Trigonometric Functions of Angles in a right triangle (C)

Recall from the Pythagorean Theorem that a triangle is a right triangle if and only if the sum of the squares on two of its sides equals the square on its third side.

Suppose that ΔABC is a triangle and that $\angle ABC$ is a right angle. Let $||AB||$ denote the length of the side AB, etc, and let θ denote the angle $\angle BCA$. The sine of θ, denoted $sin(\theta)$, is defined to be

$$\sin(\theta) = \frac{||AB||}{||AC||} = \frac{\text{opposite}}{\text{hypoteneuse}}$$

and the cosine of θ, denoted by $\cos(\theta)$, is defined to be

$$\cos(\theta) = \frac{||BC||}{||AC||} = \frac{\text{adjacent}}{\text{hypoteneuse}}$$

We define the sine of a right angle to be 1, and the cosine of a right angle to be 0. If two angles sum to a straight angle, we define there sines to be equal and their cosines to sum to zero. Finally, we define the sine of a straight angle to to 0 and the cosine of a straight angle to to -1. These conventions define sine and cosine for all angles which may occur as angles of triangles.

It then follows from the Pythagorean Theorem that

$$(sin(\theta))^2 + (cos(\theta))^2 = 1$$

This identity is sometimes called the **Pythagorean Identity**, of for reasons that will become clear below, the **Circle Identity**.

Four additional functions of θ, the tangent (tan), secant (sec), cotangent (cot), and cosecant (csc), are defined as

$$\tan(\theta) = \frac{||AB||}{||BC||}, \quad \cot(\theta) = \frac{||BC||}{||AB||}, \quad \sec(\theta) = \frac{||AC||}{||BC||}, \quad \csc(\theta) = \frac{||AC||}{||AB||}.$$

$$\tan(\theta) = \frac{||AB||}{||BC||}, \cot(\theta) = \frac{||BC||}{||AB||}, \sec(\theta) = \frac{||AC||}{||BC||}, \csc(\theta) = \frac{||AC||}{||AB||}$$

Exercises

1. Suppose that the sides of a triangle have lengths 3, 4 and 5. Verify that this triangle is a a right triangle and give the sine, cosine, tangent, cotangent, secant and cosecant of each angle which is not a right angle.
2. Suppose that the sides of a triangle have lengths 5, 12 and 13. Verify that this triangle is a a right triangle and give the sine, cosine, tangent, cotangent, secant and cosecant of each angle which is not a right angle.
3. Suppose that the hypotenuse of a right triangle has length 6 and one side has length 3. How long is the other side?
4. Suppose that one side of a right triangle has length 4, and one of the angles has tangent equal to 2. How long is the hypotenuse of this triangle?
5. Suppose that the hypotenuse of a right triangle has length 7 and one angle has sine equal to 2/5. How long are the sides of this triangle?