Amplitude and Phase Shift (C)

However, these formulae are extremely useful. For example, in the analysis of simple harmonic motion one encounters expressions such as $3\cos(2t) + 4\sin(2t)$. These are shifted cosine waves. To see why, observe that the ordered pair

$$\left(\frac{3}{3^2+4^2}, \frac{4}{3^2+4^2}\right) = \left(\frac{3}{5}, \frac{4}{5}\right)$$

is a point on the circle $x^2 + y^2 = 1$. Therefore there is an angle $\phi \in [0, 2\pi]$ so that

$$(\cos(\phi), \sin(\phi)) = \left(\frac{3}{5}, \frac{4}{5}\right).$$

Therefore

$$\begin{aligned} 3\cos(2t) + 4\sin(2t) &= 5\left(\tfrac{3}{5}\cos(2t) + \tfrac{4}{5}\sin(2t)\right) \\ &= 5(\cos(\phi)\cos(2t) + \sin(\phi)\sin(2t)) \\ &= 5\cos(2t - \phi). \end{aligned}$$

The coefficient 5 is the **amplitude** of the wave, and ϕ is the **phase shift**.

Exercises:

Find the amplitude and the sine and cosine of the phase shift for each of the following. Sketch a graph of the wave as a function of t:

1. $4\cos(4t) + 3\sin(4t)$.

2. $7\cos(5t) + 24\sin(5t)$.

3. $5\cos(t/3) - 12\sin(t/3)$.