Inverse Trigonometric Functions (C)

The equation *sin(x)* =1/2 has many solutions, as you can tell from the fact that the line y = 1/2 intersects the graph of $y = sin(x)$ at many places. However, if we restrict our attention to the portion of the graph of $y = sin(x)$ for $x \in [-\pi/2, \pi/2]$ we see that there is only one such point of intersection. Indeed, for any number $u \in [-1,1]$ there is exactly one number $v \in [-\pi/2, \pi/2]$ so that $sin(v) = u$. We denote this number v by *arcsin(u)*. More generally, we define the function *arcsin* : [-1..1]→[-π/2..π/2] by the rule

$$arcsin(u) = v \text{ if } sin(v) = u \text{ and } v \in [-\pi/2,\pi/2]$$

It should be noted that $sin(arcsin(u)) = u$ for every $u \in [-1,1]$ but $arcsin(sin(x)) = x$ only if $x \in [-\pi/2,\pi/2]$. The quantity *arcsin*(*u*) has the following geometric interpretation. Consider the circle $x^2 + y^2 = 1$ and a point on that circle with coordinates $(\sqrt{1-u^2},\ u)$. If $u \geq 0$ then *arcsin(u)* is the length of the counter clockwise arc on that circle from (1,0) to $(\sqrt{1-u^2},\ u)$. If $u \leq 0$ then $-arcsin(u)$ is the length of the clockwise arc of the circle from (1,0) to $(\sqrt{1-u^2},\ u)$.

Similarly, we define the arccosine function, *arccos*, by considering the graph of $y = cos(x)$ for $x \in [0, \pi]$ and considering the solutions of $cos(v)$ = u for $u \in [-1,1]$ and v restricted to [0, π]. Formally, the function *arccos* is defined by *arcsin* : [-1..1]→[-π/2..π/2] with the rule

$$arccos(u) = v \text{ if } cos(v) = u \text{ and } v \in [0,\pi]$$

The arccosine function has the following geometric interpretation. Consider the point $(u, \sqrt{1-u^2})$ on the circle $x^2 + y^2 = 1$. The quantity *arccos(x)* is the length of the counter-clockwise arc from (1,0) to $(u, \sqrt{1-u^2})$ on the circle $x^2 + y^2 = 1$.

The functions arcsine and arccosine are related by the identity

$$arcsin(u) + arccos(u) = \pi/2.$$

By analogy, there are functions arctangent (arctan), arccotangent (arccot), arcsecant (arcsec) and arccosecant (arccsc). They are defined as follows:

arctan:

arctan : (-∞, ∞) → (-π/2..π/2) by the rule

$$arctan(u) = v \text{ if } tan(v) = u \text{ and } v \in (-\pi/2,\pi/2)$$

Note that the range of arctangent is the range of arcsine without the endpoints. On most calculators you will not find a key sequence for arccotangent. It is approximated by relying on the identity arccot(u) + arctan (u) = π/2 or the identity arccot(u) = arctan(1/u) for x≠0.

arccot:

arccot : (-∞, ∞) → (0,π) by the rule

$$arccot(u)\ v \text{ if } cot(v) = u \text{ and } v \in (0,\pi/2)$$

Note that the range of arccotangent is the range of arccosine without the endpoints.

arcsec:

arcsec : (-∞, -1] ∪ [1,∞) → [0,π/2) ∪ (π/2,π] by the rule

$$arcsec(u) = v \text{ if } sec(v) \text{ and } v \in [0,\pi/2) \cup (\pi/2,\pi].$$

Note that the range of arcsecant is the range of arccosine without the point $\pi/2$. In fact, since $sec(x) = 1/cos(x)$ we have $arcsec(u) = arccos(1/u)$. Generally there is no key sequence for arcsecant on a calulator, and this identity must be used to approximate its values.

arccsc:

arcsec : $(-\infty, -1] \cup [1,\infty) \rightarrow [-\pi/2,0) \cup (0,\pi/2]$ by the rule

$$arccsc(u) = v \text{ if } csc(v) = u \text{ and } v \in [\pi/2,0) \cup (0,\pi/2].$$

Note that the range of arccosecant is the range of arcsine without the point . In fact, since $csc(x) = 1/sin(x)$ we have $arccsc(u) = arcsin(1/u)$. Generally there is no key sequence for arccosecant on a calculator, and this identity must be used to approximate its values.