Calculus Survival Facts: Trigonometry

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Abstract

This is the bare minimum of trigonometry you need to be successful in the first semester of calculus.

1 Definitions

1. Radian measure. Suppose that an angle has vertex 0. The radian measure of this angle is the length of the arc in intercepts in a circle of radius R centered at 0 divided by R. Radians are dimensionless.

2. Sine, Cosine and Tangent. Consider the circle of radius 1 centered at $(0,0)$, which has equation $x^2+y^2=1$. Suppose that if we move counter-clockwise a distance a along the circle from $(0,0)$ to (c,s). We then say that the sine of a, written $\sin(a)$, is s, the cosine of a, writen $\cos(a)$, is c, and the tangent of a, written $\tan(a)$, is the slope of the line passing through $(0,0)$ and (c,s). For negative values of a we move $|a|$ in the clockwise direction from $(0,0)$ and apply the same principle. If we end up at (c,s) then we have $\sin(a)=s$, $\cos(a)=c$ and $\tan(a)$ is the slope of the line joining $(0,0)$ and (c,s).

2 Functional Properties

1. Functional properties of sine. The domain of sine is all real numbers, and the range is $[-1,1]$. The sine function is periodic with period 2π, that is $\sin(a+2\pi)=\sin(a)$ for all real numbers a, and if $\sin(a+p)=\sin(a)$ for all real numbers a, then $p=2\pi k$ for some integer k.

2. Functional properties of cosine. The domain of cosine is all real numbers, and the range is $[-1,1]$. The cosine function is periodic with period 2π, that is $\cos(a+2\pi)=\cos(a)$ for all real numbers a, and if $\cos(a+p)=\cos(a)$ for all real numbers a, then $p=2\pi k$ for some integer k.

3. Functional properties of tangent. The domain of tangent is all real except those that can be written in the form $(\pi/2)+k\pi$ for some integer k, and the range is all real numbers. The cosine function is periodic with period π, that is $\tan(a+\pi)=\tan(a)$ for all real numbers a in the domain of tangent, and if $\tan(a+p)=\tan(a)$ for all real numbers a in the domain of tangent, then $p=\pi k$ for some integer k.

4. Principle values:

a	$\sin(a)$	$\cos(a)$	$\tan(a)$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	undefined

3 Identities

1. The six basic identities:

$$\begin{aligned}
\tan(a) &= \frac{\sin(a)}{\cos(a)} \\
(\sin(a))^2+(\cos(a))^2 &= 1 \\
\sin(-a) &= -\sin(a) \\
\cos(-a) &= \cos(a) \\
\sin(a+b) &= \sin(a)\cos(b)+\sin(b)\cos(a) \\
\cos(a+b) &= \cos(a)\cos(b)-\sin(a)\sin(b)
\end{aligned}$$

2. The Law of Cosines: In any triangle, if the sides measure A, B and C and the measure of the angle opposite the side of length A is a, then

$$A^2 = B^2 + C^2 - 2BC\cos(a).$$

4 Additional Trigonometric Functions

1. Cosecant. The cosecant function, denoted by csc, is defined by $\csc(a) = 1/\sin(a)$, and is defined for all a such that $\sin(a) \neq 0$.

2. Secant. The secant function, denoted by sec, is defined by $\sec(a) = 1\cos(a)$ and is defined for all a such that $\cos(a) \neq 0$.

3. Cotangent. The cotangent function, denoted by cot, is defined by $\cot(a) = \cos(a)/sin(a)$, and is defined for all a such that $\sin(a) \neq 0$.

5 Inverse Functions

1. Arccosine. The arccosine function, denoted by arccos, has domain $[-1, 1]$, range $[0, \pi]$ and is defined by $\arccos(x) = a$ if $\cos(a) = x$. Therefore $\cos(\arccos(x)) = x$ for $x \in [-1, 1]$ and $\arccos(\cos(a)) = a$ if $a \in [0, \pi]$.

2. Arcsine. The arcsine function, denoted by arcsin, has domain $[-1, 1]$, range $[-\pi/2, \pi/2]$ and is defined by $\arcsin(x) = a$ if $\sin(a) = x$. Therefore $\sin(\arcsin(x)) = x$ for $x \in [-1, 1]$ and $\arcsin((\sin(a)) = a$ if $a \in [-pi/2, \pi/2]$.

3. Arctangent. The arctangent function, denoted by arctan, has domain all the real numbers, range $(-\pi/2, \pi/2)$ and is defined by $\arctan(x) = a$ if $\tan(a) = x$. Therefore $\tan(\arctan(x)) = x$ for any real number x while $\arctan(\tan(a)) = a$ if $a \in (-\pi/2, \pi/2)$.

4. Arcsecant. The arcsecant function, denoted by arcsec is defined by $\text{arcsec}(x) = \arccos(1/x)$. Its domain is $(-\infty, -1] \cup [1, \infty)$ and its range is $[0, \pi/2) \cup (\pi/2, \pi]$.

5. Arccosecant. The arccosecant function, denoted by arccsc is defined by $\text{arccsc}(x) = \arcsin(1/x)$. Its domain is $(-\infty, -1] \cup [1, \infty)$ and its range is $[-\pi/2, 0) \cup (0, \pi/2]$.

6. Arccotangent. The arccotangent function, denoted by arccot, has domain all real numbers, range $(0, \pi)$ and $\text{arccot}(x) = a$ if $cot(a) = x$. Therefore $\cot(\text{arccot}(x)) = x$ for every real number, while $arccot(\cot(a)) = a$ if $a \in (0, \pi)$.