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Money and Dynamic Credit Arrangements with Private Information

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We construct a model with private information in which consumers write dynamic contracts with financial intermediaries. A role for money arises due to random limited participation of consumers in the financial market. Without defection constraints, a Friedman rule is optimal, the mean and variability of wealth tend to fall with inflation in the steady state, and the welfare effects of inflation are very small. With defection constraints, the effects of inflation on the distribution of welfare and consumption are large, but the effect on average welfare is still small. The relaxation of defection constraints resulting from higher inflation can cause a substantial increase in the real interest rate. *Journal of Economic Literature* Classification Numbers: D8, E4, G2. © 2000 Academic Press

1. INTRODUCTION

The main goal of this paper is to show how the theory of dynamic contracts under private information can be extended to address issues in monetary economics. We construct a dynamic risk-sharing model in which

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endowments are private information, and where there are roles for money and for long-term credit arrangements. The interaction between money and credit is analyzed, and we examine the implications of the model for long-run optimal monetary policy. Solutions are computed, and we measure the welfare effects of inflation.

It is clear that credit instruments currently play an important role in the payments systems of developed economies. Also, a typical feature of most observed credit relationships is that they are long-term. That is, it is usual for consumers and firms to establish enduring relationships with banks and other financial intermediaries, and there are good reasons for this. In particular, long-term contractual arrangements permit the use of dynamic incentives, which in turn promote efficiency. In this paper, we construct a model where dynamic credit arrangements and fiat currency are both useful in carrying out transactions, and where credit can potentially be as important as it appears to be currently in the U.S. economy.

We start with a credit paradigm, using it to derive a role for money, with money and credit coexisting. In particular, the environment we consider is in the spirit of the literature on dynamic contracts under private information [2, 3, 6, 7, 16, 27, 35]. The basic model builds on [16] in that there is no aggregate risk, but individuals face random endowment shocks which are private information. Here, a nonnegativity constraint on consumption and an endogenous interest rate guarantee that a limiting distribution of wealth with mobility will exist.[2]

Consumers make long-term credit arrangements with financial intermediaries, and they also have the opportunity to trade on a competitive money market in each period. There is a transactions role for money which arises due to random limited participation in the financial market. That is, timing within the period is such that a consumer might be in contact with the financial intermediary before full information on her current income is available. It may then be desirable for the consumer to transact on the money market in order to smooth consumption. The notion of limited participation is similar to the constructs used in [15, 21], though here this limited participation occurs at random, and it is idiosyncratic; in each period some fraction of consumers is subject to limited participation while the remaining fraction is not.

A key issue in the literature on dynamic contracts relates to the role of limited commitment. Some authors, for example, [5, 18, 19] have focused on the implications of limited commitment for imperfect insurance in the

[2] In Green's model, in the limit a vanishing fraction of the population receives all of the aggregate endowment. Limiting distributions with mobility are obtained in Atkeson and Lucas [7] with exogenous lower bounds on expected utilities, and in Phelan [23], with endogenous lower bounds.

absence of private information. Others [7, 23] have studied how limited commitment affects the distribution of consumption and wealth in private information economies. In our model, an important feature of the model is the defection constraints implied by limited commitment on the part of consumers, that arise in the dynamic contracting problem of a financial intermediary. In any period, each consumer has the option of defecting from her long-term contract with the financial intermediary, and trading in each succeeding period on the competitive money market. Effectively, the outside option for each consumer is to become a Bewley incomplete markets consumer, as in [9, 10].

An important and novel feature of the model is that the value of the outside option for consumers in our model depends on monetary policy. Specifically, the higher the inflation rate, the lower the value of defecting to money market trading only. This creates an interesting (and new to the literature) effect of long-run inflation, in that dispersion in welfare across the population tends to increase in the steady state as inflation increases, which contributes to an increase in consumption variability. The variability in consumption also rises with inflation as the result of a more conventional mechanism. That is, higher money growth and inflation implies that consumers economize too much on real money balances, so that they are less able to insure against income risk, and the variability in consumption tends to increase.

As a benchmark, we first consider a pure credit model. This is a special case where consumers are never subject to limited participation in the financial market, and so money is not valued and defection is equivalent to going to autarky. We then show that, in a version of the model without defection constraints, a version of the Friedman rule holds. In particular, it is optimal (in that the allocation is identical to the one for the pure credit allocation, which is the solution to a social planner's problem) for the money supply to grow at a rate that equates the real rate of return on money and the real interest rate, but this money growth rate does not equate the rate of return on money with the rate of time preference.

We compute solutions, again starting with the benchmark pure credit economy. Steady state allocations have the property that there is a well-defined limiting distribution with mobility, which is certainly not the case in all dynamic private information contracting models (see, for example, [6, 16, 30]). The critical features of the environment that give rise to a limiting distribution with mobility are that the interest rate is endogenous, consumption is nonnegative, and that the economy has random endowments rather than random preferences (see [2]). The defection constraints are not important for this particular result, as we obtain limiting distributions with mobility without these constraints.

Dropping defection constraints, in which case a Friedman rule is optimal, we examine the quantitative effects of inflation on the distribution of consumption and welfare. Higher inflation tends to reduce the variance of consumption and welfare for the population, though the variance of consumption conditional on wealth tends to rise since the money market is less useful for smoothing consumption at high rates of inflation. However, these effects (except for those on the variability of consumption) are small. For sufficiently high money growth rates, money will cease to be held in this model, but the cost of eliminating currency altogether is approximately 3% of consumption for the average consumer. Money will not be held given inflation rates in excess of about 1500 percent per annum.

Now, for the economy with defection constraints on intermediary contracts, the results are quite different. Here, higher inflation tends to increase (rather than decrease) the variability of welfare and the variability of consumption increases much more with inflation than when there are no defection constraints. This is due to the fact that higher inflation relaxes the defection constraints in a quantitatively important way. Though the quantitative effects of inflation on the distribution of welfare and consumption are much larger when taking into account the ability of consumers to opt out of financial intermediary contracts, the effect of inflation on the welfare of the average consumer remains small. However, average steady state expected utility for the population is now maximized at about 15% annual inflation rather than the Friedman rule, as is the case with no defection constraints.

This is not the first paper to examine money and distribution in a dynamic model with private information. For example, [22] compares the distributional implications of cash-in-advance vs a setup as in [6]. Also, [29] studies a model with dynamic private information and risk neutrality where the optimal allocation can be supported with money or with a bond market.

In Section 2 the model economy is constructed, and Section 3 looks at equilibrium allocations. Section 4 contains a discussion of a pure credit economy as a benchmark, and optimal monetary policy is derived. Section 5 discusses calibration and computational methods, Section 6 looks at quantitative results, and Section 7 is a conclusion.

2. THE MODEL

There is a continuum of consumers with unit mass, each with preferences given by

$$E_0(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t),$$

where $0<\beta<1$, c_t is consumption, and $u(\cdot)$ is the period utility function. Assume that $u(\cdot)$ is strictly increasing and strictly concave with $u(0)=0$. We have $c_t \geqslant 0$ for all t. In each period t, a consumer receives a random endowment y_t, where $y_t \in \{y_0, y_1\}$ with $0 \leqslant y_0 < y_1$. Here, $\Pr[y_t = y_1] = \pi$, where $0<\pi<1$. Endowments are i.i.d. over time and across consumers, and they are private information.

At $t=0$, consumers form coalitions, which we will denote financial intermediaries. We assume that the intermediary is able to observe consumers' assets (here, their money balances) at the beginning of each period. Let consumers be indexed by $i \in [0, 1]$. During any period, there are two possible modes of interaction between consumer i and the financial intermediary, dictated by the realization of the idiosyncratic random variable $s_t^i \in \{0, 1\}$, which is public information at the beginning of the period. Assume that s_t^i is i.i.d. over time and across consumers, with $\Pr[s_t^i = 1] = \rho$, where $0 \leqslant \rho \leqslant 1$. If $s_t^i = 1$, then the consumer receives his/her current endowment y_t at the beginning of the period, and then makes a report $z_t^i \in \{y_0, y_1\}$ to the financial intermediary concerning his/her endowment, following which the intermediary makes a transfer of consumption goods τ_t to the consumer. Alternatively, if $s_t^i = 0$, then at the beginning of period t the consumer receives y_0 units of the consumption good (in all states of the world), then obtains the goods transfer from the financial intermediary, and then finally receives $y_1 - y_0$ units of the consumption good with probability π and zero units with probability $1-\pi$. Note that in the second case the financial intermediary cannot make the transfer contingent on the total endowment the consumer receives during the period. A positive transfer can be viewed as a withdrawal from the consumer's account with the financial intermediary, while a negative transfer can be interpreted as a deposit.

The way in which consumers interact with financial intermediaries in this model is similar to the limited participation structures in [15, 21]. For example, in Lucas's model the member of the Lucas household who purchases consumption goods is not able to access the financial market after a random open market operation occurs in the financial market during the period. In our model, limited participation occurs at random for individuals, and a constant fraction of the population, $1-\rho$, is subject to limited participation in any given period. Random limited participation is here intended to capture the idea that money is held to insure against random circumstances where it is very costly or impossible to access the technology which would allow a credit transaction.

Consumer i enters period t with M_t^i units of fiat money, where M_0^i is given. After receiving transfers from the financial intermediary, consumers can trade money for consumption goods on a competitive market, where the price of consumption goods in terms of money is p_t. The monetary authority makes a transfer of T_t units of fiat money per consumer to financial intermediaries at the beginning of the period.

In any period, after contacting the financial intermediary the consumer may abandon the long-term credit contract. The alternative is to trade in the current period, and each subsequent period, on the competitive money market, i.e. the consumer then behaves in the same way as an agent in a Bewley-type incomplete markets economy where money is the only asset (see [9, 10]). On defection, it is not possible for the consumer to negotiate a contract with another intermediary; intermediary contracts can be agreed to only at the first date. The possibility of defection from the long-term contract leads to a set of defection constraints that the contract must satisfy. In the subsequent analysis, we will determine steady state equilibrium allocations with and without these defection constraints.

3. EQUILIBRIUM ALLOCATIONS

Letting $\bar{M}_t$ denote the per capita stock of fiat money at the end of the period, the monetary authority must meet the constraint

$$\frac{(\bar{M}_t - \bar{M}_{t-1})}{p_t} = \frac{T_t}{p_t},$$

or, defining $\bar{m}_t \equiv \bar{M}_t/p_t$, $\omega_t \equiv T_t/p_t$, and $\gamma_t \equiv p_t/p_{t-1}$,

$$\bar{m}_t - \frac{\bar{m}_{t-1}}{\gamma_t} = \omega_t. \tag{1}$$

That is, current per capita real balances minus per capita real balances in the previous period divided by the gross rate of return on money must equal the per capita money transfer to consumers in units of consumption goods.

We suppose that each intermediary can trade on a bond market, facing the sequence of prices $\{q_t\}_{t=0}^{\infty}$ and the sequence of money prices of consumption goods $\{p_t\}_{t=0}^{\infty}$. We can think of the bond market involving trade in one-period bonds, which sell at price $q_t/(1-q_t)$ in period t, and pay off $1/(1-q_{t+1})$ units of the consumption good in period $t+1$. Consumers cannot trade on the bond market. Defining $m_t = M_t/p_t$, the real balances of a consumer, we suppose that the consumers who are members of the financial intermediary agree on an initial distribution $\psi_0(w_0, m_0)$, i.e., a distribution of expected utility entitlements w_0 and real money balances m_0 among the members of the financial intermediary. Here, the initial distribution of money balances across consumers is given, and expected utility entitlements are to be met through the design by the financial intermediary of a transfer policy and a specification of trading by consumers on the money market.

Along the lines of [2, 3, 6, 7], we think of the financial intermediary as solving a set of component problems to determine the optimal contractual arrangements with each of its members. Specifically, there is a separate cost minimization problem that the intermediary solves for each initial (w_0, m_0), facing $\{q_t\}_{t=0}^{\infty}$ and $\{p_t\}_{t=0}^{\infty}$. We confine attention to steady states, where $q_t = q$, $\gamma_t = \gamma$, and $\omega_t = \omega$ for all t, where q, γ, and ω are constants. Also, the distribution of expected utilities and real money balances across consumers, $\psi(w, m)$, is constant in the steady state. Given that any intermediary has a positive measure of consumers, each intermediary faces the same steady state distribution $\psi(w, m)$, and we can analyze this economy as if there were only one representative financial intermediary.

The component contracting problems can be specified in recursive form by treating w, the consumer's expected utility, and m, his/her real money balances, as state variables, and applying Green's notion [16] of temporary incentive compatibility. Note that w has an interpretation as the account balance of the consumer with the financial intermediary. Letting $v(w, m)$ denote the expected discounted cost to the intermediary of delivering a level of expected utility w at the current date to a consumer holding m units of real balances at that date, the intermediary's problem can be formulated in terms of the following Bellman equation.

$$v(w, m) = \min \left\{ \begin{array}{l} (1-q)[\rho\pi\tau_1(w, m) + \rho(1-\pi)\,\tau_0(w, m) + (1-\rho)\,\tau(w, m)] \\ \qquad + q \left[\begin{array}{l} \rho\pi v[w_{11}(w, m), m_{11}(w, m)] \\ \quad + \rho(1-\pi)\, v[w_{10}(w, m), m_{10}(w, m)] \\ \quad + (1-\rho)\,\pi v[w_{01}(w, m), m_{01}(w, m)] \\ \quad + (1-\rho)(1-\pi)\, v[w_{00}(w, m), m_{00}(w, m)] \end{array} \right] \end{array} \right\} \tag{2}$$

subject to

$$w = (1-\beta) \left[\begin{array}{l} \rho\pi u(y_1 + m + \tau_1(w, m) - \gamma m_{11}(w, m)) \\ \quad + \rho(1-\pi)\, u(y_0 + m + \tau_0(w, m) - \gamma m_{10}(w, m)) \\ (1-\rho)\,\pi u(y_1 + m + \tau(w, m) - \gamma m_{01}(w, m)) \\ (1-\rho)(1-\pi)\, u(y_0 + m + \tau(w, m) - \gamma m_{00}(w, m)) \end{array} \right]$$

$$+ \beta \left[\begin{array}{l} \rho\pi w_{11}(w, m) + \rho(1-\pi)\, w_{10}(w, m) \\ \quad + (1-\rho)\,\pi w_{01}(w, m) + (1-\rho)(1-\pi)\, w_{00}(w, m) \end{array} \right] \tag{3}$$

$$(1-\beta)\,u(y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m))+\beta w_{1i}(w,m)$$
$$\geqslant(1-\beta)\,u(y_i+m+\tau_j(w,m)-\gamma m_{1j}(w,m))+\beta w_{1j}(w,m),$$
$$(i,j)=(1,0),(0,1), \tag{4}$$

$$(1-\beta)\,u(y_i+m+\tau(w,m)-\gamma m_{0i}(w,m))+\beta w_{0i}(w,m)$$
$$\geqslant(1-\beta)\,u(y_i+m+\tau(w,m)-\gamma m_{0j}(w,m))+\beta w_{0j}(w,m),$$
$$(i,j)=(1,0),(0,1), \tag{5}$$

$$(1-\beta)\,u(y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m))+\beta w_{1i}(w,m)$$
$$\geqslant\delta(y_i+m+\max_{j\in\{0,1\}}\tau_j(w,m)),\qquad i=0,1, \tag{6}$$

$$(1-\beta)\,u(y_i+m+\tau(w,m)-\gamma m_{0i}(w,m))+\beta w_{0i}(w,m)$$
$$\geqslant\delta(y_i+m+\tau(w,m)),\qquad i=0,1, \tag{7}$$

$$\begin{aligned} y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m)\geqslant 0,\qquad i=0,1\\ y_i+m+\tau(w,m)-\gamma m_{0i}(w,m)\geqslant 0,\qquad i=0,1\end{aligned} \tag{8}$$

$$m_{ij}(w,m)\geqslant 0,\qquad i,j=0,1. \tag{9}$$

Here, the transfer when the endowment is high [low] and $s_t^i=1$ is $\tau_1(w,m)$ [$\tau_0(w,m)$], while the transfer when $s_t^i=0$ is $\tau(w,m)$. The consumer is assigned an expected utility for the following period, which is $w_{jk}(w,m)$ when $s_t^i=j$ and $y_t=y_k$, for $j,k=0,1$. The intermediary also recommends a quantity of real balances that the consumer is to carry into the next period, $m_{jk}(w,m)$, where the subscripts have the same meaning as for the expected utility assignment. Recommended real balances then imply a recommended transaction for the consumer on the money market.

The financial intermediary minimizes the present discounted value of goods transfers to the consumer. The first constraint in the problem above, (3), is the promise-keeping constraint, which states that contingent transfers, continuation expected utilities, and recommend future money balances are consistent with the consumer receiving expected utility w in the current period. Constraints (4) and (5) are incentive compatibility constraints, which state that it not be in the consumer's interest to misreport his/her endowment to the financial intermediary. The constraints (6) and (7) are defection constraints. That is, in each period, given the consumer's initial money balances, endowment, and transfer from the financial intermediary, it should not be in the consumer's interest to defect from the long-term contract with the financial intermediary and trade in each subsequent period on the money market. Note that if the consumer chooses to defect, she will make an income report to the financial intermediary that maximizes her transfer. The function $\delta(y)$ is the value of defecting with assets y and is defined by the functional equation

$$\delta(y) = \max_{m'} \{(1-\beta)\, u(y - \gamma m') + \beta[\pi\delta(y_1 + m') + (1-\pi)\, \delta(y_0 + m')]\} \tag{10}$$

subject to

$$y - \gamma m' \geqslant 0, \tag{11}$$

$$m' \geqslant 0. \tag{12}$$

Here, m' is the quantity of real balances that the consumer would take into the next period if she defected from the contract with the financial intermediary, (11) is a nonnegativity constraint on consumption in the current period, and (12) is a nonnegativity constraint on real balances. Finally, in the financial intermediary's problem, (8) and (9) are nonnegativity constraints on consumption and money balances, respectively.

Note that the expected utility the consumer receives should she defect from the contract with the financial intermediary is essentially expected utility in a Bewley-type economy [9, 10]. That is, the long-term contract must guarantee the consumer a path for expected utility that never falls below what could be achieved if the consumer can only trade in the money market each period. It is immediately apparent from (10) that the expected utility the consumer receives by defecting is decreasing in the gross inflation rate, γ. That is, $\delta(y)$ is decreasing in γ, so that an increase in γ will tend to relax the defection constraints (6) and (7).

Now, the optimal decision rules $m_{ij}(w, m)$, $w_{ij}(w, m)$, $i, j = 0, 1$, from the solution to (2) subject to (3)–(9) define a mapping, T, where $T\colon S \to S$. Here, S is the set of joint distribution functions on $\Omega \times R^+$, where Ω denotes the set of feasible expected utility assignments. That is, the solution to the financial intermediary's contracting problem implies stochastic laws of motion for the assets of individuals in the population, and therefore tells us how the joint distribution of expected utilities and money balances will evolve over time.

Definition 1. A steady state equilibrium consists of a cost function $v(w, m)$, decision rules $\tau_i(w, m)$, $\tau(w, m)$, $m_{ij}(w, m)$, $w_{ij}(w, m)$, for $i, j = 0, 1$, a value function $\delta(\cdot)$, a price q, and a distribution $\psi(w, m)$, such that, given γ,

(i) $v(w, m)$, $\tau_i(w, m)$, $\tau(w, m)$, $m_{ij}(w, m)$, $w_{ij}(w, m)$, for $i, j = 0, 1$, solve (2) subject to (3)–(9) given q and $\delta(\cdot)$;

(ii) $\delta(y)$ solves (10) subject to (11) and (12);

(iii)

$$\iint \begin{bmatrix} \rho\pi[m+\tau_1(w,m)-\gamma m_{11}(w,m)] \\ \quad +\rho(1-\pi)[m+\tau_0(w,m)-\gamma m_{10}(w,m)] \\ \quad +(1-\rho)\,\pi[m+\tau(w,m)-\gamma m_{01}(w,m)] \\ \quad +(1-\rho)(1-\pi)[m+\tau(w,m)-\gamma m_{00}(w,m)] \end{bmatrix} d\psi(w,m)=0; \quad (13)$$

(iv) $T\psi(w,m)=\psi(w,m)$.

In the definition of a steady state equilibrium, (i) states that the financial intermediary optimizes given prices and the alternatives that consumers have to the intermediary contract; (ii) states that consumers trade optimally in the money market should they defect from the intermediary contract; (iii) is a market-clearing condition stating that the net transfers consumers receive from the financial intermediary and in the money market are zero; and (iv) is the steady state condition, i.e. the distribution of expected utilities and money balances across consumers is invariant under the transformation implied by the optimal decision rules in the intermediary contract. Note that (iii) implies that the government budget constraint, (1), is satisfied.

The steady state equilibrium allocation is a stationary allocation, since if the economy starts in period 0 with a price q and a distribution $\psi(w,m)$ satisfying the definition, then that price and distribution are constant forever. It is hard to imagine though, what optimization problem for the financial intermediary would yield the stationary distribution as the outcome at date 0. However, an alternative interpretation of the problem (2) subject to (3)–(9) is that this is the component planning problem solved by a social planner. That is, we could imagine a continuum of planners, where any individual planner is responsible for the consumers with a given (w,m), can trade at the price q, and treats γ and $\delta(\cdot)$ as given. There is a coordinating social planner, who is restricted to choosing a constant price q satisfying the resource constraint (13), and to choosing a stationary distribution $\psi(w,m)$. All of these choices are made treating γ, the gross inflation rate, as being fixed. There is a higher stage of the problem at which this coordinating social planner chooses γ, but we will leave that stage of the problem aside for the time being. Given this alternative notion of how the "credit" system works, the above definition of a steady state equilibrium allocation becomes the definition of a stationary efficient allocation. The stationary efficient allocation from the planner's point of view, given γ, is identical to the steady state equilibrium allocation with financial intermediation.

One could imagine solving for a steady state equilibrium allocation as follows. First, given γ, find the function $\delta(y)$ that solves (10) subject to (11) and (12). Then, given $\delta(\cdot)$ and γ, choose a value for q and solve (2) subject to (3)–(9). The optimal decision rules for this problem then imply a steady state distribution $\psi(w, m)$, which we then need to plug into (13) to check whether the aggregate resource constraint holds. If it does not hold, then q needs to be adjusted appropriately, and (2) subject to (3)–(9) solved again, etc. A higher q puts a higher price on future transfers to consumers relative to current transfers, so that higher values for q will tend to decrease the net claims on consumption goods that the financial intermediary has to meet in the steady state. Thus, if the left-hand side of (13) is positive (negative) when we check the resource constraint, then q should be adjusted up (down). This is a sketch of the algorithm we use for computing solutions in Sections 5 and 6.

Note that money plays two roles here. First, money balances allow consumers to self-insure against random limited participation in the financial market. This can be interpreted as a transactions role for money. Second, current money balances communicate to the intermediary the endowment shock of the consumer in the previous period, so that money acts as a record-keeping device.[3]

Equilibrium allocations will have the property that $q \leqslant \gamma$, as otherwise (13) would not hold. That is, if $q > \gamma$, financial intermediaries could make infinite profits by borrowing on the bond market and having consumers acquire higher-yielding money on the competitive money market. In the case where $q < \gamma$ (i.e., money is strictly dominated in rate of return), we will have $m_{ij}(m, w) = 0$ for $i = 1$ and for $(i, j) = (0, 0)$. To see this, suppose that $m_{ij}(m, w) > 0$ for $i = 1$ and for some j. Then, when $s_t^i = 1$ and $y_t = y_j$, the consumer trades off claims to current consumption for claims to future consumption by holding money balances. However, this can be done more efficiently by the financial intermediary, which can reduce the consumer's transfers today and increase future transfers, while facing a higher interest rate than the consumer. The intermediary can thus meet all the constraints in the above optimization problem while reducing the value of the objective function, so it cannot be optimal to have $m_{ij}(m, w) > 0$ for $i = 1$. A similar argument holds for $(i, j) = (0, 0)$, given that incentive compatibility requires that $m_{01}(m, w) \geqslant m_{00}(m, w)$.

While the problem (2) subject to (3)–(9) may appear formidable, it is possible to simplify it considerably. We have the following proposition.

PROPOSITION 1. *In problem* (2) *subject to* (3)–(9), *the cost function takes the form*

$$v(w, m) = -(1-q)\, m + \theta(w),$$

[3] The record-keeping role of money has been explored by Townsend [32, 33] in two-period environments with spatial separation.

and the optimal decision rules are $\tau_i(m, w) = \tau_i^*(w) - m$, *for* $i = 0, 1$, $\tau(m, w) = \tau^*(w) - m$, *and* $w_{ij}(w, m) = w_{ij}^*(w)$, $m_{ij}(w, m) = m_{ij}^*(w)$ *for* $i, j = 0, 1$, *where* $\theta(w)$, $\tau_i^*(w)$, $\tau^*(w)$, $w_{ij}^*(w)$, $m_{ij}^*(w)$, $i, j = 0, 1$, *solve*

$$\theta(w) = \min \left\{ \begin{array}{l} (1-q)[\rho\pi\tau_1^*(w) + \rho(1-\pi)\,\tau_0^*(w) + (1-\rho)\,\tau^*(w)] \\ \quad + q \left[\begin{array}{l} \rho\pi[\theta(w_{11}^*(w)) - (1-q)\,m_{11}^*(w)] \\ \quad + \rho(1-\pi)[\theta(w_{10}^*(w)) - (1-q)\,m_{10}^*(w)] \\ \quad + (1-\rho)\,\pi[\theta(w_{01}^*(w)) - (1-q)\,m_{01}^*(w)] \\ \quad + (1-\rho)(1-\pi)[\theta(w_{00}^*(w)) - (1-q)\,m_{00}^*(w)] \end{array} \right] \end{array} \right\} \tag{14}$$

subject to

$$w = (1-\beta) \left[\begin{array}{l} \rho\pi u(y_1 + \tau_1^*(w) - \gamma m_{11}^*(w)) \\ \quad + \rho(1-\pi)\,u(y_0 + \tau_0^*(w) - \gamma m_{10}^*(w)) \\ (1-\rho)\,\pi u(y_1 + \tau^*(w, m) - \gamma m_{01}^*(w)) \\ (1-\rho)(1-\pi)\,u(y_0 + \tau^*(w, m) - \gamma m_{00}^*(w)) \end{array} \right]$$

$$+ \beta \left[\begin{array}{l} \rho\pi w_{11}^*(w) + \rho(1-\pi)\,w_{10}^*(w) \\ \quad + (1-\rho)\,\pi w_{01}^*(w) + (1-\rho)(1-\pi)\,w_{00}^*(w) \end{array} \right] \tag{15}$$

$$\begin{aligned} &(1-\beta)\,u(y_i + \tau_i^*(w) - \gamma m_{1i}^*(w)) + \beta w_{1i}^*(w) \\ &\quad \geqslant (1-\beta)\,u(y_i + \tau_j^*(w, m) - \gamma m_{1j}^*(w)) + \beta w_{1j}^*(w), \\ &\quad (i, j) = (1, 0), (0, 1), \end{aligned} \tag{16}$$

$$\begin{aligned} &(1-\beta)\,u(y_i + \tau^*(w) - \gamma m_{0i}^*(w)) + \beta w_{0i}^*(w) \\ &\quad \geqslant (1-\beta)\,u(y_i + \tau^*(w) - \gamma m_{0j}^*(w)) + \beta w_{0j}^*(w), \\ &\quad (i, j) = (1, 0), (0, 1), \end{aligned} \tag{17}$$

$$\begin{aligned} &(1-\beta)\,u(y_i + \tau_i^*(w) - \gamma m_{1i}^*(w)) + \beta w_{1i}^*(w) \\ &\quad \geqslant \delta(y_i + \max_{j \in \{0, 1\}} \tau_j^*(w)), \qquad i = 0, 1, \end{aligned} \tag{18}$$

$$\begin{aligned} &(1-\beta)\,u(y_i + \tau^*(w) - \gamma m_{0i}^*(w)) + \beta w_{0i}^*(w) \\ &\quad \geqslant \delta(y_i + \tau^*(w)), \qquad i = 0, 1, \end{aligned} \tag{19}$$

$$\begin{aligned} y_i + \tau_i^*(w) - \gamma m_{1i}^*(w) \geqslant 0, \qquad i = 0, 1 \\ y_i + \tau^*(w) - \gamma m_{0i}^*(w) \geqslant 0, \qquad i = 0, 1 \end{aligned} \tag{20}$$

$$m_{ij}^*(w) \geqslant 0, \qquad i, j = 0, 1. \tag{21}$$

Proof. In the problem (2) subject to (3)–(9), conjecture that the cost function takes the form $v(w, m) = -(1-q)\, m + \theta(w)$, and substitute this on the left-hand side and right-hand side of (2). Then, substitute $\tau_i(m, w) = \tau_i^*(w) - m$, for $i = 0, 1$, $\tau(m, w) = \tau^*(w) - m$, and $w_{ij}(w, m) = w_{ij}^*(w)$, $m_{ij}(w, m) = m_{ij}^*(w)$ for i, $j = 0, 1$, in (2)–(9). Rearranging, we get (14) subject to (15)–(21).

Proposition 1 states that the problem (2) subject to (3)–(9), which involves two state variables, can be collapsed to a single-state-variable problem, (14) subject to (15)–(21). The key to this result is that the optimal transfer policy for the financial intermediary involves, as a starting point, confiscating the consumer's money balances at the beginning of each period. Then, the current money balances of the consumer are irrelevant for determining the consumer's optimal consumption allocation, continuation expected utility, and money balances as of the beginning of the next period, and the cost function $v(w, m)$ is linear in current money balances. For computational purposes, the reduction from two to one in the dimension of the state space is very important.

4. A PURE CREDIT ECONOMY

In this section we wish to consider the special case where $\rho = 1$, and where money is not valued in equilibrium. If $\rho = 1$ and money has no value, the expected utility from deviating from the long-term contract with the financial intermediary is the current utility from consuming in the present plus the discounted expected utility from autarky, i.e. $\delta(y) = (1-\beta)\, u(y) + \beta[\pi u(y_1) + (1-\pi)\, u(y_0)]$ given assets y. We can think of this special case as a pure credit economy.

The financial intermediary's problem when $\rho = 1$ and money is not valued reduces to a problem which is similar to the one considered in [16], except that here we have a nonnegativity constraint on consumption, the interest rate is endogenous, and we impose defection constraints. The problem is

$$z(w) = \min \left\{ \begin{array}{l} (1-q)[\pi\tau_1(w) + (1-\pi)\, \tau_0(w)] \\ \quad + q[\pi z(w_1(w)) + (1-\pi)\, z(w_0(w))] \end{array} \right\} \tag{22}$$

subject to

$$w = (1-\beta)[\pi u(y_1 + \tau_1(w)) + (1-\pi)\, u(y_0 + \tau_0(w))] + \beta[\pi w_1(w) + (1-\pi)\, w_0(w)] \tag{23}$$

$$(1-\beta)\, u(y_1+\tau_1(w))+\beta w_1(w) \geqslant (1-\beta)\, u(y_1+\tau_0(w))+\beta w_0(w) \tag{24}$$

$$(1-\beta)\, u(y_0+\tau_0(w))+\beta w_0(w) \geqslant (1-\beta)\, u(y_0+\tau_1(w))+\beta w_1(w) \tag{25}$$

$$(1-\beta)\, u(y_1(w))+\beta w_1(w) \geqslant (1-\beta)\, u(y_1+\max_{j\in\{0,1\}} \tau_j(w))+\beta[\pi u(y_1)+(1-\pi)\, u(y_0)] \tag{26}$$

$$(1-\beta)\, u(y_0+\tau_0(w))+\beta w_0(w) \geqslant (1-\beta)\, u(y_0+\max_{j\in\{0,1\}} \tau_j(w))+\beta[\pi u(y_1)+(1-\tau)\, u(y_0)] \tag{27}$$

$$y_i+\tau_i(w)\geqslant 0, \qquad \text{for} \quad i=0, 1. \tag{28}$$

Here, $z(w)$ is the cost function, $\tau_1(w)$ $[\tau_0(w)]$ is the transfer in the high [low] endowment state, and $w_1(w)$ $[w_0(w)]$ is the expected utility entitlement in the following period when the current endowment is high [low]. Equation (23) is the promise-keeping constraint, inequalities (24) and (25) are the incentive constraints, (26) and (27) are the defection constraints, and (28) are nonnegativity constraints on consumption.

The definition of a steady state equilibrium here is the direct analogue of Definition 1, except that here the condition

$$\int [\pi\tau_1(w)+(1-\pi)\, \tau_0(w)]\, d\psi(w)=0 \tag{29}$$

replaces (13), where $\psi(w)$ is the steady state distribution of expected utility entitlements across consumers.

The pure credit economy is a useful benchmark, as we know that the steady state equilibrium allocation for this economy is efficient. That is, the problem (22)–(28) is the component planning problem for a social planner charged with efficiently delivering expected utility w to a consumer, given q. The steady state allocation is then the limiting distribution achieved by a social planner whose goal is to deliver some initial distribution of expected utilities in an efficient manner, subject to meeting the aggregate resource constraint each period (as in [7]). Alternatively, the pure credit steady state equilibrium allocation is the efficient distribution of expected utilities attained by a social planner restricted to choosing from stationary expected utility distributions.

We will define a steady state allocation to be "efficient" if it is the limiting distribution chosen by the social planner. The steady state equilibrium allocation for the pure credit economy weakly dominates any steady state equilibrium allocation with $\rho\neq 1$ in efficiency terms. There are two reasons for this. First, if $\rho\neq 1$ and a consumer is subject to random limited participation, then consumption smoothing can only be achieved by having the consumer trade on the money market, as it is impossible to smooth consumption by way of transfers from the financial intermediary. Now suppose, on the one hand, that the financial intermediary wishes the consumer to consume less in the current period and more in the future. Since there is rate-of-return-dominance of money, it is always at least as efficient to accomplish this intertemporal reallocation of consumption by having the financial intermediary acquire bonds in the current period and sell them in the future, rather than having the consumer acquire money and sell it in the future. Supposing, on the other hand, that the financial intermediary wishes the consumer to consume more today and less in the future, if this is done through the financial intermediary then the intermediary can accomplish this by borrowing on the bond market. The consumer is not able to do the same thing on the money market due to the nonnegativity constraint on money balances.

The second reason the pure credit allocation weakly dominates any steady state equilibrium allocation with $\rho \neq 1$ is that monetary exchange in general makes defection from the intermediary contract more attractive. For any $\gamma > 0$ the value of defection will weakly dominate autarky, and strictly dominate in the case where the consumer would choose to hold positive cash balances in some future state of the world if defection occurs. That is, autarky is always an option if the consumer defects from the intermediary contract, but in general the consumer can do better than autarky by trading on the money market. Thus, the defection constraints will be more severe when $\rho \neq 1$, and therefore some allocations which could be achieved under pure credit cannot be achieved when $\rho \neq 1$.

The first cause of the deviation from the benchmark pure credit allocation is a standard type of distortion caused by inflation, which will be reflected in a suboptimal "quantity of money," and a positive nominal interest rate. The second cause is more unconventional. Because consumers have an alternative to the long-term contract with the financial intermediary, this puts constraints on the types of contracts that intermediaries can offer. Further, since the alternative involves trading on the money market, inflation will affect the value of this outside option.[4]

Optimal Money Growth with No Defection Constraints

The first step in evaluating the welfare effects of inflation in this model is to consider how γ affects the steady state equilibrium allocation when we drop the defection constraints from the financial intermediary's contracting

[4] Corbae and Blume [12] have explored the idea that an outside option of trading on a competitive money market can affect the efficiency of long-term contracts in an environment without private information. They show that money can be valued for no other reason than that it provides this outside option, but the fact that it provides it implies that valued fiat money is inefficient.

problem. This allows us to focus on the more standard effects of inflation, and to highlight more clearly why the non-standard effects of inflation in the model (due to the dependence of the value of defecting on the inflation rate) are interesting.

In the financial intermediary's problem, (2) subject to (3)–(9), we drop the defection constraints (6) and (7). Also, in the benchmark pure credit version of the financial intermediary's problem, (22) subject to (23)–(28), we drop the defection constraints (26) and (27).

PROPOSITION 2. *Dropping defection constraints, if* $\rho \neq 1$ *and* $\gamma = q$ *then the steady state equilibrium allocation is identical to the pure credit steady state equilibrium allocation (the steady state equilibrium allocation when* $\rho = 1$*).*

Proof. First, conjecture that when $\gamma = q$, a solution to (2) subject to (3)–(9) (absent (6) and (7)) is $v(w, m) = z(w) - (1-q)\, m$, $\tau_1(w, m) = \tau_1(w) - m$, $\tau_0(w, m) = \tau_0(w) - m$, $\tau(w, m) = \tau_0(w) - m$, $m_{01}(w, m) = (\tau_0(w) - \tau_1(w))/q$, $m_{ij}(w, m) = 0$ for $(i, j) = (1, 1), (1, 0), (0, 0)$, $w_{ij}(w, m) = w_j(w)$ for $i, j = 0, 1$, where $[z(w), \tau_1(w), \tau_0(w), w_1(w), w_0(w)]$ is the solution to the pure credit problem, (22) subject to (23)–(28), dropping (26) and (27). We have already shown that $m_{ij}(w, m) = 0$ for $(i, j) = (1, 1), (1, 0), (0, 0)$. Now, substituting in the Bellman equation (2), we obtain

$$z(w) - (1-q)\, m$$
$$= \min \left\{ \begin{array}{l} (1-q)[\rho\pi\tau_1(w) + \rho(1-\pi)\, \tau_0(w) + (1-\rho)\, \tau_0(w) - m] \\ + q \left[\begin{array}{c} \pi z[w_1(w)] + (1-\pi)\, z[w_0(w)] \\ -(1-\rho)\, \pi \dfrac{(1-q)[\tau_0(w) - \tau_1(w)]}{q} \end{array} \right] \end{array} \right\}.$$

Simplifying, we get

$$z(w) = \min \left\{ \begin{array}{l} (1-q)[\pi\tau_1(w) + (1-\pi)\, \tau_0(w)] \\ \quad + q[\pi z[w_1(w)] + (1-\pi)\, z[w_0(w)]] \end{array} \right\},$$

which is (22). Similarly, substituting in the constraints (3)–(8), we obtain the constraints (23)–(28), and constraint (9) is also satisfied. Thus, the solution to the financial intermediary's problem is the same for any ρ when $\gamma = q$, and given the market-clearing value of q for the pure credit economy in the steady state, $q = q^*$. Now, it remains to be shown that in the steady state the aggregate resource constraint, (13), is satisfied given $q = q^*$. Substituting in (13), and given (29), it is straightforward to show that this is the case.

Now, since we know that the pure credit steady state allocation is efficient, Proposition 2 tells us that, when $\rho \neq 1$ and there are no defection constraints, $\gamma = q$ implies that the steady state equilibrium allocation is efficient. Thus, a Friedman rule is optimal here when there are no defection constraints. That is, since the only monetary distortion in this special case is the standard type of intertemporal distortion caused by inflation which exists in many monetary models (cash-in-advance and money-in-the-utility-function models, for example), it would be surprising if a Friedman rule were not optimal. However, note that this Friedman rule is modified somewhat from its form in [14]. Here, it is optimal to equate the real returns on bonds (accessible only to financial intermediaries) and money, as is consistent with [14], but this need not involve equating the rate of time preference of consumers with the market rate of return. In general, it will be the case that $q > \beta$ for the pure credit allocation, as in [3, 7].[5] Note that the logic of the proof of Proposition 2 suggests that a Friedman rule would be optimal if we considered equilibria where the bond price and the distribution of assets were not constant over time. However, we have not proved this.

For the economy with the possibility of deviation from intermediary contracts, there is no Friedman rule result as above. This is due to the fact that, if we consider the pure credit equilibrium allocation, which is the solution to (22) subject to (23)–(28), the defection constraints (6) and (7) will not collapse to (26) and (27) when $q = \gamma$. That is, at the Friedman rule rate of inflation, if a consumer defects she can do better than autarky because she is able to smooth consumption by trading on the money market. We therefore cannot replicate the pure credit allocation by driving the nominal interest rate to zero in the steady state, if there are defection constraints.[6]

5. CALIBRATION AND COMPUTATION

We use the economy without defection constraints as a benchmark, setting parameters so that the steady state allocation matches observed

[5] Note that transfers and real money balances are not uniquely determined when $\gamma = q$, since the financial intermediary is indifferent when trading off current for future consumption, between giving the consumer less transfers in the present and more transfers in the future, and requiring the agent to acquire money balances in the present and spend them in the future. Thus, we get the standard type of indeterminacy result that holds under a Friedman rule in many monetary models.

[6] This is not to say, however, that there is not some optimal inflation rate when there are defection constraints. It is difficult to define optimality in this case however, as it is not clear whether the approach of breaking a planner's problem down into component problems is valid when there are defection constraints that depend on the value of money on the competitive market.

features of the U.S. economy. We interpret a period as one quarter, and set y_0, y_1, and π so as to match the variability in quarterly household income. Using data from the Panel Study of Income Dynamics (PSID), [1] argues that a first-order autoregression closely matches the time series properties of annual earnings, with a range of 0.23 to 0.53 for the first-order serial correlation coefficient, and a coefficient of variation in unconditional earnings of 20 to 40 percent. We model the endowment process of an individual consumer in the model as i.i.d., as it is not tractable to introduce serial correlation in endowments. Then, taking a period to be one quarter, if the coefficient of variation of income is 30 percent for annual data, then for quarterly data the coefficient of variation would be 60 percent. Thus, we set $\pi = 0.5$, $y_0 = 1 - \varepsilon$, and $y_1 = 1 + \varepsilon$, with $\varepsilon = 0.6$. The utility function we use is $u(c) = 1 - e^{-\alpha c}$, with $\alpha = 1$, which implies a coefficient of relative risk aversion of unity at the mean endowment. The constant absolute risk aversion utility function is convenient for computational purposes here as it is bounded. The remaining parameters, ρ, β, and γ, were set so as to produce an equilibrium steady state allocation which would match observed average real interest rates, inflation rates, and the observed use of currency relative to credit in transactions. Rom the real business cycle literature [25], the real interest rate is taken to be 1% per quarter, so in a steady state we want $q = 0.99$. A survey of households by the Federal Reserve [8], conducted in 1984, finds that 24% of the current value of household transactions is carried out in currency. In the model, the steady state quantity of currency transactions is

$$\tfrac{1}{2}\iint \{[1-(1-\rho)\,\pi](m+\omega)$$
$$+(1-\rho)\,\pi\,|m+\omega+-\gamma m_{01}(m,w)|\}\; d\psi(m,w),$$

and the steady state quantity of credit transactions is

$$\tfrac{1}{2}\iint [\rho\pi\,|\tau_1(m,w)| + \rho(1-\pi)\,|\tau_0(m,w)| + (1-\rho)\,|\tau(m,w)|]\; d\psi(m,w),$$

where we multiply by $\frac{1}{2}$ in each case to avoid counting each side of a given transaction twice. When the Federal Reserve survey was done, the inflation rate was approximately 1% per quarter, so we set $\gamma = 1.01$ for calibration purposes.

Solutions were computed for the economy without defection constraints as follows. First, grids were chosen for the two state variables, w and m. The lower bound on expected utility, w, is $\pi u(y_1 - y_0) + (1-\pi)\,u(0)$, which is the minimum incentive compatible level of expected utility that can be imposed on a consumer, and the lower bound on m is zero. Since choice variables in the financial intermediary's problem are independent of

m, it is only necessary to solve the problem at each point along the w grid, and for a single value for m, say $m=0$, and then use this solution to determine what the solution is for all points on the grid. We start with an initial guess for $q \in (\beta, 1)$, and make an initial guess for the function $\theta(w)$. Then, value iteration is used to arrive at the solution for $\theta(w)$ given q. At each iteration, $\theta(w)$ is updated by fitting a third-order Chebychev polynomial (plus an additional term, $1/(1-w)$, which helped to give a good fit), to the values computed for the cost function at points on the grid on the previous iteration. When convergence is achieved given q, then the decision rules are interpolated across a finer grid, and a matrix of Markov transition probabilities for the state w is constructed as an approximation using a lottery over the two closest grid points. A limiting distribution over w is computed, the analogue of the left-hand side of (13) is evaluated, and q is updated according to a bisection method. Then value-iteration is performed again, etc.

To match the observed real interest rate and the evidence from the Federal Reserve survey on household transactions, we set $\beta=0.99$ and $\rho=0.81$, in computing the allocation for the economy without defection constraints. This implies that $q=0.99$ (actually slightly greater than β, but the difference is on the order of 10^{-5}) and that currency accounts for 24% of the value of transactions.

The procedure for computing solutions with defection constraints is similar, but first we need to use the functional equation (10) to determine the value function $\delta(y)$ (again, using value iteration, and interpolation of the value function with a Chebychev polynomial). Then, the lower bound for expected utilities for intermediary contracts is given by

$$\underline{w} = \pi\delta(y_1 - y_0) + (1-\pi)\,\delta(0).$$

Note that $\underline{w} \geqslant \pi u(y_1) + (1-\pi)\, u(y_0) > \pi u(y_1 - y_0) + (1-\pi)\, u(0)$, the lower bound on expected utilities without defection constraints. That is, without defection constraints the worst incentive compatible treatment a consumer could receive is to give up y_0 (the low endowment) each period. With defection constraints, at worst the intermediary could force the agent to give up y_0 in the current period, with the consumer defecting to the money market thereafter. Once the consumer abandons the long-term contract, she can do at least as well as autarky, which is better than giving up y_0 in each state at each date.

6. COMPUTATIONAL RESULTS

We first consider the pure credit case (i.e., $\rho=1$) as a benchmark, noting that the allocations here will be identical to those with $\rho \neq 1$ and $\gamma = q$

(a Friedman rule) when there are no defection constraints. We then compare these benchmark allocations to allocations with various inflation rates greater than the inflation rate for the Friedman rule allocation, for the case with no defection constraints. Finally, we consider the case with defection constraints, and look at similar inflationary experiments.

Pure Credit Economy

To get an idea of some of the general features of the solutions, it helps to look at allocations with low discount factors. We first consider an economy with $\rho = 1$ (the pure credit case), $\beta = 0.5$, and no defection constraints, with the other parameters set as discussed in Section 5. The results are in Figs. 1–3. Figure 1 shows $w_1(w)$ ($w1$ in the figure) and $w_0(w)$ ($w0$ in the figure). Here, note that, for low levels of w, future expected utility remains at the lower bound on expected utilities, $\pi u(y_1 - y_0)$, when the consumer receives a low endowment in the current period. In general, given the binding incentive constraint [(24) binds at the optimum, but (25) does not], expected utility rises when the consumer receives a high endowment, and falls when there is a low endowment. Figure 2 shows the limiting distribution of consumers across expected utility entitlements. Here, note that

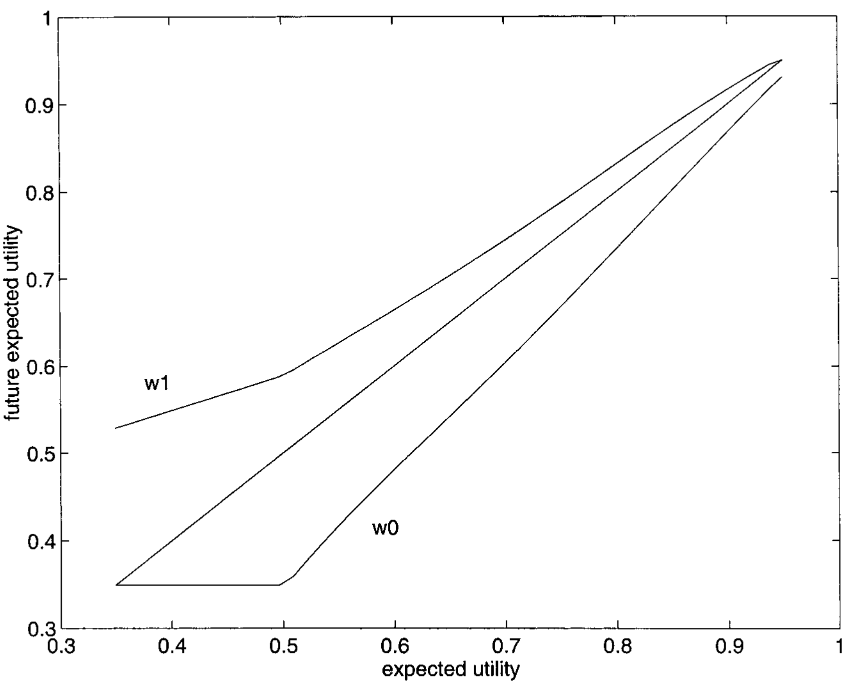

FIG. 1. Rho = 1, beta = 0.5, no defection.

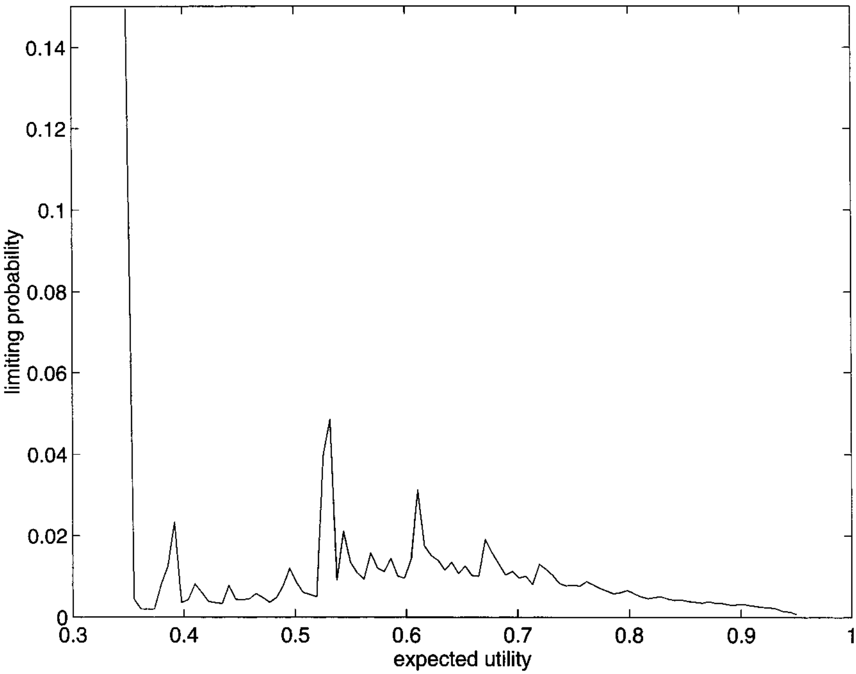

FIG. 2. Rho = 1, beta = 0.5, no defection.

a significant fraction of consumers, over 14%, are effectively credit-constrained in that they are at the lower bound on expected utilities. Half of these consumers, those with low endowments, will consume zero, as we see in Fig. 3. Note in Fig. 3 that the gap between consumption in the good state ($c1$) and consumption in the bad state ($c0$) is larger at the low end of the distribution, where the binding lower bound on expected utilities mitigates the incentives available to induce truthful reporting on the part of consumers. Due to the low discount factor, intertemporal incentives are not very good, and there is on average a fairly large gap between consumption in the high-endowment state and consumption in the low-endowment state.

We next consider pure credit economies which axe calibrated to the data. That is, we use the same parameters as for the previous example, except that $\beta = 0.99$. Note again that the allocations here will be identical, in the case where there are no defection constraints, to what we would get with $\rho \neq 1$ under a Friedman rule in place ($\gamma = q$). For the results, see Figs. 4 and 5. We do not show plots of $w_1(w)$ and $w_0(w)$ (as in Fig. 1) as these functions are very close to the 45-degree line. Figure 4 shows equilibrium steady state distributions of expected utilities (w) across the population, for

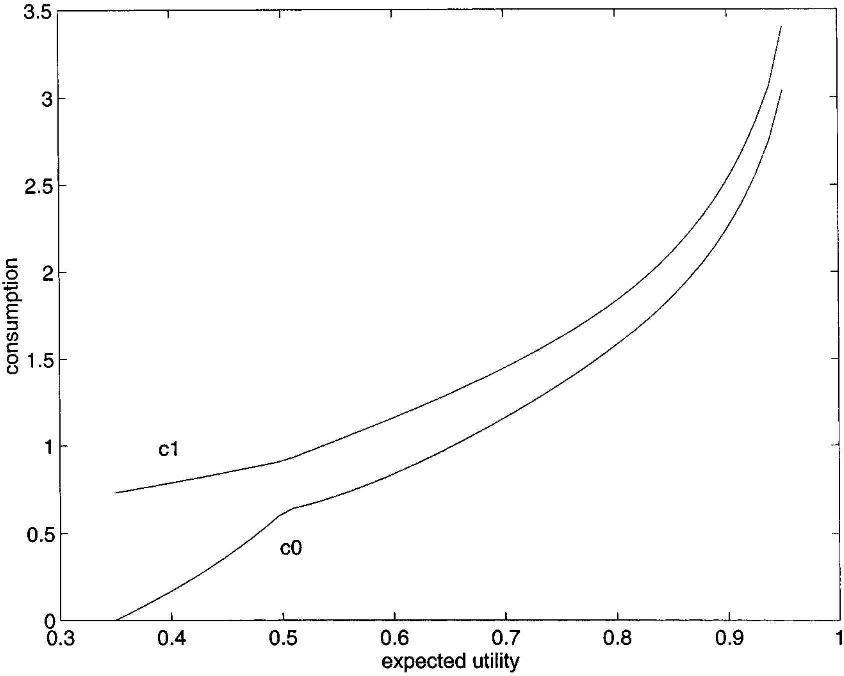

FIG. 3. Rho = 1, beta = 0.5, no defection.

the cases where the defection constraints (26) and (27) are imposed and where they are not.

For the solutions with defection constraints in place, the defection constraints bind only at low levels of expected utility. For the lowest expected utilities, the incentive constraints (24) and (25) do not bind, but the defection constraints (26) and (27) do, so that in this range insurance is constrained by the threat of defection. As expected utility increases, (24) binds, (25) does not, (26) binds and (27) does not, so that insurance is constrained in this range by the incentive constraint of the high-income consumer and the threat of defection by this agent. For expected utilities above some threshold, the defection constraints do not bind, and only the incentive constraint for the high-income consumer, (24), binds. It is straightforward to show that if we eliminate private information from the model, allowing only for possible defection from the intermediary arrangement, then given the paramaterization here a perfect insurance allocation is supported. That is, the defection constraints are satisfied given an allocation where everyone consumes unity forever. Thus, it is the defection constraints in conjunction with the private information friction that matters here; the defection constraints are irrelevant on their own.

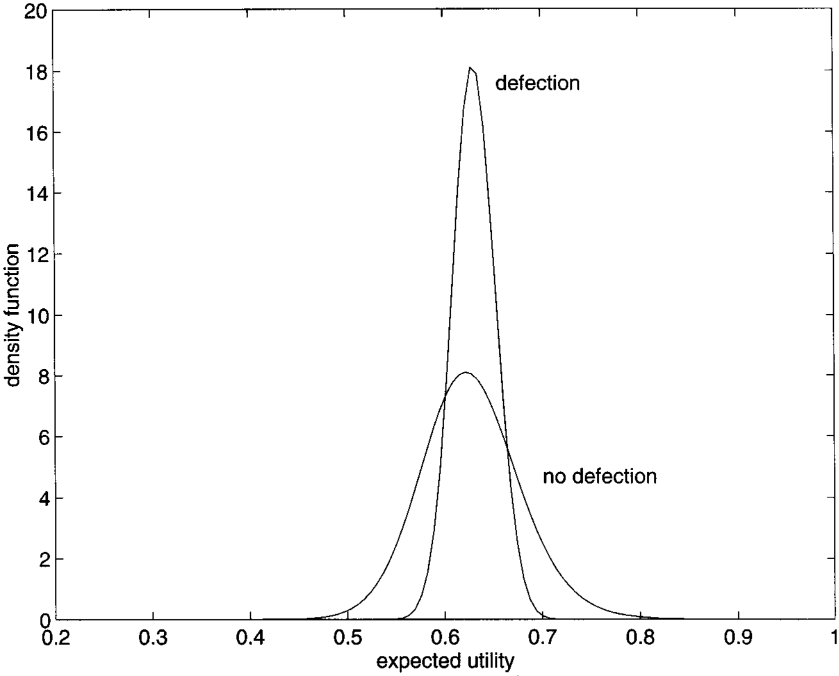

FIG. 4. Pure credit distributions.

Note that the dispersion in expected utilities is higher without the defection constraints. That is, the defection constraints impose a lower bound on expected utilities, which eliminates the lower tail of the distribution. Since the market-clearing condition (29) must hold, and since average consumption, conditional on the level of expected utility, increases with w (see Fig. 5), steady state q will tend to adjust to cut off the upper tail of the distribution as well, so that the imposition of defection constraints must reduce variability in w in the steady state. In Fig. 4, we see that there are essentially no credit-constrained consumers in the steady state either with or without defection constraints. This is due to the fact that, with a high discount factor, the range of expected utilities for which a low endowment will imply that next period's expected utility is at the lower bound, is negligible, in contrast to Fig. 1. In Fig. 5, where $c0$ ($c1$) denotes consumption in the low (high) endowment state when there are no defection constraints, there is very little variability in consumption, conditional on expected utility, i.e. in this sense the solution is very close to perfect insurance. This reflects the fact that, since consumers do not discount the future much, intertemporal incentives work very well. However, note in Fig. 4 that the variability in expected utilities is substantial. Therefore, since the slopes of

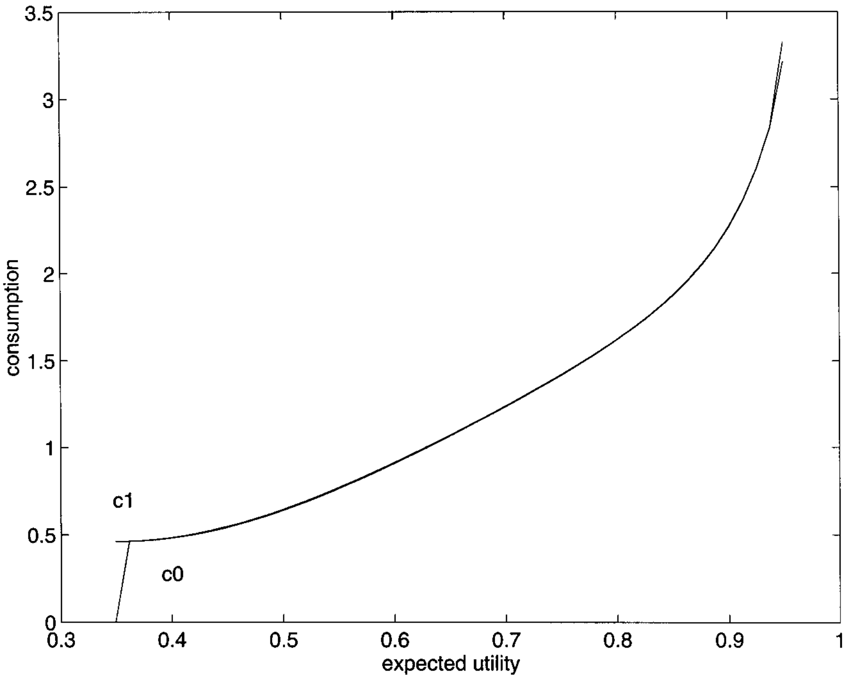

FIG. 5. Rho = 1, beta = 0.99, no defection.

the consumption profiles in Fig. 5 are quite steep, this will lead to high variability in consumption across the population. In the pure credit steady state, the unconditional coefficient of variation of consumption is about 15%, while the unconditional coefficient of variation of income is 60%, when there are no defection constraints. With defection constraints, the unconditional coefficient of variation of consumption is about 9%.

Money and Credit with No Defection Constraints

Here, we examine the effects of inflation for the calibrated economy with no defection constraints. We consider steady state equilibrium allocations for inflation rates running from the Friedman rule inflation rate ($\gamma = q$) to an incipient inflation rate sufficiently high to rule out a steady state monetary equilibrium. This latter equilibrium is essentially an inefficient pure credit allocation where, with probability $1 - \rho$, the transfer to the consumer from the financial intermediary is noncontingent on income.

The results are summarized in Table I. The second column in the table lists the mean level of expected utility across the population in the steady state for each inflation rate, where inflation rates are annualized. Entries in the third column are the standard deviations of expected utilities across the

TABLE I

$\rho = 0.81$, No Defection Constraints

Annual inflation rate	Mean E.U.	S.D. of E.U.	S.D. of cons.	Welfare cost
−3.94%	0.6270	0.0519	0.1508	0
10%	0.6270	0.0515	0.1549	≈0
100%	0.6262	0.0500	0.1908	0.0021
1500%	0.6173	0.0452	0.2964	0.0257
> Non-mon. threshold	0.6156	0.0450	0.3066	0.0301

population, and entries in the fourth column are the standard deviations of consumption for the population. In the last column, we have welfare costs of inflation, measured by the quantity of consumption, relative to the mean level of consumption (equal to unity here), required in each state of the world to make the average agent in an economy with a given inflation rate indifferent between living in that economy and an economy with the Friedman rule rate of inflation in the steady state. That is, given the utility function we have assumed, with the coefficient of absolute risk aversion equal to one, if U denotes expected utility of the average consumer in the steady state under the inflation rate in question (i.e., the entry in column 2 of Table I), U^* denotes the expected utility of the average consumer under the Friedman rule (0.6270 in Table I), and λ is the welfare cost in the last column of Table I, then

$$\lambda = \ln\left(\frac{1-U}{1-U^*}\right).$$

Note in Table I that mean expected utility falls with inflation, the standard deviation of expected utility falls, and the standard deviation of consumption rises substantially. The last row of the table consists of results for an inflation rate in excess of what is required to drive money out of the system, a rate which is somewhat greater than 1500% per annum. These results reflect the fact that, as the inflation rate rises, money becomes a less efficient store of value, and is therefore used less for insurance purposes. There is then less consumption smoothing, the variability of consumption rises, and welfare falls. Figure 6 shows consumption profiles with 10% inflation in the state where the consumer is subject to the limited participation problem, and cannot get insurance through the financial intermediary ($c01$ is consumption when income is high, and $c00$ is consumption when income is low). Here, note the difference in the variability in consumption, conditional on expected utility, relative to Fig. 5. Due to the fact that there

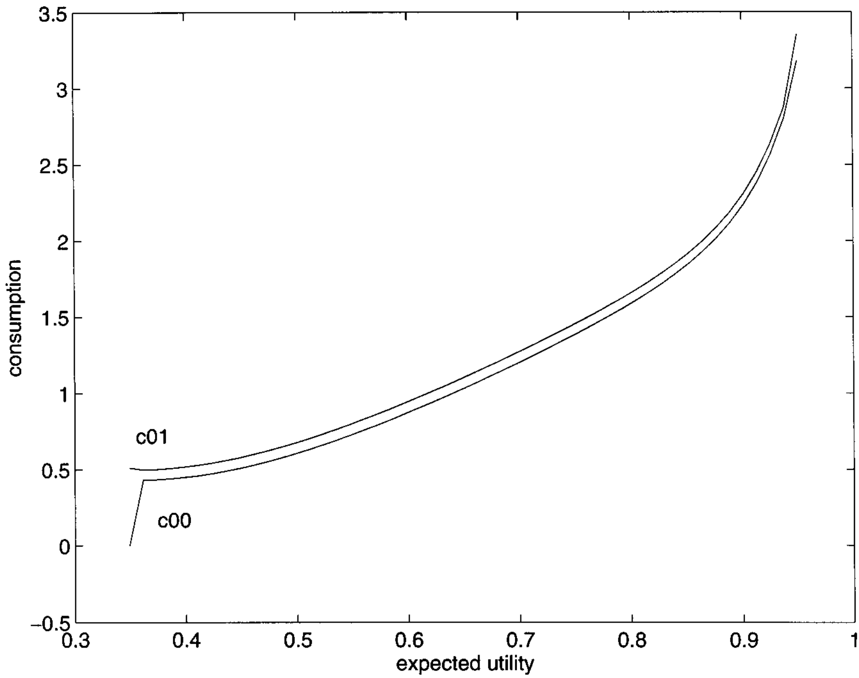

FIG. 6. Consumption with limited participation, 10% inflation.

is less insurance, intertemporal incentives are used less, and this tends to reduce the variability in expected utilities across the population.

In Table I, inflation has very small effects on the distribution of expected utilities, in that there is little change in the mean and standard deviation of expected utility (columns 2 and 3) as inflation increases. Figure 7 shows the distribution of expected utilities under the Friedman rule, and given an incipient inflation rate high enough to drive out currency. The welfare effects of inflation are also small; in particular the cost of inflation in terms of consumption is an order of magnitude lower than the cost measured by [11] in a standard cash-in-advance model. Part of the difference in welfare costs is due to the fact that, in this version of the model, only 24% of the value of transactions is accounted for by cash transactions (at 4% inflation per annum), whereas all consumption in Cooley and Hansen's model is purchased with cash held in advance. As well, there is no effect of inflation on the labor-leisure choice [11], growth [13], or the costs of credit [20]. Note further that the model here gives a measure of the welfare cost associated with doing without currency in the economy, which is the entry in the last row and last column of Table I. This welfare cost is approximately 3% of consumption.

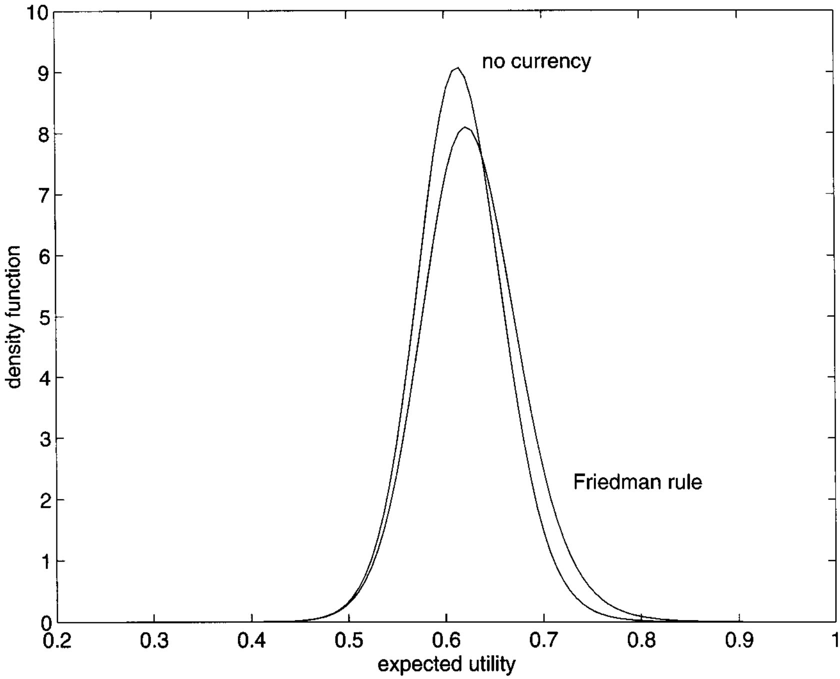

FIG. 7. No defection.

In spite of the fact that inflation has small effects on the distribution of expected utilities and on welfare, the effect on the variability in consumption is substantial. For example if inflation increases from the Friedman rule rate to an incipient rate sufficient to drive out currency, then the coefficient of variation in consumption approximately doubles, from about 15% to about 30%.

Money and Credit with Defection Constraints

Table II shows results for the case with defection constraints imposed. Here, note the differences from Table I. As was the case for the pure credit allocations discussed above, the variability in expected utility (column 3) and the variability in consumption (column 4) are considerably smaller with defection constraints than without. Also, in contrast to Table I, in Table II the standard deviation of expected utilities across the population increases as the inflation rate increases. This is due to the fact that higher inflation implies that the value of defecting from the intermediary contract decreases (note the difference between Figs. 7 and 8). Therefore, the defection constraints bind for lower levels of expected utility, and dispersion in expected utilities tends to increase.

TABLE II

$\rho = 0.81$, with Defection Constraints

Annual inflation rate	Mn. E.U.	SD(EU)	SD(Cons)	Real rate	Welfare cost
−0.48%	0.6310	0.0039	0.0722	0.48%	0
10%	0.6313	0.0048	0.0670	2.93%	−0.0008
15%	0.6314	0.0053	0.0662	3.64%	−0.0011
100%	0.6304	0.0095	0.0914	3.78%	0.0016
1500%	0.6209	0.0115	0.2535	3.93%	0.0270
> Non-mon. threshold	0.6192	0.0115	0.2742	3.94%	0.0315

The standard deviation in expected utilities increases as the inflation rate increases not only because lower levels of expected utility are permitted due to the relaxation of the defection constraints, but because higher levels of expected utility are permitted in equilibrium. That is, since consumption is monotonically increasing with expected utility, if there are more low-expected-utility consumers in the population in the steady state, then to have market clearing, there must also be more high-expected-utility consumers. These high-expected-utility consumers arise in the steady state

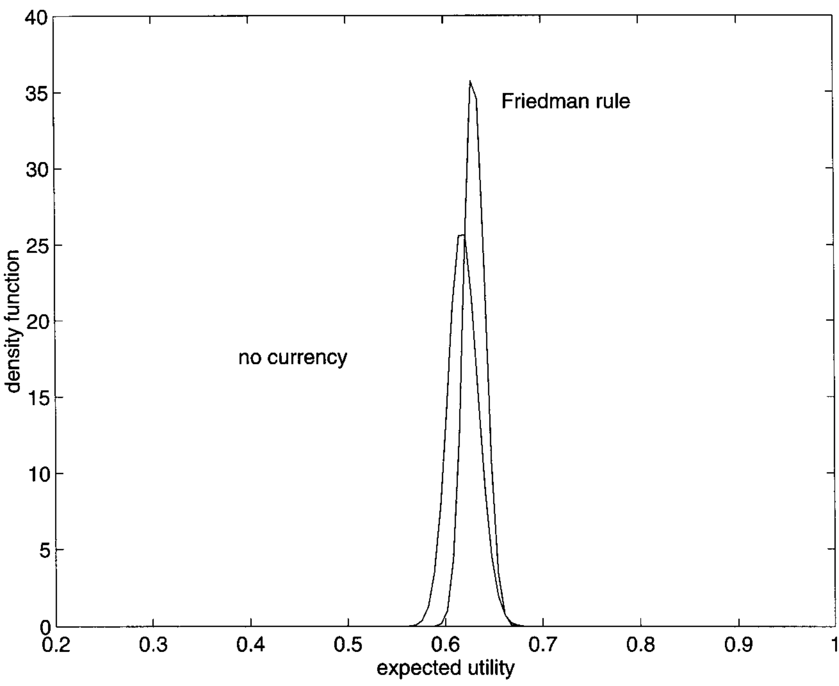

FIG. 8. Defection.

because prices on the bond market adjust such that it is efficient for consumers to postpone consumption, i.e. the real interest rate must rise. We see this effect in Table II, column 5, where the real rate of interest rises with inflation. Without defection constraints, there was a very small negative effect on the real interest rate from an increase in the inflation rate.

This positive effect of inflation on the real interest rate occurs for a novel reason, which can be contrasted with results from standard monetary models. In models where the portfolio substitution effects of long-run monetary policy dominate, for example [31] or standard overlapping generations models [34], an increase in the rate of inflation results in substitution away from money towards alternative assets, which tends to drive down the real rates of return on the alternatives. Inflation also sometimes has no effect on the real rate of interest, as in [26], or causes the marginal product of capital to increase because, for example, of a cash-in-advance constraint on the purchases of investment goods [28].

In column 4 of Table II, the effects of inflation on the standard deviation of consumption are non-monotonic. There are three effects at work here. First, with the relaxation in defection constraints that comes with an increase in inflation, the variability in expected utilities increases, which tends to increase the variability in consumption, since consumption increases on average with expected utility across the population. Second, with higher inflation consumers cannot smooth consumption as well in the state where money is useful for insurance purposes, since they hold lower real cash balances. Third, as the defection constraints are relaxed with higher inflation, this allows for better risk-sharing for the low levels of expected utility where the defection constraints bind. At low inflation rates, the third effect tends to dominate the other two, and at high inflation rates this effect is relatively small, so that the variability in consumption decreases with inflation at low inflation rates and increases at high inflation rates.

In spite of the fact that the relaxation of defection constraints produces large effects of inflation on the distribution of welfare and consumption across the population, the quantitative effect on the welfare of the average consumer is much the same as when the defection constraints are not in place. That is, the welfare costs in the last column of Table II are similar in magnitude to the corresponding entries in Table I, except that welfare increases with inflation in Table II for low levels of inflation. If we take mean expected utility across the population to be the welfare criterion, then the optimal inflation rate with defection constraints is about 15% per annum, whereas without defection constraints the Friedman rule is optimal. That is, with defection constraints there are two opposing effects of inflation on average welfare. First, higher inflation relaxes the defection constraints, which is good for average welfare. Second, higher inflation implies worse risk-sharing, which is bad for average welfare. At low rates of inflation, the first effect dominates, and the second effect dominates for high rates of inflation.

7. CONCLUSION

We have constructed a model of money and credit with dynamic private information, and examined the implications of this model for the effects of inflation on the distribution of consumption and welfare. Consumers have random unobservable endowments and write long-term contracts with financial intermediaries. A role for money arises due to random limited participation. That is, contact with financial intermediaries is such that consumers are not always able to smooth consumption adequately with credit, and instead must sometimes resort to currency transactions.

The key findings relate to how defection constraints, arising from the option consumers have of abandoning contracts with financial intermediaries in favor of spot trading on the money market, alter the effects of changes in the money growth rate in the steady state. In particular, in the absence of defection constraints a Friedman rule is optimal, and increases in the money growth rate and inflation tend to reduce the mean level of expected utility and the variability in expected utility across the population and to increase the variability in consumption. The effects of inflation are small. Alternatively, with defection constraints the results change dramatically. In general, defection constraints tend to reduce the variability in expected utility across the population. Higher inflation tends to increase the variability in expected utilities, while increasing the variability of consumption much more than when there are no defection constraints in force. With defection constraints, an increase in the inflation rate causes a relatively large increase in the real interest rate, at low inflation rates.

Our results are of course sensitive to the information structure we have assumed. First, if we assumed that there was private information concerning preference shocks rather than endowments, as for example in [6], then we would not obtain a steady state limiting distribution with mobility in the case without defection constraints. However, the defection constraint case would yield qualitatively similar results with preference shocks. What makes preference shocks unattractive for this application is that a preference shock model will yield allocations where consumption is more variable than income (assuming fixed incomes), which is at odds with observation. Preference shocks do have the advantage, however, that it is more plausible that they be unobservable than that incomes be unobservable.

Second, it is clearly important that we have assumed that money balances are observable. This assumption is key to the model's tractability, but it lacks superficial plausibility. In practice individual cash balances would seem to be quite difficult to observe. Further, some results in the literature (e.g., [4, 17]) suggest that, if there are unobserved private trades, then efficient allocations are identical to what can be achieved in an incomplete markets economy where all trade in contingent claims is shut down. These results, though limited to particular environments, could be interpreted as having negative consequences for private information theory. That is, we might ask why we should go to the work of analyzing a complicated private information economy rather than saving some trouble and analyzing a related, and much simpler, incomplete markets setup. In defense of our model, it might be possible to consider a setup where money can be hidden, but it is in everyone's interest to reveal their true cash balances (note that allocations have the property that higher cash balances are always associated with a higher expected utility entitlement), and where the allocation does not collapse to the incomplete markets allocation. This approach, however, is beyond what we know how to do. Of course, assets in practice are neither perfectly observable nor perfectly unobservable, so the issue of what works better for particular applications, incomplete markets models or private information models, will depend on the question, and the quantitative factors that play a role in answering the question. We intend to explore this idea further in future work.

REFERENCES

1. S. R. Aiyagari, Uninsured idiosyncratic risk and aggregate saving, *Quart. J. Econ.* **109** (1994), 659–684.
2. S. R. Aiyagari and F. Alvarez, "Stationary Efficient Distributions with Private Information and Monitoring: A Tale of Kings and Slaves," working paper, Federal Reserve Bank of Minneapolis and University of Pennsylvania, 1995.
3. S. R. Aiyagari and S. D. Williamson, Credit in a random matching model with private information, *Rev. Econ. Dynam.* **2** (1999), 36–64.
4. F. Allen, Repeated principal-agent relationships with lending and borrowing, *Econ. Lett.* **17** (1985), 27–31.
5. F. Alvarez and U. Jermann, Asset pricing when risk sharing is limited by default, manuscript, University of Chicago, 1998.
6. A. Atkeson and R. Lucas, Jr., On efficient distribution with private information, *Econ. Stud.* **59** (1992), 427–453.
7. A. Atkeson and R. Lucas, Jr., Efficiency and inequality in a simple model of efficient unemployment insurance, *J. Econ. Theory* **66** (1995), 64–88.
8. R. Avery, G. Elliehausen, A. Kennickell, and P. Spindt, The use of cash and transactions accounts by american families, *Fed. Res. Bull.* **72** (1987), 87–108.
9. T. Bewley, The optimum quantity of money, *in* "Models of Monetary Economies" (J. Kareken and N. Wallace, Eds.), pp. 169–210, Federal Reserve Bank of Minneapolis, Minneapolis, MN, 1980.

10. T. Bewley, A difficulty with the optimum quantity of money, *Econometrica* **51** (1983), 1485–1504.
11. T. Cooley and G. Hansen, The inflation tax in a real business cycle model, *Amer. Econ. Rev.* **79** (1989), 733–748.
12. D. Corbae and A. Blume, "Credit and Currency with Limited Commitment," working paper, University of Iowa, 1995.
13. M. Dotsey and P. Ireland, The welfare cost of inflation in general equilibrium, *J. Monet. Econ.* **37** (1996), 29–48.
14. M. Friedman, The optimum quantity of money, *in* "The Optimum Quantity of Money and Other Essays," pp. 1–50, Aldine, Hawthorne, NY, 1969.
15. T. Fuerst, Liquidity, loanable funds, and real activity, *J. Monet. Econ.* **29** (1992), 3–24.
16. E. Green, Lending and the smoothing of uninsurable income, *in* "Contractual Arrangements for Intertemporal Trade" (E. Prescott and N. Wallace, Eds.), pp. 3–25, Univ. of Minnesota Press, Minneapolis, MN, 1987.
17. P. Ireland and J. Lacker, "Coalition-Proof Risk Sharing with Private Information," working paper, Boston College and Federal Reserve Bank of Richmond, 1998.
18. N. Kocherlakota, Implications of efficient risk sharing without commitment, *Rev. Econ. Stud.* **63** (1996), 595–609.
19. D. Krueger and F. Perri, Risk sharing: Private insurance markets or redistributive taxes? manuscript, Federal Reserve Bank of Minneapolis, 1999.
20. J. Lacker and S. Schreft, Money and credit as means of payment, *J. Monet. Econ.* **38** (1996), 3–24.
21. R. Lucas, Jr., Liquidity and interest rates, *J. Econ. Theory* **50** (1990), 237–264.
22. R. Lucas, Jr., On efficiency and distribution, *Econ. J.* **102** (1992), 233–247.
23. C. Phelan, Repeated moral hazard and one-sided commitment, *J. Econ. Theory* **66** (1995), 488–506.
24. C. Phelan and R. Townsend, Computing multi-period information-constrained optima, *Rev. Econ. Stud.* **58** (1991), 853–881.
25. E. Prescott, Theory ahead of business cycle measurement, *Fed. Res. Bank Minn. Quart. Rev.* (1986), 9–22.
26. M. Sidrauski, Inflation and economic growth, *J. Polit. Econ.* **75** (1967), 796–810.
27. S. Spear and S. Srivastava, On repeated moral hazard with discounting, *Rev. Econ. Stud.* **54** (1987), 599–618.
28. A. Stockman, Anticipated inflation and the capital stock in a cash-in-advance economy, *J. Monet. Econ.* **8** (1981), 387–393.
29. B. Taub, Currency and credit are equivalent mechanisms, *Int. Econ. Rev.* **35** (1994), 921–956.
30. J. Thomas and T. Worrall, Income fluctuations and asymmetric information: An example of a repeated principal-agent problem, *J. Econ. Theory* **51** (1990), 367–390.
31. J. Tobin, Money and economic growth, *Econometrica* **33** (1965), 671–810.
32. R. Townsend, Economic organization with limited communication, *Amer. Econ. Rev.* **77** (1987), 954–971.
33. R. Townsend, Currency and credit in a private information economy, *J. Polit. Econ.* **97** (1989), 1323–1344.
34. N. Wallace, The overlapping generations model of fiat money, *in* "Models of Monetary Economies" (N. Wallace and J. Kareken, Eds.), pp. 49–82, Federal Reserve Bank of Minneapolis, Minneapolis, MN, 1980.
35. C. Wang, Dynamic insurance with private information and balanced budgets, *Rev. Econ. Stud.* **62** (1995), 577–595.