

Law of Cosines (C)

An extension of the Pythagorean Theorem is the following:

Theorem 800 (Law of Cosines)

Suppose that $\triangle ABC$ is any triangle. Then

$$\|AB\|^2 = \|BC\|^2 + \|AC\|^2 - 2\|BC\| \cdot \|AC\| \cos(\angle BCA)$$

Exercises

1. A triangle has sides of length 2 and 3 with an included angle of 45 degrees. How long is the other side?
2. A triangle has sides of length 5 and 3 and the included angle measures $\pi/6$ radians. How long is the remaining side?
3. A triangle has sides of length 3, 4 and 6. What are the cosines of each of its angles?
4. A triangle is formed by joining the points (2,0), (9,3) and (5,7). What are the cosines of each of its angles?
5. A triangle is formed by joining the points (2,3,4), (0,-2,9) and (11,2,0). What are the cosines of its angles?
6. A triangle is formed by joining the points (0,0,0), (a,b,c) and (x,y,z). Let θ be the angle whose vertex is (0,0,0). Show that

$$\cos(\theta) = \frac{ax + by + cz}{\sqrt{a^2 + b^2 + c^2} \sqrt{x^2 + y^2 + z^2}}$$