

## Addition formula for sine and for cosine (C)

It more difficult to see that there are identities for the sine and cosine of the sum of two angles:

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

and

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

### Exercises:

1.

If  $\cos(A) = 3/5$ ,  $\sin(A) = 4/5$ ,  $\cos(B) = 5/13$  and  $\sin(\theta) = 12/13$  find the sine and cosine of  $A+B$ .

2.

If  $\cos(A) = -3/5$ ,  $\sin(A) = 4/5$ ,  $\cos(B) = 5/13$  and  $\sin(\theta) = -12/13$  find the sine and cosine of  $A+B$ .

3.

Show that if  $h \neq 0$  then

$$\frac{\cos(A + h) - \cos(A)}{h} = -\sin(A)\frac{\sin(h)}{h} - \cos(A)\frac{1 - \cos(h)}{h}$$

and

$$\frac{\sin(A + h) - \sin(A)}{h} = -\cos(A)\frac{\sin(h)}{h} - \sin(A)\frac{1 - \cos(h)}{h}$$

4.

Using the odd and even properties of sine and cosine, derive the subtraction identities

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

and

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B).$$

This shows that the subtraction identities should not be committed to memory.

5.

If  $\cos(A) = 3/5$ ,  $\sin(A) = 4/5$ ,  $\cos(B) = 5/13$  and  $\sin(\theta) = 12/13$  find the sine and cosine of  $A-B$ .

6.

If  $\cos(A) = -3/5$ ,  $\sin(A) = 4/5$ ,  $\cos(B) = 5/13$  and  $\sin(\theta) = -12/13$  find the sine and cosine of  $A-B$ .