Simplifying Expressions Involving Trigonometric and Inverse Trigonometric Functions (C)

Identities involving trigonometric functions and their inverses

We are interested in simplifying such hair raising expressions as $tan(arcsin(x/3))$ or $sin(2arctan(x))$. There is a simple way to deal with all such expressions. Just recall that the value of an inverse trigonometric function is an angle. For example:

- $arctan(x/3)$ is an angle, call θ, whose tangent is $x/3$ and which lies in the interval $(-\pi/2, \pi/2)$. If we assume, for the moment, that x is positive, then there is a right triangle containing θ where the side opposite θ has length x and the side adjacent θ has length 3. Therefore the hypotenuse of this triangle has length $\sqrt{9+x^2}$, so $sin\left(arctan(x/3)\right) = \ x/\sqrt{9+x^2}$. All the domains and ranges have been worked out so that this calculation works even if $x < 0$, but if you want to check for yourself, place the vertex of $arctan(x/3)$ at the origin, make one side the positive x axis, and the let the other side go through $(3,x)$. This side will intersect the unit reference circle at $\left(3/\sqrt{9+x^2}, x/\sqrt{9+x^2}\right)$, so the sine of this angle is $x/\sqrt{9+x^2}$.
- Simplify $sin(2arccos(x))$. First we employ the double angle identity

  $$sin(2arccos(x)) = 2sin(arccos(x))cos(arccos(x)).$$

  One part is easy: $cos(arccos(x)) = x$ As for $sin(arccos(x))$ we proceed as in the previous example: There is a right triangle containing $arccos(x)$ where the hypotenuse of the triangle is of length 1 and whose side adjacent to $arccos(x)$ has length x. The other side of this right triangle must have length $\sqrt{1-x^2}$ so $sin\left(arccos(x)\right) = \ x\sqrt{1-x^2}$. Therefore $sin\left(2arccos(x)\right) = 2x\sqrt{1-x^2}$.