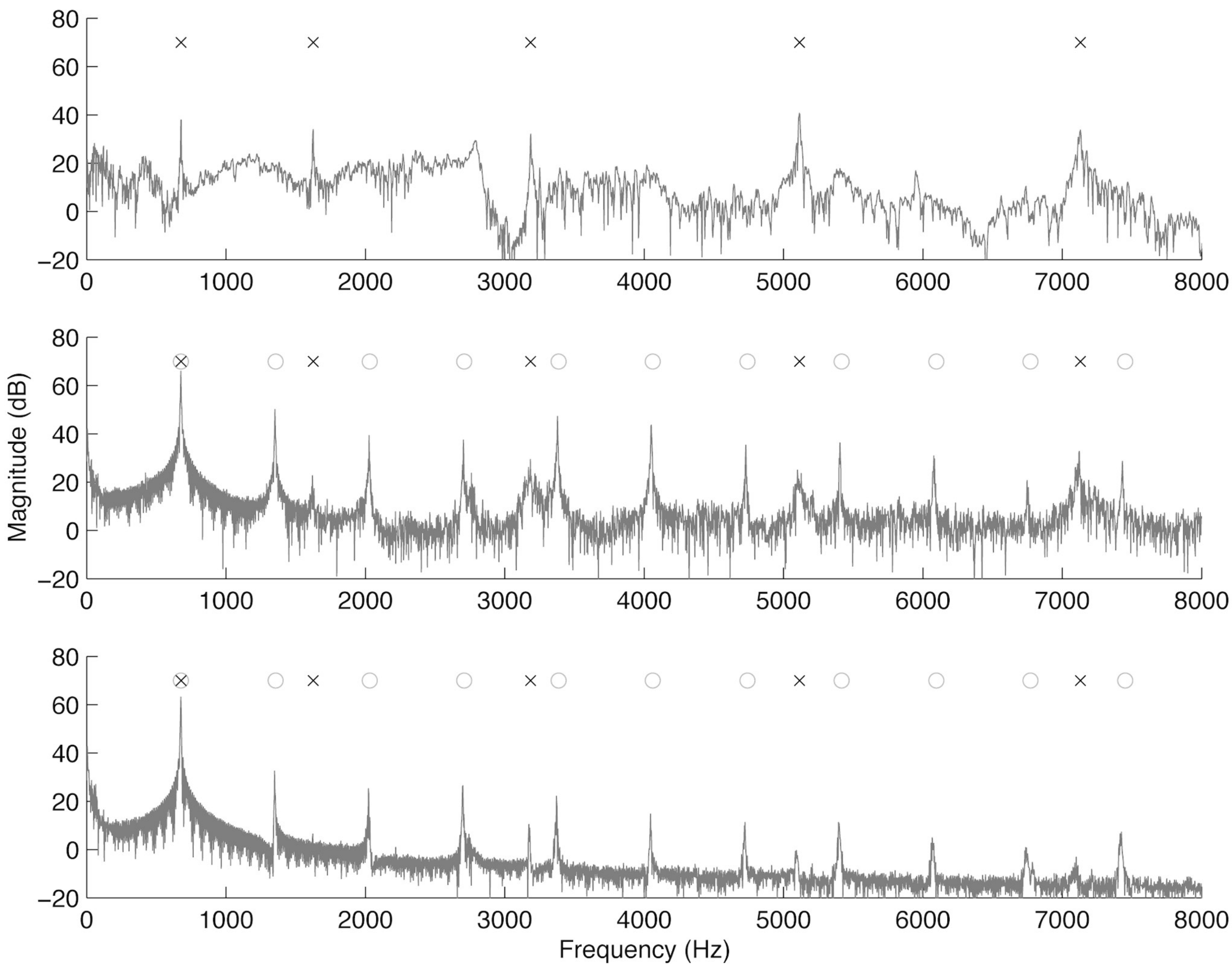
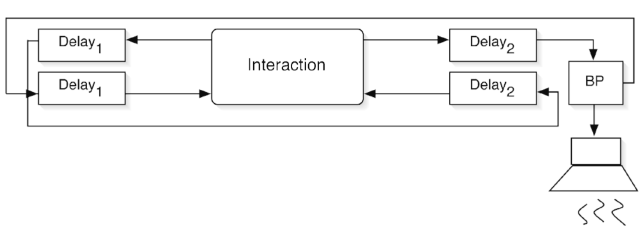


Figure 10. Digital waveguide network structure of the bowl resonator representing one mode. Each bi-directional delay line contains the waves propagating along both sides of the bowl.



in the computer music community (Wilkerson, Ng, and Serafin 2002; Burtner, Serafin, and Topper 2002; Tanaka and Knapp 2002).

Oral tradition dates the singing bowl back to 560–180 BCE in Tibet. These bowls have been found in temples, monasteries, and meditation halls throughout the world. Traditionally, bowls are made of metal and are hand-hammered round to produce beautiful tones and vibrations. Today, they are also available in glass and can be made using machine-manufacturing processes. They are used in music, relaxation, meditation, and healing.

The Tibetan singing bowl's modes are geometri-

Figure 11. The Tibetan singing bowl used for the recordings.

cally close to spherical segments. In typical performance, the bowl is rubbed with a wooden stick (which may be wrapped in a thin sheet of leather) along its rim. Depending on the rubbing velocity and initial state of the bowl (i.e., certain modes may be already ringing), various modes can be excited.

Modeling the Tibetan Bowl

To create a virtual bowl that faithfully reproduces its real counterpart, we recorded the Tibetan bowl shown in Figure 11 while hit at eight different positions as shown in Figure 12. For each position, the resulting spectra are shown in Figure 13.

Clearly, a number of higher modes lie close together, yielding audible beating. The beating can be seen more clearly in Figure 14.

Considering the strong similarity between the structure of the bowl and the wine glasses, we used circular banded waveguides to implement the bowl model. As explained in Essl et al. (2003), beatings can be implemented using detuned banded waveguides. The spectrum of the synthetic bowl is shown in Figure 15. Note the long decay time and the beatings. (The characteristic rubbing interaction can be added to the model the same way as was done for the wine glass described earlier. The interaction in Figure 10 then uses a friction model instead of a impulsive striking model.)

Indian Tabla Drums

Banded-waveguide synthesis can also be used for two-dimensional structures. As an example, the

Figure 12. Eight different positions at which the Tibetan bowl was struck.

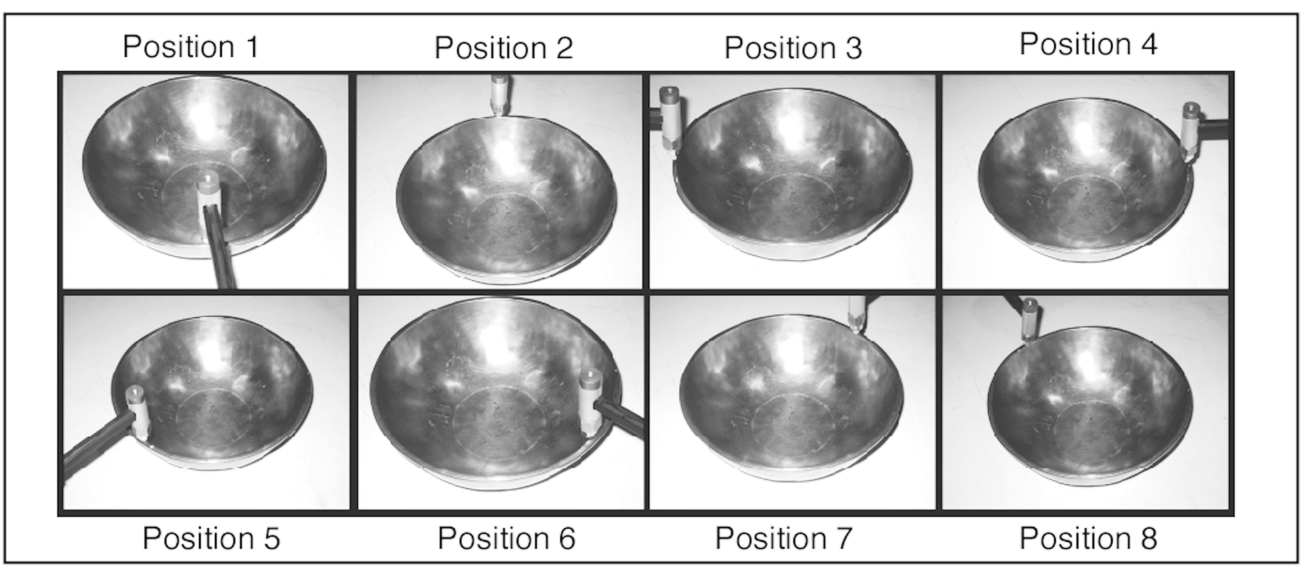
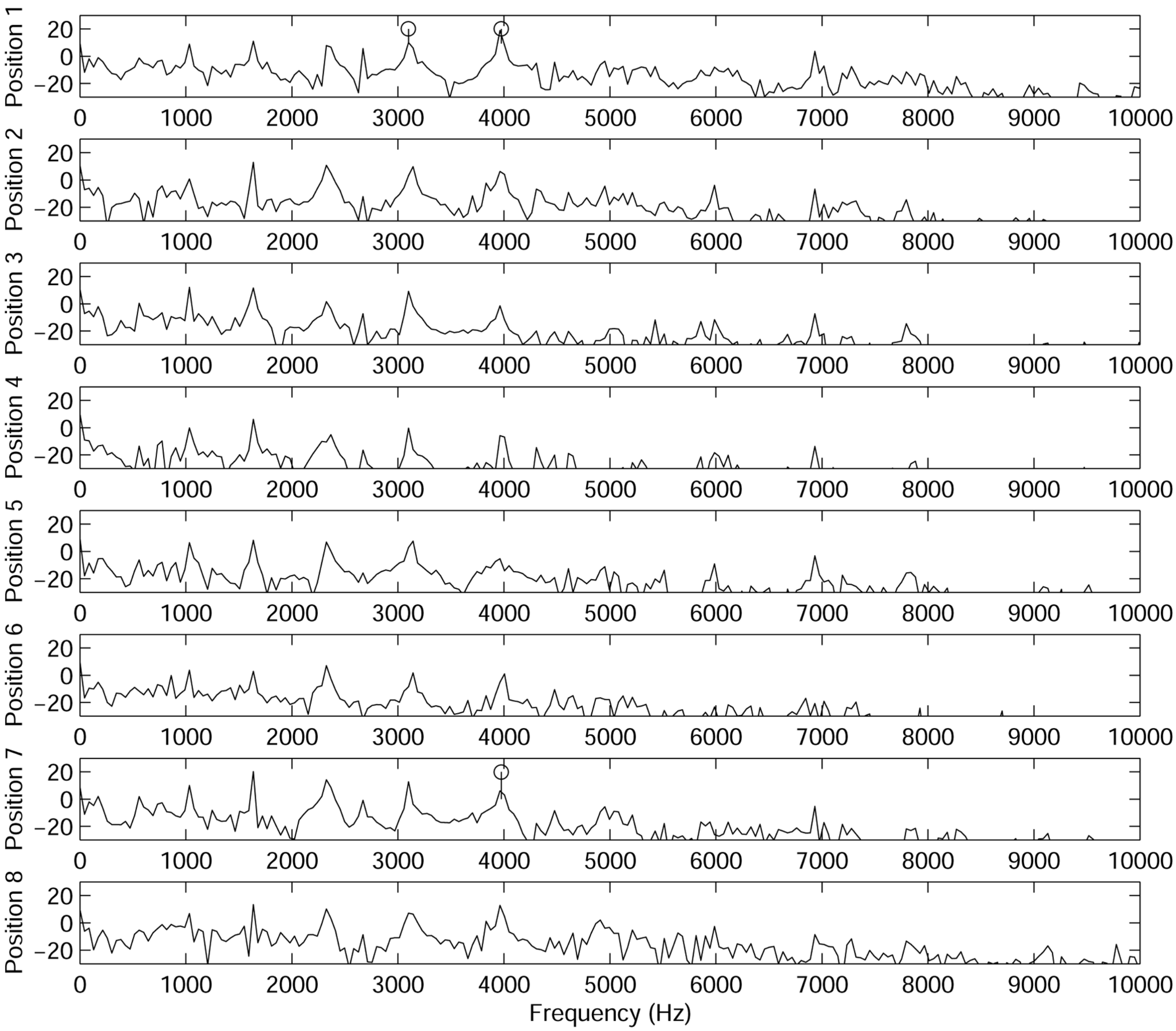


Figure 13. Spectra of the eight different positions at which the Tibetan bowl was hit.



Indian Tabla drums as depicted in Figure 16 are discussed in Essl (2002) and have been used in conjunction with a novel electronic Tabla controller (Kapur et al. 2002).

The Tabla is a pair of drums with a number of interesting characteristics. The modes of the first four to six partials are harmonic, unlike what one might expect from a circular membrane. To achieve this harmonic tuning, the Tabla drums are manufactured using membranes of non-uniform thickness (Rossing 2000). There are a number of typical performance strokes to Tablas. One interesting stroke is a modulating form of the ''Ga'' stroke, which is performed on the larger, right drum, called ''bayan.'' The palm of the hand rests on the drum. After the drum has been excited with a quick impact from the fingertips, the player pushes the palm down and toward the center of the drum, thereby achieving a characteristic upward pitch-bending sound (Rossing 2000). The small drum is called ''dahina.''

Figure 14. Beating upper partials in spectrogram of a recorded Tibetan bowl.

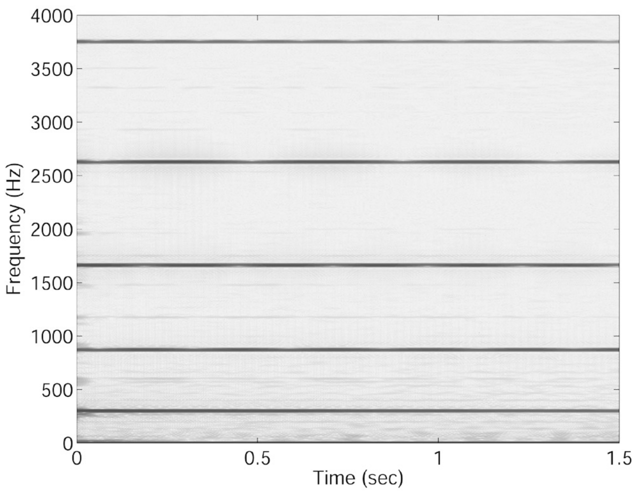


Figure 15. Spectrogram of the synthetic Tibetan bowl.

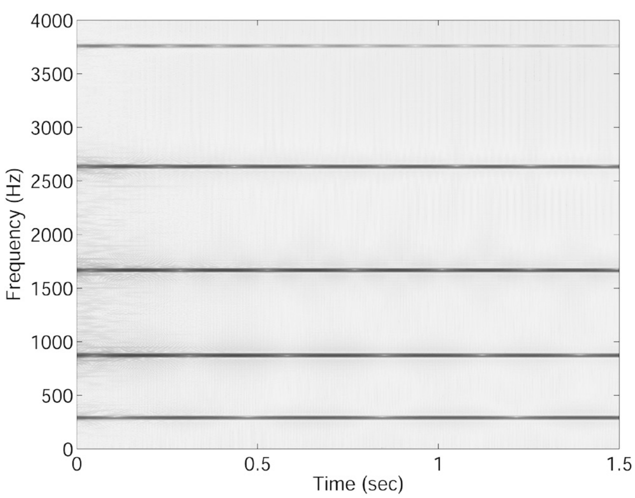


Figure 16. The Indian Tabla Drum consisting of the larger bayan (left) and the smaller dahina (right).

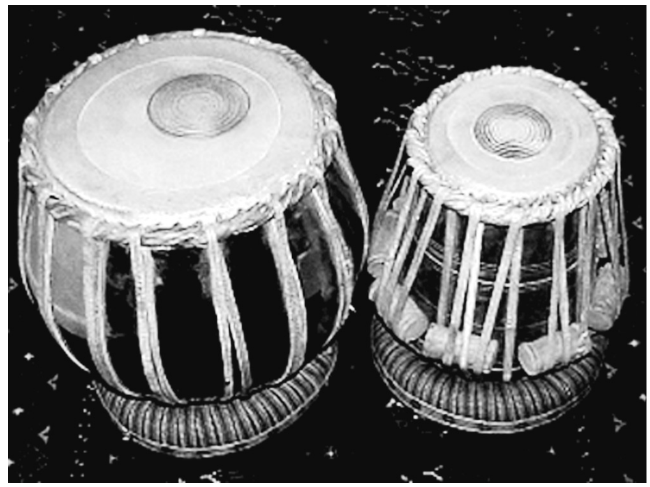


Table 1. Spectral Frequencies of Dominant Partials of Measured and Simulated Tablas Given as f_n:f_1 (i.e., as a Ratio Relative to the Fundamental Frequency)

	Bayan		*Dahina*	
n	*measured*	*simulated*	*measured*	*simulated*
2	2.00	2.02	2.89	2.87
3	3.01	3.03	4.95	5.01
4	4.01	4.05	6.99	6.73
5	4.69	4.72	8.01	8.00
6	5.63	5.65	9.02	8.70

Modeling the Tabla

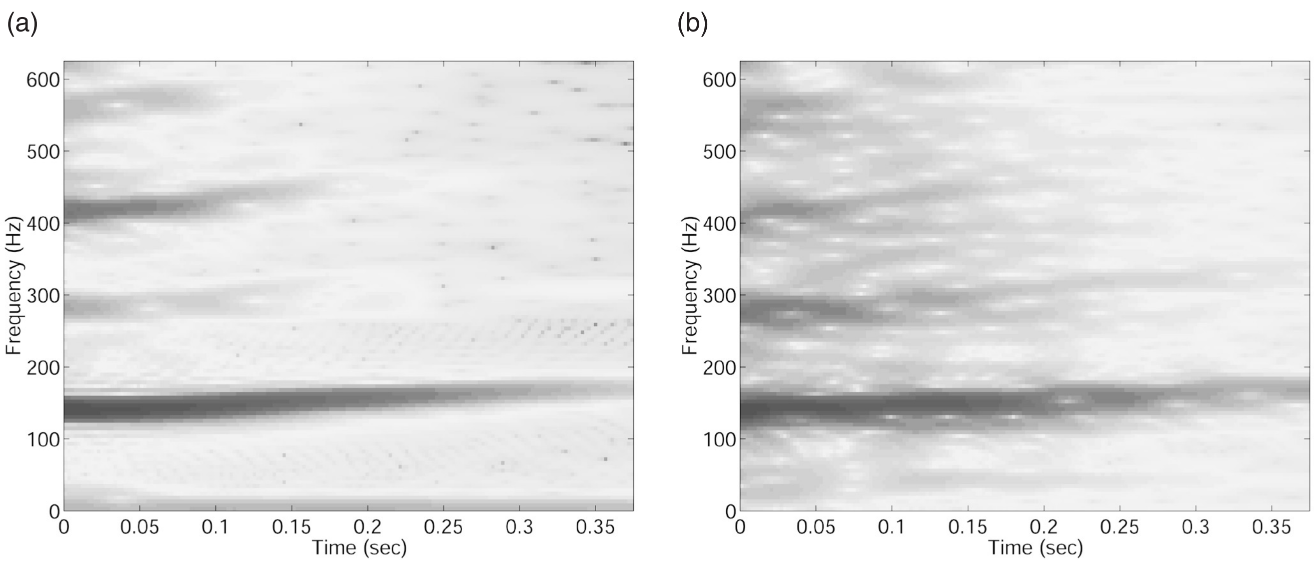
As discussed in the companion article, the trajectories that lead to closed paths can be constructed for circular membranes. Even in dimensions higher than one, closed paths lead to modes of vibration using the principle of closed wavetrains (Essl et al. 2003).

The results of modal comparison between real drums and propagation simulations can be found in Table 1. The strokes performed are open-membrane strokes in the center on both the bayan and the dahina. This was in turn modeled as impulsive excitation.

Using this principle of closed wavetrains, we can infer how dynamical interactions of strokes relate to pitch changes through path-length changes. Here, we are particularly interested in the ''Ga'' stroke. In this case, the pitch-bending technique directly corresponds to shortening the physical path of waves traveling on the membrane, which can be directly implemented in a banded-waveguide model.

The results for more complicated pitch-bending strokes can be seen in Figure 17. The simulation sounds comparable to the recorded stroke. It should be noted that the simulation method is robust to the pitch-bending manipulation. In fact, much more extreme bends than the one depicted here are

Figure 17. Spectrogram showing the upward bending of a modulated Ga stroke. The fundamental bends from 136 to 162 Hz (left, measured) and 134 to 171 Hz (right, simulated).



possible. High-pitched, large-scale bends on our propagational model perceptually resemble water-drop sounds, suggesting a much wider range of interesting application for behaviors of this type.

Cymbals

As our final example, we discuss the modeling of cymbals using banded-waveguide meshes. As described in Fletcher (1994), the vibration of a cymbal is very similar to the vibration of flat, circular plates. Although modes are clearly distinguishable at low-frequency modes, they often mix with one another at high frequencies.

The nonlinear coupling between vibrational modes, moreover, is quite strong, which makes many partials quickly appear in the spectrum. This is true no matter how the cymbal is excited.

Fletcher (1994) investigated nonlinearities in cymbals. The results of exciting a cymbal with a sinusoidal shaker show that, while at low frequencies the radiated sound is concentrated at the fundamental of the exciting frequency, increasing the amplitude also increases the relative levels of all partials. At a critical excitation amplitude, the spectrum develops a complete set of sub-harmonics, and transitions to fully chaotic behavior can appear.

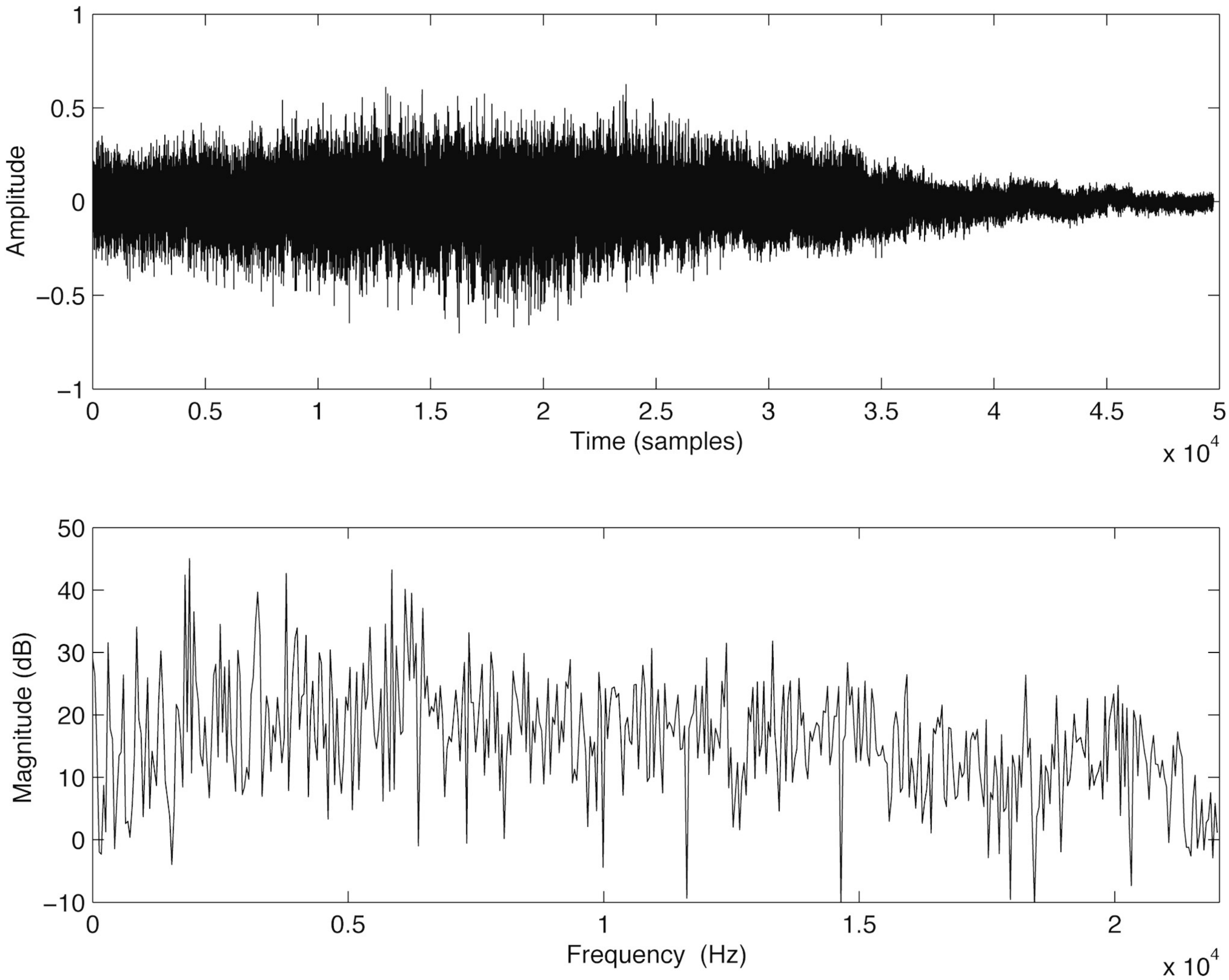
The mathematical problem of analyzing cymbal behavior in detail is rather complex. The frequency response of a bowed cymbal presents a large number of potentially active modes.

Figure 18 shows the frequency response of an orchestral cymbal of diameter 41 cm bowed with a violin bow. The recording was made in a quiet room, and the microphone was placed about 0.3 m from the cymbal. Several prominent peaks comprise the more steady oscillation of the cymbal, and there is still much energy at high frequencies where modes are very dense.

Modeling the Bowed Cymbal

Bowed cymbals and bowed plates lend themselves well to being modeled with a banded-waveguide mesh structure as described in the companion article. Low modes are excited by the bowing, and energy is transferred nonlinearly to high-frequency modes, which are chaotically coupled. The manner of excitation of these strong lower modes relies on a detailed mechanical interaction of the bow and the rim, and thus an interface between the bowing and the resonating plate that preserves the loca-

Figure 18. An example of a bowed cymbal. Top: time-domain waveform without the attack. Bottom: frequency response.



tions of the bow-cymbal contact is needed. Banded waveguides allow individual modes to be controlled in time, frequency, and space. The shimmering, noise-like high-frequency modes are not a direct consequence of the bow excitation, so a banded-waveguide mesh can be used as an approximation of a dense modal region, as explained in Essl et al. (2003).

Bowed cymbals can produce a wide range of sounds with small variations in bowing force, velocity, and position. In certain cases, the cymbal produces a noisy growl, and modes are very dense throughout the spectrum. In this case, a waveguide mesh with sufficient mode density at the lower frequency range would be too large to be implemented in real-time with present technology.

Banded-waveguide structures allow exact tuning of partial frequencies and hence avoid problems of waveguide meshes with grid dispersion and the related difficulty of tuning modes exactly (van Duyne and Smith 1993; Savioja and Välimäki 2000). In this application, owing to the density of modes in the range modeled by the mesh, these difficulties can be neglected.

Conclusion

In this article, we described physical models of different musical instruments based on banded waveguides. These models have been implemented in the Synthesis Toolkit (Cook and Scavone 1999), Pure Data (Puckette 1997), and Max/MSP platforms (Zicarelli 1998), and can be downloaded online at ccrma-www.stanford.edu/software/stk and ccrma-www.stanford.edu/~serafin/software.html.

Acknowledgments

The authors would like to thank Matthew Burtner, Patty Huang, Ajay Kapur, Carmen Ng, Sølvi Ystad, and Carr Wilkerson for their help in modeling many of the instruments proposed in this article and for their creativity in using the banded waveguides in musical applications. We also thank Ed Gaida for the kind permission to use his photograph of the glass harmonica. The authors are also grateful for the many helpful comments by three anonymous reviewers and the editor, which greatly helped to improve this article and the companion article.

References

Bork, I. 1995. ''Practical Tuning of Xylophone Bars and Resonators.'' *Applied Acoustics* 46:103–127.

Bork, I., et al. 1999. ''Comparison Between Modal Analysis and Finite Element Modeling of a Marimba Bar.'' *Acustica united with Acta Acustica* 85(2):258–266.

Bretos, J., C. Santamaría, and J. A. Moral. 1999. ''Finite Element Analysis and Experimental Measurements of Natural Eigenmodes and Random Responses of Wooden Bars Used in Musical Instruments.'' *Applied Acoustics* 56:141–156.

Burtner, M., S. Serafin, and D. Topper. 2002. ''Real-Time Spatial Processing and Trasformations of a Singing Bowl.'' *Proceedings of the 2002 Conference on Digital Audio Effects.* Hamburg, Germany: University of the Federal Armed Forces Hamburg, Department of Signal Processing and Communications, pp. 123–126.

Chaigne, A., and V. Doutaut. 1997. ''Numerical Simulations of Xylophones I: Time-Domain Modeling of the Vibrating Bars.'' *Journal of the Acoustical Society of America* 101(1):539–557.

Cook, P. R. 1997. ''Physically Informed Sonic Modeling (PhISM): Synthesis of Percussive Sounds.'' *Computer Music Journal* 21(3):38–49.

Cook, P. R. 2002. *Real Sound Synthesis for Interactive Applications*. Natick, Massachusetts: A. K. Peters.

Cook, P. R., and G. Scavone. 1999. ''The Synthesis Toolkit (STK).'' *Proceedings of the 1999 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 164–166.

Doutaut, V., D. Matignon, and A. Chaigne. 1998. ''Numerical Simulations of Xylophones II: Time-Domain Modeling of the Resonator and of the Radiated Sound Pressure.'' *Journal of the Acoustical Society of America* 104(3):1633–1647.

Essl, G. 2002. *Physical Wave Propagation Modeling for Real-Time Synthesis of Natural Sounds*. Ph.D. thesis, Princeton University, Department of Computer Science.

Essl, G., and P. Cook. 2002. ''Banded Waveguides on Circular Topologies and of Beating Modes: Tibetan Singing Bowls and Glass Harmonicas.'' *Proceedings of the 2002 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 49–52.

Essl, G., and P. R. Cook. 1999. ''Banded Waveguides: Towards Physical Modeling of Bowed Bar Percussion Instruments.'' *Proceedings of the 1999 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 321–324.

Essl, G., and P. R. Cook. 2000. ''Measurements and Efficient Simulations of Bowed Bars.'' *Journal of the Acoustical Society of America* 108(1):379–388.

Essl, G., et al. 2003. ''Theory of Banded Waveguides.'' *Computer Music Journal* 28(1):37–50.

Fletcher, N. H. 1994. ''Nonlinear Dynamics and Chaos in Musical Instruments.'' *Complexity International* 1. Available online at www.complexity.org.au/ci/vol01/fletch01.

Fletcher, N. H., and T. D. Rossing. 1998. *The Physics of Musical Instruments*, 2nd ed. New York: Springer-Verlag.

Kapur, A., et al. 2002. ''The Electronic Tabla Controller.'' *Proceedings of the 2002 Conference on New Instruments for Musical Expression*. Limerick, Ireland: University of Limerick, Department of Computer Science and Information Systems, pp. 77–81.

Laakso, T. I., et al. 1996. ''Splitting the Unit Delay: Tools for Fractional Delay Filter Design.'' *IEEE Signal Processing Magazine* 13(1):30–60.

Moore, J. L. 1970. *Acoustics of Bar Percussion Instruments*. Ph.D. thesis, Ohio State University.

O'Brien, J. F., P. R. Cook, and G. Essl. 2001. ''Synthesizing Sounds from Physically Based Motion.'' *Proceedings of SIGGRAPH 2001*. New York: ACM Press, pp. 529–536.

Orduña Bustamante, F. 1991. ''Nonuniform Beams with Harmonically Related Overtones for Use in Percussion Instruments.'' *Journal of the Acoustical Society of America* 90(6):2935–2941. Erratum published:*Journal of the Acoustical Society of America* 91(6):3582–3583, 1992.

Puckette, M. 1997. ''Pure Data.'' *Proceedings of the 1997 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 224–227.

Rossing, T. D. 1976. ''Acoustics of Percussion Instruments: Part I.'' *The Physics Teacher* 14:546–556.

Rossing, T. D. 1994. ''Acoustics of the Glass Harmonica.'' *Journal of the Acoustical Society of America* 95(2):1106–1111.

Rossing, T. D. 2000. *Science of Percussion Instruments*. Singapore: World Scientific Publishing.

Savioja, L., and V. Välimäki. 2000. ''Reducing the Dispersion Error in the Digital Waveguide Mesh Using Interpolation and Frequency-Warping Techniques.'' *IEEE Transaction on Speech and Audio Processing* 8(2):184–194.

Scott, J. F., and J. Woodhouse. 1992. ''Vibration of an Elastic Strip with Varying Curvature.'' *Philosophical Transactions of the Royal Society of London A* 339: 587–625.

Serafin, S., et al. 2002. ''Analysis and Synthesis of Unusual Friction Driven Musical Instruments.'' *Proceedings of the 2002 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 57–60.

Serafin, S., C. Wilkerson, and J. O. Smith. 2002. ''Modeling Bowl Resonators Using Circular Waveguide Networks.'' *Proceedings of the 2002 Conference on Digital Audio Effects*. Hamburg, Germany: University of the Federal Armed Forces Hamburg, Department of Signal Processing and Communications, pp. 117–121.

Serra, X. 1986. ''A Computer Model for Bar Percussion Instruments.'' *Proceedings of the 1986 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 257–262.

Tanaka, A., and B. R. Knapp. 2002. ''Multimodal Interaction in Music Using the Electromyogram and Relative Position Sensing.'' *Proceedings of the 2002 Conference on New Interfaces for Musical Expression*. Limerick, Ireland: University of Limerick, Department of Computer Science and Information Systems, pp. 43–48.

van den Doel, K., and D. K. Pai. 1998. ''The Sound of Physical Shapes.'' *Presence* 7(4):382–395.

van Duyne, S., and J. O. Smith. 1993. ''Physical Modeling With the 2D Waveguide Mesh.'' *Proceedings of the 1993 International Computer Music Conference*. San Francisco: International Computer Music Association, pp. 40–47.

Wawrzynek, J. 1989. ''VLSI Models for Sound Synthesis.'' In M. Mathews and J. Pierce, eds. *Current Directions in Computer Music Research*. Cambridge, Massachusetts: MIT Press, pp. 113–148.

Wilkerson, C., C. Ng, and S. Serafin. 2002. ''The Mutha Rubboard Controller: Interactive Heritage.'' *Proceedings of the 2002 Conference on New Instruments for Musical Expression*. Limerick, Ireland: University of Limerick, Department of Computer Science and Information Systems, pp. 82–85.

Zicarelli, D. 1998. ''An Extensible Real-Time Signal Processing Environment for Max.'' *Proceedings of the 1998 International Computer Music Conference*. San Francisco, International Computer Music Association, pp. 463–466.