

Circle Identity (C)

It follows from the definition of cosine and sine as the coordinates of points on the unit circle that for any angle θ ,

$$(\cos(\theta))^2 + (\sin(\theta))^2 = 1$$

This identity is called the **Circle Identity**. If the quadrant of an angle is known, this identity may be used to find the cosine of an angle if the sine is given, and vice versa.

Example: If θ is in the second quadrant and $\sin(\theta) = 5/13$, what is the cosine of θ ?

Solution: Since θ is in the second quadrant its cosine is negative. Also,

$$(\cos(\theta))^2 + (5/13)^2 = 1,$$

so $(\cos(\theta))^2 = 144/169 = (12/13)^2$. Therefore, $\cos(\theta) = -12/13$.

Exercises In each case find the indicated sine or cosine.

1. θ in the first quadrant, $\sin(\theta) = 3/5$. Find $\cos(\theta)$.
2. θ in the fourth quadrant, $\cos(\theta) = 3/5$. Find $\sin(\theta)$.
3. θ in the third quadrant, $\cos(\theta) = -3/5$. Find $\sin(\theta)$.
4. θ in the second quadrant, $\sin(\theta) = 1/5$. Find $\cos(\theta)$.
5. θ in the first quadrant, $\sin(\theta) = 7/25$. Find $\cos(\theta)$.
6. θ in the second quadrant, $\cos(\theta) = -24/25$. Find $\sin(\theta)$.
7. θ in the third quadrant, $\cos(\theta) = -7/25$. Find $\sin(\theta)$.
8. θ in the fourth quadrant, $\sin(\theta) = -1/25$. Find $\cos(\theta)$.