Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 135 (2007) 171 – 195
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Money, credit and banking

Aleksander Berentsen[a], Gabriele Camera[b], Christopher Waller[c,*]

[a] University of Basel, Switzerland
[b] Purdue University, USA
[c] University of Notre Dame, USA

Received 7 October 2005; final version received 3 March 2006
Available online 19 October 2006

Abstract

In monetary models where agents are subject to trading shocks there is typically an ex post inefficiency since some agents are holding idle balances while others are cash constrained. This problem creates a role for financial intermediaries, such as banks, who accept nominal deposits and make nominal loans. In general, financial intermediation improves the allocation. The gains in welfare come from the payment of interest on deposits and not from relaxing borrowers' liquidity constraints. We also demonstrate that when credit rationing occurs increasing the rate of inflation can be welfare improving.
© 2006 Elsevier Inc. All rights reserved.

JEL classification: E41; E50

Keywords: Money; Credit; Rationing; Banking

1. Introduction

In monetary models where agents are subject to trading shocks there is typically an ex post inefficiency since some agents are holding idle balances while others are cash constrained.[1] Given this inefficiency a credit market that reallocates money across agents would reduce or eliminate this inefficiency. While this seems obvious at first glance, it overlooks a fundamental tension between money and credit. A standard result in monetary theory is that for money to be essential in exchange there must be an absence of record keeping. In contrast, credit requires record keeping. This tension has made it inherently difficult to introduce credit into a model where

* Corresponding author. Fax: +1 574 631 4963.
E-mail address: cwaller@nd.edu (C. Waller).

[1] Models with this property include [4,8,9,15,23,24,26].

0022-0531/$ - see front matter © 2006 Elsevier Inc. All rights reserved.
doi:10.1016/j.jet.2006.03.016

money is essential. [2] Furthermore, once credit exists the issues of repayment and enforcement naturally arise.

In this paper we address the following questions. First, how can money and credit coexist in an environment where money is essential? Second, can financial intermediation improve the allocation? Third, what is the optimal monetary policy if all trades are voluntary, i.e. when there is no enforcement?

To answer these questions, we introduce financial intermediation into a monetary model based on the Lagos and Wright [22] framework. We call financial intermediaries 'banks' because they accept nominal deposits and make nominal loans. Banks have a record keeping technology that allows them to keep track of financial histories but agents still trade with each other in anonymous goods markets. Hence, there is no record keeping of good market trades. Consequently, the existence of financial record keeping does not eliminate the need for money as a medium of exchange. We characterize the monetary equilibria in two cases: with and without enforcement. By enforcement we mean that banks can force repayment at no cost, which prevents any default, and the monetary authority can impose lump-sum taxes. In an environment with no enforcement, the monetary authority cannot tax agents and the only penalty for default is exclusion from the financial system.

With regard to the second question, we show that the equilibrium with credit improves the allocation. The gain in welfare comes from payment of interest to agents holding idle balances and not from relaxing borrowers' liquidity constraints. With respect to the third question, the answer depends on whether enforcement is feasible or not. If enforcement is feasible, the Friedman rule attains the first-best allocation. The intuition is that under the Friedman rule agents can perfectly self-insure against consumption risk by holding money at no cost. Consequently, there is no need for financial intermediation—the allocation is the same with or without credit. In contrast, without enforcement, deflation cannot be implemented nor can banks force agents to repay loans. In this situation, we show that price stability is not the optimal policy since some inflation can be welfare improving. The reason is that inflation makes holding money more costly, which increases the punishment from being excluded from the financial system and thus the incentives to repay loans. [3]

How does our approach differ from the existing literature? Other mechanisms have been proposed to address the inefficiencies that arise when some agents are holding idle balances while others are cash constrained. These mechanisms involve either trading cash against some other illiquid asset [20], collateralized trade credit [29] or inside money [10–12,16].

In our model, the role of credit is similar to that of 'illiquid' bonds in Kocherlakota [20]—credit allows the transfer of money from those with a low marginal value of consumption to those with a high valuation. The key difference is that in [20] agents adjust their portfolios by trading assets while in our model agents acquire one asset, namely money, by issuing liabilities. Although both approaches have the same implications for the allocation, in general, the presence of illiquid bonds does not eliminate the role of credit. The reason is that some agents may hold so little money and bonds that they would like to borrow additional money to acquire goods. Finally, Kocherlakota never explains why bonds are illiquid. This is not a problem for us because in our environment the interest-bearing debt instruments are held by agents who do not want to consume.

[2] By essential we mean that the use of money expands the set of allocations (see [19,33]).

[3] This confirms the intuition in Aiyagari and Williamson [1] for why the optimal inflation rate is positive when enforcement is not feasible.

The mechanisms of [11,12,29] are related to ours since some buyers are able to relax their cash constraint by issuing personal liabilities directly to sellers and this improves the allocation. However, the inefficiency associated with holding idle cash balances is not eliminated. In our model agents can either borrow to relax their cash constraints or lend their idle cash balances and earn interest. More importantly, contrary to these other models, we find that the welfare gain associated with financial intermediation is not due to relaxing buyers' cash constraints. Instead it comes from generating a positive rate of return on idle cash balances. [4]

Another key difference between our analysis and the existing literature is that, with divisible money, we can study how changes in the growth rate of the money supply affect the allocation. [5] Also, in terms of pricing, we use competitive pricing rather than bargaining (although we do study bargaining for comparison purposes). [6] Furthermore, unlike [11,12] and related models we do not have bank claims circulating as medium of exchange nor do we have goods market trading histories observable for any agent. Finally, in contrast to [16], there is no security motive for depositing cash in the bank.

The paper proceeds as follows. Section 2 describes the environment and Section 3 the agents' decision problems. In Section 4 we derive the equilibrium when banks can force repayment at no cost and in Section 5 when punishment for a defaulter is permanent exclusion from the banking system. The last section concludes.

2. The environment

The basic framework we use is the divisible money model developed in Lagos and Wright [22]. This model is useful because it allows us to introduce heterogenous preferences for consumption and production while still keeping the distribution of money balances analytically tractable. [7] Time is discrete and in each period there are two perfectly competitive markets that open sequentially. There is a $[0, 1]$ continuum of infinitely lived agents and one perishable good produced and consumed by all agents.

At the beginning of the first market agents get a preference shock such that they can either consume or produce. With probability $1-n$ an agent can consume but cannot produce while with probability n the agent can produce but cannot consume. We refer to consumers as buyers and producers as sellers. Agents get utility $u(q)$ from q consumption in the first market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. Furthermore, we assume the elasticity of utility $e(q) = \frac{qu'(q)}{u(q)}$ is bounded. Producers incur utility cost $c(q)$ from producing q units of output with $c'(q) > 0$, $c''(q) \geqslant 0$. To motivate a role for fiat money, we assume that all goods trades are anonymous so agents cannot identify their trading partners. Consequently, trading histories of

[4] This shows that being constrained is not per se a source of inefficiency. In any general equilibrium model all agents face a budget constraint. Nevertheless, the equilibrium is efficient because all gains from trade are exploited.

[5] Recently, [13] has also developed a model of banking in which money and goods are divisible. His banks serve a very different purpose than modeled here and they have records of goods markets trades between individuals, hence it is doubtful that money is essential in his model.

[6] Competitive pricing in the Lagos–Wright framework has been introduced by [27] and further investigated in [3,5,21].

[7] An alternative framework would be [30] which we could amend with preference and technology shocks to generate the same results.

agents are private information and sellers require immediate compensation meaning buyers must pay with money. [8]

In the second market all agents consume and produce, getting utility $U(x)$ from x consumption, with $U'(x) > 0$, $U'(0) = \infty$, $U'(+\infty) = 0$ and $U''(x) \leqslant 0$. The difference in preferences over the good sold in the last market allows us to impose technical conditions such that the distribution of money holdings is degenerate at the beginning of a period. Agents can produce one unit of the consumption good with one unit of labor which generates one unit of disutility. The discount factor across dates is $\beta \in (0, 1)$.

We assume a central bank exists that controls the supply of fiat currency. The growth rate of the money stock is given by $M_t = \gamma M_{t-1}$ where $\gamma > 0$ and M_t denotes the per capita money stock in t. Agents receive lump-sum transfers $\tau M_{t-1} = (\gamma - 1)M_{t-1}$ over the period. Some of the transfer is received at the beginning of market 1 and some during market 2. Let $\tau_1 M_{t-1}$ and $\tau_2 M_{t-1}$ denote the transfers in market 1 and 2, respectively, with $\tau_1 + \tau_2 = \tau$. Moreover, $\tau_1 = (1 - n)\,\tau_{\mathrm{b}} + n\tau_{\mathrm{s}}$ since the central bank might wish to treat buyers and sellers differently. This transfer scheme is merely an analytical device to see whether or not a policy of differential lump-sum transfers based on an individual's relative need for cash can replicate the same allocation that occurs with banking. Note that although buyers and sellers get different transfers they are lump sum in nature since they do not affect marginal decisions. For notational ease variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1.

If there is enforcement, the central bank can levy nominal taxes to extract cash from the economy, then $\tau < 0$ and hence $\gamma < 1$. Implicitly this means that the central bank can force agents to trade. However, this does not mean that it can force agents to produce or consume certain quantities in the good markets nor does it mean that it knows the identity of the agents. If the central bank does not have this power, lump-sum taxes are not feasible so $\gamma \geqslant 1$. We will derive the equilibrium for both environments.

Banks and record keeping: We model credit as financial intermediation done by perfectly competitive firms who accept nominal deposits and make nominal loans. For this process to work we assume that there is a technology that allows record keeping of financial histories but not trading histories in the goods market. Firms that operate this record keeping technology can do so at zero cost. We call them banks because the financial intermediaries who perform these activities—taking deposits, making loans, keeping track of credit histories—are classified as 'banks' by regulators around the world. Since record keeping can only be done for financial transactions, trade credit between buyers and sellers is not feasible. Moreover, since there is no collateral in our model bilateral trade credit cannot be supported as in [29]. Record keeping does not imply that banks can issue tangible objects such as inside money. Hence, we assume that there are no bank notes in circulation. This ensures that outside fiat currency is still used as a medium of exchange in the goods market. [9] Finally, we assume that loans and deposits are not rolled over. Consequently, all financial contracts are one-period contracts. One-period debt contracts are optimal in these environments because of the quasi-linear preferences. Unlike standard dynamic contracting models, with linear disutility of production in market 2, there is no gain from spreading

[8] There is no contradiction between assuming Walrasian markets and anonymity. To calculate the market clearing price, a Walrasian auctioneer only needs to know the aggregate excess demand function and not the identity of the individual traders.

[9] Alternatively, we could assume that our banks issue their own currencies but there is a 100% reserve requirement in place. In this case the financial system would be similar to narrow banking [32].

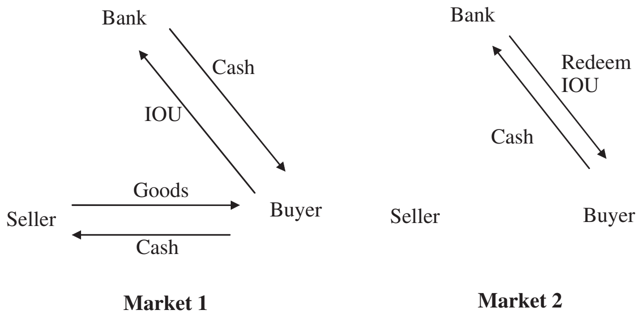

Fig. 1. Cash and credit.

out repayment of loans or redemption of deposits across periods in order to smooth the disutility of production. [10]

Although all goods transactions require money, buyers do not face a standard cash-in-advance constraint. Before trading, they can borrow cash from the bank to supplement their money holdings but do so at the cost of the nominal interest rate as illustrated in Fig. 1, which describes the flow of goods, credit and money in our model for markets 1 and 2. Note the absence of links between the seller and the bank. [11] The missing link is consistent with the assumption that there is no record-keeping in the goods market due to anonymity. For example, it rules out the following mechanism. At the end of each period, every agent reports to the bank the identity of the trading partners and the quantities traded. If the report of an agent does not match the report of his trading partner, then the bank punishes both agents by excluding them from the banking system. This mechanism requires that the agents in the good market can accurately identify their trading partners, which violates our assumption of anonymity. [12]

Default: In any model of credit, default is a serious issue. We first assume that banks can force repayment at no cost. In such an environment, default is not possible so agents face no borrowing constraints. In this case, banks are nothing more than cash machines that post interest rates for deposits and loans. In equilibrium these posted interest rates clear the market. We then consider an environment where banks cannot force agents to repay. The only punishment available is that a borrower who fails to repay his loan is excluded from the financial sector in all future periods. Given this punishment, we derive conditions to ensure voluntary repayment and show that this may involve binding borrowing constraints, i.e. credit rationing.

Welfare: At the beginning of a period before types are realized, the expected steady state lifetime utility of the representative agent is

$$(1-\beta)\,\mathcal{W} = (1-n)\,u\,(q_{\mathrm{b}}) - nc\,(q_{\mathrm{s}}) + U(x) - x, \tag{1}$$

where q_{b} is consumption and q_{s} production in market 2. We use (1) as our welfare criterion.

[10] In a stationary equilibrium, the Lagos–Wright framework turns the economy into a sequence of repeated static problems. Hence, a one-period contract is sufficient to deal with any trading frictions occurring within the period.

[11] At the beginning of market 1 the seller deposits money at the bank which is redeemed in market 2. These two transactions, however, are independent from the flow of money between buyer and bank and buyer and seller as described in Fig. 1.

[12] It also excludes commonly used forms of payments such as credit card or check payments. Such payments require that all agents must identify themselves and the value of their goods transactions to the banking system. With this information, money is not essential since all exchange can be done via record keeping.

To derive the welfare maximizing quantities we assume that all agents are treated symmetrically. The planner then maximizes (1) subject to the feasibility constraint

$$(1-n)\, q_{\mathrm{b}} = n q_{\mathrm{s}}. \tag{2}$$

The first-best allocation satisfies

$$\begin{aligned} &U'\left(x^*\right) = 1 \quad \text{and} \\ &u'\left(q^*\right) = c'\left(\frac{1-n}{n} q^*\right), \end{aligned} \tag{3}$$

where $q^* \equiv q_{\mathrm{b}}^* = \frac{n}{(1-n)} q_{\mathrm{s}}^*$. These are the quantities chosen by a social planner who could force agents to produce and consume.

3. Symmetric equilibrium

The timing in our model is as follows. At the beginning of the first market agents observe their production and consumption shocks and they receive the lump-sum transfers $\tau_1 M_{-1}$. Then, the banking sector opens and agents can borrow or deposit money. Finally, the banking sector closes and agents trade goods. In the second market agents trade goods and settle financial claims.

In period t, let ϕ be the real price of money in the second market. We focus on symmetric and stationary equilibria where all agents follow identical strategies and where real allocations are constant over time. In a stationary equilibrium end-of-period real money balances are time-invariant

$$\phi M = \phi_{+1} M_{+1}. \tag{4}$$

Moreover, we restrict our attention to equilibria where γ is time invariant which implies that $\phi/\phi_{+1} = P_{+1}/P = M_{+1}/M = \gamma$.[13]

Consider a stationary equilibrium. Let $V(m)$ denote the expected value from trading in market 1 with m money balances at time t. Let $W(m, \ell, d)$ denote the expected value from entering the second market with m units of money, ℓ loans, and d deposits at time t. In what follows, we look at a representative period t and work backwards from the second to the first market.

3.1. The second market

In the second market agents produce h goods and consume x, repay loans, redeem deposits and adjust their money balances. If an agent has borrowed ℓ units of money, then he pays $(1+i)\,\ell$ units of money, where i is the nominal loan rate. If he has deposited d units of money, he receives $(1+i_d)\,d$, where i_d is the nominal deposit rate. The representative agent's program is

$$\begin{aligned} &W(m, \ell, d) = \max_{x,h,m_{+1}} \left[U(x) - h + \beta V_{+1}(m_{+1})\right] \\ &\text{s.t.} \quad x + \phi m_{+1} = h + \phi\,(m + \tau_2 M_{-1}) + \phi\,(1+i_d)\,d - \phi\,(1+i)\,\ell, \end{aligned} \tag{5}$$

[13] This eliminates stationary equilibria where γ is stochastic.

where m_{+1} is the money taken into period $t+1$ and ϕ is the real price of money. Rewriting the budget constraint in terms of h and substituting into (5) yields

$$W(m,\ell,d) = \phi\left[m + \tau_2 M_{-1} - (1+i)\,\ell + (1+i_d)\,d\right] + \max_{x,m_{+1}}\left[U(x) - x - \phi m_{+1} + \beta V_{+1}(m_{+1})\right].$$

The first-order conditions are $U'(x) = 1$ and

$$\phi = \beta V'_{+1}(m_{+1}), \tag{6}$$

where $V'_{+1}(m_{+1})$ is the marginal value of an additional unit of money taken into period $t+1$. Notice that the optimal choice of x is the same across time for all agents and the m_{+1} is independent of m. As a result, the distribution of money holdings is degenerate at the beginning of the following period. The envelope conditions are

$$W_m = \phi, \tag{7}$$

$$W_\ell = -\phi(1+i), \tag{8}$$

$$W_d = \phi(1+i_d). \tag{9}$$

3.2. The first market

Let q_{b} and q_{s}, respectively, denote the quantities consumed by a buyer and produced by a seller trading in market 1. Let p be the nominal price of goods in market 1. While we use competitive pricing for most of what we do, we also consider matching and bargaining later on to compare the allocation with financial intermediation to the allocation in Lagos and Wright [22].

It is straightforward to show that agents who are buyers will never deposit funds in the bank and sellers will never take out loans. Thus, $\ell_{\mathrm{s}} = d_{\mathrm{b}} = 0$. In what follows we let ℓ denote loans taken out by buyers and d deposits of sellers. We also drop these arguments in $W(m,\ell,d)$ where relevant for notational simplicity.

An agent who has m money at the opening of the first market has expected lifetime utility

$$V(m) = (1-n)\left[u(q_{\mathrm{b}}) + W(m + \tau_{\mathrm{b}} M_{-1} + \ell - pq_{\mathrm{b}}, \ell)\right] + n\left[-c(q_{\mathrm{s}}) + W(m + \tau_{\mathrm{s}} M_{-1} - d + pq_{\mathrm{s}}, d)\right], \tag{10}$$

where pq_{b} is the amount of money spent as a buyer and pq_{s} the money received as a seller. Once the preference shock occurs, agents become either a buyer or a seller. Note that sellers cannot deposit receipts of cash, pq_{s}, earned from selling in market 1. In short, the bank closes before the onset of trading in market 1.

Sellers' decisions: If an agent is a seller in the first market, his problem is

$$\max_{q_{\mathrm{s}},d}\left[-c(q_{\mathrm{s}}) + W(m + \tau_{\mathrm{s}} M_{-1} - d + pq_{\mathrm{s}}, d)\right]$$
$$\text{s.t.}\quad d \leqslant m + \tau_{\mathrm{s}} M_{-1}.$$

The first-order conditions are

$$-c'(q_{\mathrm{s}}) + pW_m = 0,$$
$$-W_m + W_d - \lambda_d = 0,$$

where λ_d is the multiplier on the deposit constraint. Using (7), the first equation reduces to

$$c'(q_s) = p\phi. \tag{11}$$

Sellers produce such that the ratio of marginal costs across markets $(c'(q_s)/1)$ is equal to the relative price $(p\phi)$ of goods across markets. Due to the linearity of the envelope conditions, q_s is independent of m and d. Consequently, sellers produce the same amount no matter how much money they hold or what financial decisions they make. Finally, it is straightforward to show that for any $i_d > 0$ the deposit constraint is binding and so sellers deposit all their money balances.

Buyers' decisions: If an agent is a buyer in the first market, his problem is

$$\max_{q_b,\ell}\left[u(q_b) + W(m + \tau_b M_{-1} + \ell - pq_b, \ell)\right]$$
$$\text{s.t.}\quad pq_b \leqslant m + \tau_b M_{-1} + \ell,\quad \ell \leqslant \bar{\ell}.$$

Notice that buyers cannot spend more cash than they bring into the first market, m, plus their borrowing, ℓ, and the transfer $\tau_b M_{-1}$. They also face the constraint that the loan size is bounded above by $\bar{\ell}$. They take this constraint as given. However, in equilibrium it is determined endogenously.

Using (7), (8) and (11) the buyer's first-order conditions reduce to

$$u'(q_b) = c'(q_s)(1 + \lambda/\phi), \tag{12}$$
$$\phi i = \lambda - \lambda_\ell, \tag{13}$$

where λ is the multiplier on the buyer's cash constraint and λ_ℓ on the borrowing constraint. If $\lambda = 0$, then (12) reduces to $u'(q_b) = c'(q_s)$ implying trades are efficient.

For $\lambda > 0$, these first-order conditions yield

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i + \lambda_\ell/\phi.$$

If $\lambda_\ell = 0$, then

$$\frac{u'(q_b)}{c'(q_s)} = 1 + i. \tag{14}$$

In this case the buyer borrows up to the point where the marginal benefit of borrowing equals the marginal cost. He spends all his money and consumes $q_b = (m + \tau_b M_{-1} + \ell)/p$. Note that for $i > 0$ trades are inefficient so a positive nominal interest rate acts as tax on consumption.

Finally, if $\lambda_\ell > 0$

$$\frac{u'(q_b)}{c'(q_s)} > 1 + i. \tag{15}$$

In this case the marginal value of an extra unit of a loan exceeds the marginal cost. Hence, a borrower would be willing to pay more than the prevailing loan rate. However, if banks are worried about default, then the interest rate may not rise to clear the market and credit rationing occurs. Consequently, the buyer borrows $\bar{\ell}$, spends all of his money and consumes $q_b = \left(m + \tau_b M_{-1} + \bar{\ell}\right)/p$.

Since all buyers enter the period with the same amount of money and face the same problem, q_b is the same for all of them. The same is true for the sellers. Finally, market clearing implies

$$q_s = \frac{1-n}{n} q_b. \tag{16}$$

Banks: Banks accept nominal deposits, paying the nominal interest rate i_d, and make nominal loans ℓ at nominal rate i. The banking sector is perfectly competitive with free entry, so banks take these rates as given. There is no strategic interaction among banks or between banks and agents. In particular, there is no bargaining over terms of the loan contract. Finally, we assume that there are no operating costs or reserve requirements.

The representative bank solves the following problem per borrower:

$$\max_{\ell} (i - i_d)\,\ell$$
$$\text{s.t.} \quad \ell \leqslant \bar{\ell}, \quad u(q_\text{b}) - (1+i)\,\ell\phi \geqslant \Gamma,$$

where Γ is the reservation value of the borrower. The reservation value is the borrower's surplus from receiving a loan at another bank. We investigate two assumptions about repayment. In the first case, banks can force repayment at no cost so the borrowing constraint is $\bar{\ell} = \infty$. In the second case, we assume that a borrower who fails to repay his loan will be shut out of the banking sector in all future periods. Given this punishment, we need to derive conditions to ensure voluntary repayment which determines $\bar{\ell}$.

The first-order condition is

$$i - i_d - \lambda_\text{L} + \lambda_\Gamma \left[u'(q_\text{b}) \frac{dq_\text{b}}{d\ell} - (1+i)\,\phi \right] = 0,$$

where λ_L and λ_Γ are the Lagrange multipliers on the lending constraint and participation constraint of the borrower, respectively. For $i - i_d > 0$ the bank would like to make the largest loan possible to the borrower. Thus, the bank will always choose a loan size such that $\lambda_\Gamma > 0$.

With free entry banks make zero profits so $i = i_d$. Since, $dq_\text{b}/d\ell = \phi/c'(q_\text{s})$ we have

$$\frac{u'(q_\text{b})}{c'(q_\text{s})} = 1 + i + \frac{\lambda_\text{L}}{\lambda_\Gamma \phi}.$$

If $\lambda_\text{L} = 0$ the loan offered by the bank implies (14) so repayment is not an issue. If $\lambda_\text{L} > 0$ the constraint on the loan size is binding and implies (15). In a symmetric equilibrium all buyers borrow the same amount, ℓ, and sellers deposit the same amount, d, so loan market clearing requires

$$(1-n)\,\ell = nd. \tag{17}$$

Marginal value of money: Using (10) the marginal value of money is

$$V'(m) = (1-n)\frac{u'(q_\text{b})}{p} + n\phi\,(1+i_d)\,.$$

In the appendix we show that the value function is concave in m so the solution to (6) is well defined.

Using (11) $V'(m)$ reduces to

$$V'(m) = \phi \left[(1-n)\,\frac{u'(q_\text{b})}{c'(q_\text{s})} + n\,(1+i_d) \right]. \tag{18}$$

The marginal value of money has two components. If the agent is a buyer he receives $u'(q_\text{b})\,/c'(q_\text{s})$ from spending the marginal unit of money. This effect is standard. Now, if he is a seller he can lend the unit of money and receive $1 + i_d$. Thus, financial intermediation increases the marginal value of money because sellers can deposit idle cash and earn interest.

4. Equilibrium with enforcement

In this section, as a benchmark, we assume that the monetary authority can impose lump-sum taxes and banks can force repayment of loans at no cost. This does not imply that the banks or the monetary authority can dictate the terms of trade between private agents in the goods market.

In any stationary monetary equilibrium use (6) lagged one period to eliminate $V'(m)$ from (18). Then use (4) and (16) to get

$$\frac{\gamma-\beta}{\beta}=(1-n)\left[\frac{u'(q_{\mathrm{b}})}{c'\left(\frac{1-n}{n}q_{\mathrm{b}}\right)}-1\right]+ni_d. \tag{19}$$

The right-hand side measures the value of bringing one extra unit of money into the first market. The first term reflects the net benefit (marginal utility minus marginal cost) of spending the unit of money on goods when a buyer and the second term is the value of depositing an extra unit of idle balances when a seller.

Since banks can force agents to repay their loans, agents are unconstrained so $\bar{\ell}=\infty$. This implies that (14) holds. Using it in (19) yields [14]

$$\frac{\gamma-\beta}{\beta}=(1-n)\,i+ni_d. \tag{20}$$

Now, the first term on the right-hand side reflects the interest saving from borrowing one less unit of money when a buyer.

Zero profit implies $i=i_d$ and so

$$\frac{\gamma-\beta}{\beta}=i. \tag{21}$$

We can rewrite this in terms of q_{b} using (14) to get

$$\frac{\gamma-\beta}{\beta}=\frac{u'(q_{\mathrm{b}})}{c'\left(\frac{1-n}{n}q_{\mathrm{b}}\right)}-1. \tag{22}$$

Definition 1. When repayment of loans can be enforced, a monetary equilibrium with credit is an interest rate i satisfying (21) and a quantity q_{b} satisfying (22).

Proposition 1. *Assume repayment of loans can be enforced. Then if $\gamma>\beta$, a unique monetary equilibrium with credit exists. Equilibrium consumption is decreasing in γ, and satisfies $q_{\mathrm{b}}<q^*$ with $q_{\mathrm{b}}\to q^*$ as $\gamma\to\beta$.*

It is clear from (22) that money is neutral, but not super-neutral. Increasing its stock has no effect on q_{b}, while changing the growth rate γ does. Moreover, the Friedman rule ($\gamma=\beta$) generates the first-best allocation.

[14] This equation implies that if nominal bonds could be traded in market 2, their nominal rate of return, i_{b}, would be $(1-n)\,i+ni_d$. Thus agents would be indifferent between holding a nominal bond or holding a bank deposit.

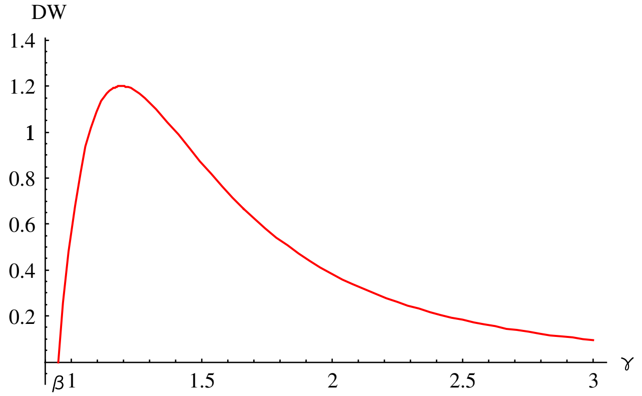

Fig. 2. Difference in welfare.

How does this allocation differ from the allocation in an economy without credit? Let $\tilde{q}_b$ denote the quantity consumed when there is no financial intermediation. It is straightforward to show that $\tilde{q}_b$ solves (19) with $i_d = 0$, i.e.,

$$\frac{\gamma - \beta}{\beta} = (1 - n)\left[\frac{u'(\tilde{q}_b)}{c'\left(\frac{1-n}{n}\tilde{q}_b\right)} - 1\right]. \tag{23}$$

Comparing (23) to (22), it is clear that $\tilde{q}_b < q_b$ for any $\gamma > \beta$. Thus, we have proved the following:

Corollary 1. *For $\gamma > \beta$ financial intermediation improves the allocation and welfare.*

The key result of this section is that financial intermediation improves the allocation away from the Friedman rule. The greatest impact on welfare is for moderate values of inflation. The reason is that near the Friedman rule there is little gain from redistributing idle cash balances while for high inflation rates money is of little value anyway. At the Friedman rule agents can perfectly self-insure against consumption risk because the cost of holding money is zero. Consequently, there is no welfare gain from financial intermediation. [15]

The welfare implications of financial intermediation are displayed in Fig. 2. The graph shows the difference ($DW = W^b - W^{nb}$) in the expected lifetime utilities with financial intermediation (W^b) and without (W^{nb}) as a function of the inflation rate γ. Note that the difference is equal to zero when $\gamma = \beta$ (at the origin), converges to zero when $\gamma \to \infty$ and is maximal for some intermediate rate of inflation. Fig. 2 is drawn with the utility function $u(q) = q^{0.8}/0.8$, cost function $c(q) = q$, discount factor $\beta = 0.95$ and measure of sellers $n = 0.4$.

Given that financial intermediation improves the allocation away from the Friedman rule, is it because it relaxes borrowers' liquidity constraints or because it allows payment of interest to depositors? The following proposition answers this question.

[15] With extensive margin externalities as in [6] or [30], being away from the Friedman rule can be optimal. In this case, financial intermediation clearly is welfare improving.

Proposition 2. *The gain in welfare from financial intermediation is due to the fact that it allows payment of interest to depositors and not from relaxing borrowers' liquidity constraints.*

According to Proposition 2 the gain in welfare comes from payment of interest to agents holding idle balances. To prove this claim we show in the proof of Proposition 2 that in equilibrium, agents are indifferent between borrowing to finance equilibrium consumption or bringing in a sufficient amount of cash to finance the same consumption. The only importance of borrowing is to sustain payment of interest to depositors. That is, even though each individual agent is indifferent between borrowing and not borrowing, agents taking out loans are needed to finance the interest received by the depositors.

As a final proof of this argument, we consider a systematic government policy that redistributes cash in market 1 by imposing lump-sum taxes on sellers and giving the cash as lump-sum transfers to buyers. This clearly relaxes the liquidity constraints of the buyers while paying no interest to depositors. However, inspection of (23) reveals that neither τ_1 nor τ_{b} appear in this equation. Hence, varying the transfer across the two markets or by redistributing cash from sellers to buyers in a predictable, lump-sum fashion has no effect on $\tilde{q}_{\mathrm{b}}$. It only affects the equilibrium price of money in the last market. Agents simply change the amount of money they bring into market 1 and so the demand for money changes in market 2, which alters the price of money ϕ. Note also that this implies that the allocation with credit cannot be replicated by government policies using lump-sum transfers or taxes.

Finally, we would like to see how the equilibrium allocation in our credit model compares with Lagos and Wright [22]. Towards this end, we now consider matching and bargaining in market 1 as an alternative to Walrasian pricing.

Bargaining: We assume $n = 1/2$ and $\tau_{\mathrm{b}} = \tau_{\mathrm{s}} = \tau$. The timing is similar as before in that agents observe their preference shocks, then go to the bank to borrow and deposit funds. The bank then closes. The difference is that buyers now are paired with sellers and they bargain according to the generalized Nash protocol over the quantity of goods and money to be exchanged. The decisions in market 2 are unaffected.

The value function for each agent at the opening of market 1 is

$$\begin{aligned} V(m) = & \tfrac{1}{2}\left[u(q_{\mathrm{b}}) + W(m + \tau M_{-1} + \ell - z_{\mathrm{b}}, \ell)\right] \\ & + \tfrac{1}{2}\left[-c(q_{\mathrm{s}}) + W(m + \tau M_{-1} - d + z_{\mathrm{s}}, d)\right], \end{aligned}$$

where z_{b} is the amount of money given up when a buyer and z_{s} is the amount received when a seller.

Due to the linearity of $W(m, \ell, d)$, the bargaining problem is

$$\begin{aligned} & \max_{q,z} \left[u(q) - \phi z\right]^{\theta}\left[-c(q) + \phi z\right]^{1-\theta} \\ & \quad \text{s.t.} \quad z \leqslant m + \tau M_{-1} + \ell, \end{aligned}$$

where θ is the buyer's bargaining weight. As in Lagos and Wright [22], the solution to the bargaining problem yields

$$\phi z = g(q) \equiv \frac{\theta c(q) u'(q) + (1-\theta) u(q) c'(q)}{\theta u'(q) + (1-\theta) c'(q)},$$

where $g'(q) > 0$. If $\theta < 1$, then $g'(q) > c'(q)$ while for $\theta = 1$, $g'(q) = c'(q)$. In any monetary equilibrium, the buyer will spend all of his cash holdings so $z = m + \tau M_{-1} + \ell$. It then follows that $dz/dm = 1$ and $\partial q/\partial m = \phi/g'(q) > 0$.

To determine borrowing and lending choices, we still have that sellers will deposit all of their money in the bank. Those who are buyers now maximize

$$\max_{\ell} u(q) + W(m + \tau M_{-1} + \ell - z, \ell)$$
$$\text{s.t.} \quad z \leqslant m + \tau M_{-1} + \ell.$$

It is straightforward to show that

$$\frac{u'(q)}{g'(q)} = 1 + i. \tag{24}$$

Differentiating $V(m)$ and rearranging gives

$$V'(m) = \frac{\phi}{2}\left[\frac{u'(q)}{g'(q)} + 1 + i_d\right].$$

Using (6) lagged one period, (24) and $i = i_d$ yields

$$\frac{\gamma - \beta}{\beta} = \frac{u'(q)}{g'(q)} - 1 = i. \tag{25}$$

The interesting aspect of this result is that the nominal interest rate with bargaining is exactly the same as it is with competitive pricing. It then follows from (24) that the quantity traded in all matches under bargaining is lower than under competitive pricing since $g'(q) > c'(q)$. This is due to the holdup problem that occurs on money demand under bargaining.

How does the allocation here compare to the allocation in Lagos and Wright [22]? Their equilibrium value of q solves [16]

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2}\left[\frac{u'(q)}{g'(q)} - 1\right]. \tag{26}$$

It is clear that the quantity solving (25) is greater than the quantity solving (26). Thus the existence of a credit market increases output and welfare even with bargaining. [17]

5. Equilibrium without enforcement

In the previous section enforcement occurred in two occasions. First, the monetary authority could impose lump-sum taxes. Second, banks could force repayment of loans. Here, we assume away any enforcement. The first implication is that the monetary authority cannot run a deflation. Consequently, $\gamma \geqslant 1$. The second is that those who borrow in market 1 have an incentive to default in market 2. To offset this short-run benefit we assume that if an agent defaults on his loan then the only punishment is permanent exclusion from the banking system. This is consistent with the requirement that all trades are voluntary since banks can refuse to trade with private

[16] This is the expression if one sets $\sigma = 1/2$ in their model.

[17] A similar result is found in [14] using a model of competitive search where market makers can charge differential entry fees for buyers and sellers.

agents. Furthermore, it is in the banks' best interest to share information about agents' repayment histories.

For credit to exist, it must be the case that borrowers prefer repaying loans to being banished from the banking system. Given this punishment, the real borrowing constraint $\phi\bar{\ell}$ is endogenous and we need to derive conditions to ensure voluntary repayment. In what follows, since the transfers only affect prices, we set $\tau_b = \tau_s = \tau_1 > 0$.

For buyers entering the second market with no money and who repay their loans, the expected discounted utility in a stationary equilibrium is

$$W(m) = U\left(x^*\right) - h_b + \beta V_{+1}(m_{+1}),$$

where h_b is a buyer's production in the second market if he repays his loan.

Consider the case of a buyer who defaults on his loan. The benefit of defaulting is that he has more leisure in the second market because he does not work to repay the loan. The cost is that he is out of the banking system, meaning that he cannot borrow or deposit funds for the rest of his life. He cannot lend because the bank would confiscate his deposits to settle his loan arrears. Thus, a deviating buyer's expected discounted utility is

$$\widehat{W}(m) = U(\widehat{x}) - \widehat{h}_b + \beta\widehat{V}_{+1}\left(\hat{m}_{+1}\right),$$

where the hat indicates the optimal choice by a deviator. The value of being in the banking system $W(m)$ as well as the expected discounted utility of defection $\widehat{W}(m)$ depend on the growth rate of the money supply γ. This puts constraints on γ that the monetary authority can impose without destroying financial intermediation.

Existence of a monetary equilibrium with credit requires that $W(m) \geqslant \widehat{W}(m)$, where the real borrowing constraint $\phi\bar{\ell}$ satisfies

$$W(m) = \widehat{W}(m). \tag{27}$$

Given a borrowing constraint there are two possibilities: (i) the borrowing constraint is non-binding for all agents or (ii) it binds for some agents. In an unconstrained equilibrium with credit we have $\phi\ell < \phi\bar{\ell}$ and in a constrained equilibrium $\phi\ell = \phi\bar{\ell}$. The following lemma is used for the remainder of this section.

Lemma 3. *The real borrowing constraint $\phi\bar{\ell}$ satisfies*

$$\phi\bar{\ell} = \frac{\beta}{(1+i)(1-\beta)}\left\{(1-n)\Psi(q_b, \widehat{q}_b) + c'(q_s)\left(\frac{\gamma-\beta}{\beta}\right)[\widehat{q}_b - (1-n)q_b]\right\}, \tag{28}$$

where

$$\Psi(q_b, \widehat{q}_b) = u(q_b) - u(\widehat{q}_b) - c'(q_s)(q_b - \widehat{q}_b) \geqslant 0.$$

In any equilibrium with $i > 0$, banks lend out all of their deposits so real lending satisfies

$$\phi\ell = \frac{n}{1-n}\phi M.$$

To guarantee repayment in a constrained equilibrium banks charge a nominal loan rate, $\bar{\imath}$, that is below the market clearing rate.[18]

Definition 2. A monetary equilibrium with unconstrained credit is a triple $(q_b, \widehat{q}_b, i)$ satisfying

$$\frac{\gamma-\beta}{\beta} = (1-n)\left[\frac{u'(q_b)}{c'(q_s)} - 1\right] + ni, \tag{29}$$

$$\frac{\gamma-\beta}{\beta} = (1-n)\left[\frac{u'(\widehat{q}_b)}{c'(q_s)} - 1\right], \tag{30}$$

$$\frac{u'(q_b)}{c'(q_s)} = 1+i \tag{31}$$

such that $0 < \phi\ell = nc'(q_s)\,q_b < \phi\bar{\ell}$, where $q_s = \frac{1-n}{n}q_b$.

Definition 3. A monetary equilibrium with constrained credit is a triple $(\bar{q}_b, \widehat{q}_b, \bar{\imath})$ satisfying (28), (29) and (30) where $nc'(\bar{q}_s)\,\bar{q}_b = \phi\bar{\ell}$ and $\bar{q}_s = \frac{1-n}{n}\bar{q}_b$.

Proposition 4. *There exists a critical value $\tilde{\beta}$ such that if $\beta \geqslant \tilde{\beta}$ there is a $\hat{\imath} > 0$ such that the following is true*:

(i) *If $i > \hat{\imath}$, a unique monetary equilibrium with unconstrained credit exists.*
(ii) *If $0 < i \leqslant \hat{\imath}$, a monetary equilibrium with constrained credit may exist.*
(iii) *If $i = 0$, no monetary equilibrium with credit exists.*

According to Proposition 4, existence of a monetary equilibrium with credit requires that there is some inflation. The reason for this is quite intuitive. If a borrower works to repay his loan in market 2, he is strictly worse off than when he defaults since the outside option (trading with money only) yields almost the efficient consumption q^* in all future periods. With zero inflation, agents are able to self-insure at low cost, thus having access to financial markets is of no value. As a consequence, borrowers will not repay their loans and so financial intermediation is impossible. This result is related to Aiyagari and Williamson [1] who also report a break-down of financial intermediation close to the Friedman rule in a dynamic contracting model with private information.

For low rates of inflation credit rationing occurs. Again, in this case the cost of using money to self-insure is low. To induce repayment banks charge a below market-clearing interest rate since this reduces the amount borrowers have to repay. In short, with an endogenous borrowing constraint, the interest rate is lower than would occur in an economy where banks can force repayment.[19]

One aspect that is puzzling about this result is that the incentive to default is *higher* for low nominal interest rates and *lower* for high nominal interest rates. This seems counter-intuitive

[18] This may seem counter-intuitive since one would think that banks would reduce $\bar{l}$ to induce repayment. However, this cannot be an equilibrium since it would imply that banks are not lending out all of their deposits. If banks are not lending out all of their deposits then zero profits would require $i_d = (1-\mu)\,i$ where μ is the fraction of deposits held idle by the bank. If all banks were to choose a triple (i, i_d, μ) with $\mu > 0$ such that they earned zero profits, then a bank could capture the entire market and become a monopolist by raising i_d by an infinitesimal amount and lowering μ and i by an infinitesimal amount. Since all banks can do this, in a constrained equilibrium, the only feasible solution is $\mu = 0$ and $i = i_d = \bar{\imath}$.

[19] Similar results occur in [17,18] or [2].

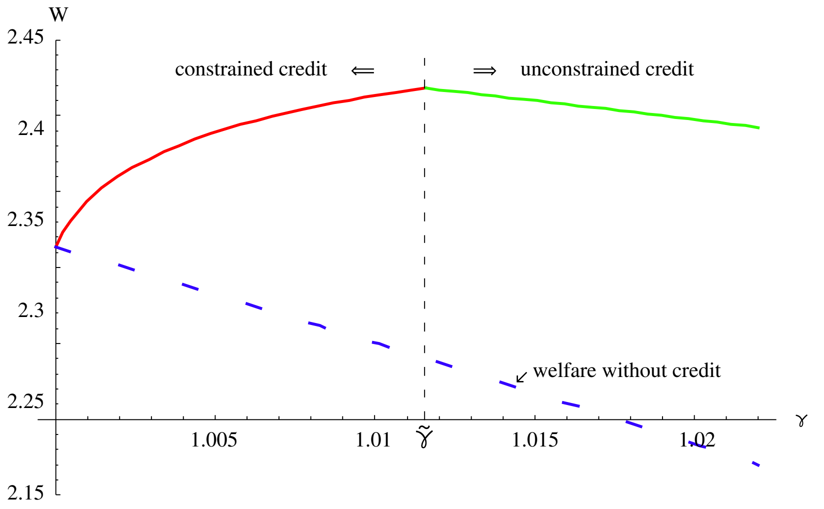

Fig. 3. Welfare with endogenous borrowing constraint.

at first glance since standard credit-rationing models, such as [31], suggest that the likelihood of default increases as interest rates rise. The reason for the difference is that standard credit rationing models focus on *real* interest rates, while our model is concerned with *nominal* interest rates. In our model, nominal rates rise because of perfectly anticipated inflation, which acts as a tax on a deviator's wealth since he carries more money for transactions purposes. This reduces the incentive to default thereby alleviating the need to ration credit. Consequently, a key contribution of our analysis is to show how credit rationing can arise from changes in *nominal* interest rates.

Is inflation welfare improving? In an unconstrained borrowing equilibrium, it is straightforward to show that inflation is always welfare reducing since it reduces the real value of money balances and consumption for all agents. However, in a constrained borrowing equilibrium, it may be optimal for the monetary authority to set $\gamma > 1$ since inflation increases the cost of being excluded from the banking system. This relaxes the borrowing constraint and creates a first-order welfare gain. We would like to know under what conditions the optimal inflation rate is positive. We can thus state the following:

Proposition 5. *In a constrained credit equilibrium, if $\beta > (1+n)^{-1}$, then a positive steady state inflation rate maximizes welfare.*

Thus, as long as agents are sufficiently patient, inflation is welfare improving. By relaxing the budget constraint, inflation allows the nominal interest rate to increase towards the market clearing level. This increases the compensation sellers receive for bringing in idle money balances yet it does not crowd out consumption by buyers since they are credit constrained. Consequently, the demand for money in market 2 increases, which raises the real value of money and q_b. To illustrate this proposition we solve the model numerically and the results are contained in Fig. 3.

In Fig. 3 the equilibrium with credit exists and is unique. The solid line is the equilibrium with credit while the dashed line is the equilibrium without credit. At the origin, $\gamma = 1$, the constrained credit equilibrium breaks down so the allocation is the same as the equilibrium without credit.

When $\gamma < \widetilde{\gamma}$, where $\widetilde{\gamma} = \beta\left(1 + \hat{\imath}\right)$, the borrowing constraint is binding and welfare is increasing in γ and when $\gamma > \widetilde{\gamma}$ the borrowing constraint is not binding and welfare is decreasing in γ. Note that the equilibrium with credit has higher welfare than the equilibrium without credit even if there is rationing.

6. Conclusion

In this paper we have shown how money and credit can coexist in a model where money is essential. Our main findings are that reallocating idle cash via financial intermediation can expand output and improve welfare away from the Friedman rule but not at the Friedman rule. Furthermore, such an improvement cannot be achieved through a central bank policy of lump-sum taxes and transfers. Interestingly, financial intermediation is most valuable for moderate rates of inflation. Also, when voluntary repayment is an issue, credit rationing may arise and in this situation, inflation improves welfare.

Our framework is open to many extensions such as private bank note issue, financing of investment instead of consumption, and longer term financial contracts. We could also extend the model to investigate the role of banks in transmitting aggregate shocks and study the optimal response of a central bank to these shocks as in [7]. Finally, the interaction of financial market regulation and stabilization policies would allow analysis of different monetary arrangements such as those expressed by the real-bills doctrine or the quantity theory as studied in [28].

Acknowledgments

We would like thank David Andolfatto, Ken Burdett, Allen Head, Nobuhiro Kiyotaki, Guillaume Rocheteau, Shouyong Shi, Ted Temzelides, Neil Wallace, Warren Weber, Randall Wright and three anonymous referees for very helpful comments. The paper has also benefitted from comments by participants at several seminar and conference presentations. Much of the paper was completed while Berentsen was visiting the University of Pennsylvania. We also thank the Federal Reserve Bank of Cleveland, the Kellogg Institute and the CES-University of Munich for research support. G. Camera acknowledges financial support from the NSF Grant DMS-0437210.

Appendix

Proof that $V(m)$ is concave $\forall m$. Differentiating (10) with respect to m

$$V'(m) = (1-n)\left[u'(q_{\rm b})\frac{\partial q_{\rm b}}{\partial m} + W_m\left(1 - p\frac{\partial q_{\rm b}}{\partial m} + \frac{\partial \ell}{\partial m}\right) + W_\ell \frac{\partial \ell}{\partial m}\right]$$
$$+n\left[-c'(q_{\rm s})\frac{\partial q_{\rm s}}{\partial m} + W_m\left(1 + p\frac{\partial q_{\rm s}}{\partial m} - \frac{\partial d}{\partial m}\right) + W_d \frac{\partial d}{\partial m}\right].$$

Recall from (7), (8), and (9) that $W_m = \phi$, $W_\ell = -\phi(1+i)$ and $W_d = \phi(1+i_d)$ $\forall m$. Furthermore, $\partial q_{\rm s}/\partial m = 0$ because the quantity a seller produces is independent of his money holdings. We also know that $\partial d/\partial m = 1$ since a seller deposits all his cash when $i > 0$. Hence,

$$V'(m) = (1-n)\left[u'(q_{\rm b})\frac{\partial q_{\rm b}}{\partial m} + \phi\left(1 - p\frac{\partial q_{\rm b}}{\partial m} + \frac{\partial \ell}{\partial m}\right) - \phi(1+i)\frac{\partial \ell}{\partial m}\right] + n\phi(1+i_d).$$

Since $i > 0$ implies $pq_{\mathrm{b}} = m + \tau_{\mathrm{b}} M_{-1} + \ell$ we have $1 - p\left(\partial q_{\mathrm{b}}/\partial m\right) + \partial \ell/\partial m = 0$. Hence [20]

$$V'(m) = (1-n)\, u'(q_{\mathrm{b}})/p + n\phi\,(1+i_d).$$

In a symmetric equilibrium $q_{\mathrm{s}} = \frac{1-n}{n} q_{\mathrm{b}}$. Define $m^* = pq^*$. Then if $m < m^*$, $0 < q_{\mathrm{b}} < q^*$, implying $\partial q_{\mathrm{b}}/\partial m > 0$ so that $V''(m) < 0$. If $m \geqslant m^*$, $q_{\mathrm{b}} = q^*$ implying $\partial q_{\mathrm{b}}/\partial m = 0$, so that $V''(m) = 0$. Thus, $V(m)$ is concave $\forall m$. $\square$

Proof of Proposition 1. Because $u(q)$ is strictly concave there is a unique value q that solves (14), and for $\gamma > \beta$, $q < q^*$ where q^* is the efficient quantity solving $u'(q^*) = c'\left(\frac{1-n}{n} q^*\right)$. As $\gamma \to \beta$, $u'(q) \to c'\left(\frac{1-n}{n} q\right)$, $q \to q^*$, and from (21) $i \to 0$. In this equilibrium, the Friedman rule sustains efficient trades in the first market. Since $V(m)$ is concave, then for $\gamma > \beta$, the choice m is maximal.

We now derive equilibrium consumption and production in the second market. Recall that, due to idiosyncratic trade shocks and financial transactions, money holdings are heterogeneous after the first market closes. Therefore, if we set $m = M_{-1}$, the money holdings of agents at the opening of the second market are 0 for buyers and $\frac{1}{n}(1+\tau_1) M_{-1}$ for sellers.

Eq. (6) gives us $x^* = U'^{-1}(1)$. The buyer's production in the second market can be derived as follows:

$$h_{\mathrm{b}} = x^* + \phi\left\{m_{+1} + (1+i)\,\ell - \tau_2 M_{-1}\right\} = x^* + c'(q_{\mathrm{s}})\, q_{\mathrm{b}} + inc'(q_{\mathrm{s}})\, q_{\mathrm{b}}$$

since in equilibrium

$$\begin{aligned}
&m_{+1} = M = M_{-1} + \tau_1 M_{-1} + \tau_2 M_{-1},\\
&c'(q_{\mathrm{s}})\, q_{\mathrm{b}} = \phi\left[(1+\tau_1) M_{-1} + \ell\right],\\
&\phi\ell = nc'(q_{\mathrm{s}})\, q_{\mathrm{b}}.
\end{aligned}$$

Thus, an agent who was a buyer in market 1 has to work to recover the production cost of his consumption and the interest on his loan. The seller's production is

$$\begin{aligned}
h_{\mathrm{s}} &= x^* + \phi\left\{m_{+1} - \left[pq_{\mathrm{s}} + (1+\tau_1) M_{-1} + i_d d + \tau_2 M_{-1}\right]\right\}\\
&= x^* - c'(q_{\mathrm{s}})\, q_{\mathrm{s}} - \phi i_d d.
\end{aligned}$$

The expected hours worked h satisfies

$$h = (1-n)\, h_{\mathrm{b}} + nh_{\mathrm{s}} = x^* \tag{32}$$

since in equilibrium $q_{\mathrm{b}} = \frac{n}{1-n} q_{\mathrm{s}}$ and $i\,(1-n)\,\ell = i_d nd$. Finally, hours in market 2 can be also expressed in terms of q as in the following table:

Trading history	Production in the last market
Buy	$h_{\mathrm{b}} = x^* + c'(q_{\mathrm{s}})\,(1-n)\, q_{\mathrm{b}} + ne(q_{\mathrm{b}})\, u(q_{\mathrm{b}})$
Sell	$h_{\mathrm{s}} = x^* - \frac{(1-n)}{n}\left[c'(q_{\mathrm{s}})\,(1-n)\, q_{\mathrm{b}} + ne(q_{\mathrm{b}})\, u(q_{\mathrm{b}})\right]$

[20] Note that $u'(q_{\mathrm{b}}) \frac{\partial q_{\mathrm{b}}}{\partial m} - \phi(1+i) \frac{\partial l}{\partial m} = u'(q_{\mathrm{b}}) \frac{\partial q_{\mathrm{b}}}{\partial m} - \phi(1+i)\left[p \frac{\partial q_{\mathrm{b}}}{\partial m} - 1\right] = \frac{\partial q_{\mathrm{b}}}{\partial m}\left(u'(q_{\mathrm{b}}) - \phi(1+i)\, p\right) + \phi(1+i) = \phi(1+i) = \frac{u'(q_{\mathrm{b}})}{p}$.

Since we assumed that the elasticity of utility $e(q_b)$ is bounded, we can scale $U(x)$ such that there is a value $x^* = U'^{-1}(1)$ greater than the last term for all $q_b \in [0, q^*]$. Hence, h_s is positive for all $q_b \in [0, q^*]$ ensuring that the equilibrium exists. $\square$

Proof of Proposition 2. Assume that at some point in time t an agent at the beginning of market 2 chooses never to borrow again but continues to deposit. The first thing to note is that it is optimal for him to buy the same quantity q_b since his optimal choice still satisfies (22). This implies that his money balances are $\bar{m}_{+1} = m_{+1} + \ell_{+1}$. An agent who decides to never borrow has to carry more money but he saves the interest on loans in the future. In particular, consumption and production in the market 1 are not affected. The difference in lifetime payoffs come from difference in hours worked.

If he enters market 2 having been a buyer, the hours worked are

$$\begin{aligned}\bar{h}_b &= x^* + \phi\{m_{+1} + \ell_{+1} + (1+i)\,\ell - \tau_2 M_{-1}\}\\ &= x^* + c'(q_s)\,q_b + \phi\ell_{+1} + \phi i\ell,\end{aligned}$$

while if he sold in market 1 he works

$$\begin{aligned}\bar{h}_s &= x^* + \phi\left\{m_{+1} + \ell_{+1} - \left[pq_s + (1+i)\,d + \tau_2 M_{-1}\right]\right\}\\ &= x^* - c'(q_s)\,q_s + \phi\ell_{+1} - \phi i_d d.\end{aligned}$$

The expected hours worked satisfy

$$\bar{h} = (1-n)\,\bar{h}_b + n\bar{h}_s = x^* + \phi\ell_{+1}.$$

Consequently, from (32) the additional hours worked are

$$\bar{h} - h = \phi\ell_{+1} = \gamma n c'(q_s)\,q_b > 0 \tag{33}$$

since $\ell_{+1} = \gamma\ell$ in a steady state.

Let us next consider the hours worked in market 2 in some future period. Since he has no loan to repay the hours worked are

$$\begin{aligned}\breve{h}_b &= x^* + \phi\{m_{+1} + \ell_{+1} - \tau_2 M_{-1}\}\\ &= x^* + c'(q_s)\,q_b + nc'(q_s)\,q_b(\gamma - 1)\end{aligned}$$

if he was a buyer while if he sold in market 1 he works

$$\begin{aligned}\breve{h}_s &= x^* + \phi\left\{m_{+1} + \ell_{+1} - \left[pq_s + (1+i)\,\bar{d} + \tau_2 M_{-1}\right]\right\}\\ &= x^* + nc'(q_s)\,q_b(\gamma - 1) - c'(q_s)\,q_s - (\gamma - \beta)\,c'(q_s)\,q_b/\beta.\end{aligned}$$

The expected hours worked h satisfies

$$\breve{h} = (1-n)\,\breve{h}_b + n\breve{h}_s = x^* - nc'(q_s)\,q_b\gamma(1-\beta)/\beta.$$

The expected gain from this strategy in any future period is

$$\breve{h} - h = -nc'(q_s)\,q_b\gamma(1-\beta)/\beta < 0. \tag{34}$$

Then, from (33) and (34), the total expected gain from this deviation is

$$\bar{h} - h + \frac{\beta\left(\breve{h} - h\right)}{1-\beta} = 0.$$

So agents are indifferent to borrowing at the current rate of interest or taking in the equivalent amount of money themselves. □

Proof of Corollary 1. Neither τ_1 nor τ_b appear in (22). Therefore, (τ_1, τ_2) can only affect the equilibrium ϕ. Of course, by changing τ_1 we change τ_2, for a given rate of growth of money. To see how the transfers affect ϕ note that $\phi\ell = c'(q_s)nq_b$. Since $\ell = \frac{n}{1-n}M_{-1}(1+\tau_s) = \frac{n}{1-n}M(\frac{1+\tau_s}{1+\tau})$ and $q_s = \frac{1-n}{n}q_b$ then we have

$$\phi = \frac{c'(q_s)nq_b}{\ell} = \frac{(1-n)c'(\frac{1-n}{n}q_b)\,(1+\tau)}{M(1+\tau_s)}$$

which implies the price of money in the second market, ϕ, is affected by the timing and size of lump-sum transfers. □

Proof of Lemma 3. Since the transfers do not affect quantities set $\tau_b = \tau_s = \tau_1 = 0$ and $\tau_2 = \tau$. We now derive the endogenous real borrowing constraint $\phi\bar{\ell}$. This quantity is the maximal real loan that a borrower is willing to repay in the second market at given market prices. For buyers entering the second market with no money, who repay their loans, the expected discounted utility in a steady state is

$$W(m) = U\left(x^*\right) - h_b + \beta V_{+1}(m_{+1}),$$

where h_b is a buyer's production in the second market if he repays his loan. Consider a borrower who borrowed $\bar{\ell}$ in market 1 and is considering defaulting on his loans in market 2. A deviating buyer's expected discounted utility is

$$\widehat{W}(m) = U(\widehat{x}) - \widehat{h}_b + \beta\widehat{V}_{+1}(\widehat{m}_{+1}),$$

where the hat indicates the optimal choice by a deviator. Thus $\phi\bar{\ell}$ is the value of borrowing such that $W(m) = \widehat{W}(m)$ or

$$U\left(x^*\right) - U(\widehat{x}) + \widehat{h}_b - h_b + \beta\left[V_{+1}(m_{+1}) - \widehat{V}_{+1}(\widehat{m}_{+1})\right] = 0. \tag{35}$$

The continuation payoffs are

$$\begin{aligned}\widehat{V}_{+1}(\widehat{m}_{+1}) &= (1-\beta)^{-1}\left[(1-n)\,u(\widehat{q}_b) - nc(\widehat{q}_s) + U\left(x^*\right) - \widehat{h}\right],\\ V_{+1}(m_{+1}) &= (1-\beta)^{-1}\left[(1-n)\,u(q_b) - nc(q_s) + U\left(\hat{x}\right) - h\right].\end{aligned} \tag{36}$$

We now derive $\widehat{x}$, $\widehat{q}_b$, $\widehat{q}_s$ and $\widehat{h}_b$. In the last market the deviating buyer's program is

$$\begin{aligned}\widehat{W}(\widehat{m}) = &\max_{\widehat{x},\widehat{h}_b,\widehat{m}_{+1}} \left[U(\widehat{x}) - \widehat{h}_b + \beta\widehat{V}_{+1}(\widehat{m}_{+1})\right]\\ &\text{s.t.}\quad \widehat{x} + \phi\widehat{m}_{+1} = \widehat{h}_b + \phi\left(\widehat{m} + \tau_2 M_{-1}\right).\end{aligned}$$

The first-order conditions are $U'(\widehat{x}) = 1$ and $-\phi + \beta\widehat{V}'_{+1}\left(\hat{m}_{+1}\right) = 0$. Thus, $\widehat{x} = x^*$. In market one, it is straightforward to show that if the deviator is a seller in the first market he sells $-c'(\widehat{q}_s) + p\phi = 0$. Hence, the deviator produces the same amount as non-deviating sellers so $\widehat{q}_s = q_s = \frac{1-n}{n}q_b$.

Finally, the marginal value of the money satisfies

$$\widehat{V}'_{+1}\left(\hat{m}\right) = \phi\left[\frac{(1-n)\,u'(\widehat{q}_b)}{c'(q_s)} + n\right]$$

which means that the deviator's choice of money balances satisfies

$$\frac{\gamma-\beta}{\beta}=(1-n)\left[\frac{u'(\widehat{q}_{\mathrm{b}})}{c'(q_{\mathrm{s}})}-1\right]. \tag{37}$$

Now if we compare (37) with (22) we find that for $\gamma>\beta$

$$1-n=\frac{u'(q_{\mathrm{b}})-c'(q_{\mathrm{s}})}{u'(\widehat{q}_{\mathrm{b}})-c'(q_{\mathrm{s}})} \tag{38}$$

implying $\widehat{q}_{\mathrm{b}}<q_{\mathrm{b}}$. Thus, using (35) and (36) we obtain

$$h_{\mathrm{b}}-\widehat{h}_{\mathrm{b}}=\frac{\beta}{1-\beta}\left[(1-n)\left[u(q_{\mathrm{b}})-u(\widehat{q}_{\mathrm{b}})\right]+\widehat{h}-h\right]. \tag{39}$$

Deriving $\widehat{h}_{\mathrm{b}}-h_{\mathrm{b}}$. If the buyer repays his loans he works

$$\begin{aligned}h_{\mathrm{b}}&=x^*+\phi m_{+1}-\phi\left[m-\bar{\ell}-pq_{\mathrm{b}}\right]-\phi\tau M_{-1}+\phi(1+i)\bar{\ell}\\&=x^*+i\bar{\ell}+\phi pq_{\mathrm{b}},\end{aligned}$$

where we use the equilibrium condition $m_{+1}=m+\tau M_{-1}=\gamma m$. If he defaults on his loans, he works

$$\begin{aligned}\widehat{h}_{\mathrm{b}}&=x^*+\phi\widehat{m}_{+1}-\phi\left(m-\bar{\ell}-pq_{\mathrm{b}}\right)-\phi\tau M_{-1}\\&=x^*+\phi(\widehat{m}_{+1}-m_{+1})-\phi\bar{\ell}+\phi pq_{\mathrm{b}}\\&=x^*+\phi\gamma(\widehat{m}-m)-\phi\bar{\ell}+\phi pq_{\mathrm{b}},\end{aligned}$$

where we use the equilibrium condition that a defaulter's money balances must grow at the rate γ so $\widehat{m}_{+1}=\widehat{m}+\tau M_{-1}=\gamma\widehat{m}$. Note that how much he spent in the previous market 1 is the same whether he repays or not. Thus

$$\begin{aligned}h_{\mathrm{b}}-\widehat{h}_{\mathrm{b}}&=x^*+\phi i\bar{\ell}+\phi pq_{\mathrm{b}}-x^*-\phi(\widehat{m}_{+1}-m_{+1})+\phi\bar{\ell}-\phi pq_{\varepsilon}\\&=\phi(1+i)\bar{\ell}-\phi\gamma(\widehat{m}-m).\end{aligned} \tag{40}$$

Deriving $\widehat{h}-h$: Once the agent defaults, as a buyer he spends $p\widehat{q}_{\mathrm{b}}$ units of money so his hours worked are

$$\begin{aligned}\widehat{h}&=x^*+\phi\widehat{m}_{+1}-\phi(\widehat{m}-p\widehat{q}_{\mathrm{b}})-\phi\tau M_{-1}\\&=x^*+\phi(\widehat{m}_{+1}-\widehat{m})+\phi p\widehat{q}_{\mathrm{b}}-\phi(m_{+1}-m)\\&=x^*+(\gamma-1)\phi(\widehat{m}-m)+\phi p\widehat{q}_{\mathrm{b}}.\end{aligned}$$

For a seller we have

$$\begin{aligned}\widehat{h}_{\mathrm{s}}&=x^*+\phi\widehat{m}_{+1}-\phi(\widehat{m}+p\widehat{q}_{\mathrm{s}})-\phi\tau M_{-1}\\&=x^*+(\gamma-1)\phi(\widehat{m}-m)-\phi p\left(\frac{1-n}{n}\right)q_{\mathrm{b}}.\end{aligned}$$

So for a defaulter expected hours worked are $\widehat{h}=(1-n)\widehat{h}_{\mathrm{b}}+n\widehat{h}_{\mathrm{s}}=x^*+(\gamma-1)\phi(\widehat{m}-m)$ while if he does not deviate he works $h=x^*$ and so

$$\widehat{h}-h=(\gamma-1)\phi(\widehat{m}-m). \tag{41}$$

Substituting (40) and (41) into (39) and rearranging yields

$$\phi\bar{\ell} = \frac{\beta}{(1+i)(1-\beta)}\left\{(1-n)\,\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}}) + c'(q_{\mathrm{s}})\frac{\gamma-\beta}{\beta}\left[\widehat{q}_{\mathrm{b}} - (1-n)\,q_{\mathrm{b}}\right]\right\}, \tag{42}$$

where $\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}}) = u(q_{\mathrm{b}}) - u(\widehat{q}_{\mathrm{b}}) - c'\left(\frac{1-n}{n}q_{\mathrm{b}}\right)(q_{\mathrm{b}} - \widehat{q}_{\mathrm{b}}) > 0$ since

$$\frac{u(q_{\mathrm{b}}) - u(\widehat{q}_{\mathrm{b}})}{q_{\mathrm{b}} - \widehat{q}_{\mathrm{b}}} > u'(q_{\mathrm{b}}) > c'(q_{\mathrm{s}}) \tag{43}$$

for all $\gamma > \beta$. The RHS of (42) must be positive to have a credit equilibrium. For this to be positive, substitute (37) and (43) into (42) and rearrange to obtain

$$\frac{u(q_{\mathrm{b}}) - u(\widehat{q}_{\mathrm{b}})}{q_{\mathrm{b}} - \widehat{q}_{\mathrm{b}}} > u'(q_{\mathrm{b}})\,\frac{q_{\mathrm{b}} - \widehat{q}_{\mathrm{b}}\frac{u'(q_{\mathrm{b}})}{u'(\widehat{q}_{\mathrm{b}})}}{q_{\mathrm{b}} - \widehat{q}_{\mathrm{b}}}.$$

The RHS of this inequality is less than $u'(q_{\mathrm{b}})$ since $u'(q_{\mathrm{b}}) < u'(\widehat{q}_{\mathrm{b}})$ while from (43) the LHS is greater than $u'(q_{\mathrm{b}})$. Thus $\phi\bar{\ell} > 0$ for $\gamma > \beta$. $\square$

Proof of Proposition 4. *Unconstrained credit equilibrium*. We need $c'(q_{\mathrm{s}})\,nq_{\mathrm{b}} = \phi\ell < \phi\bar{\ell}$. Since $i = (\gamma-\beta)/\beta$ in an unconstrained equilibrium from (42) we have

$$(1-\beta)(1+i)\,c'(q_{\mathrm{s}})\,nq_{\mathrm{b}} < \beta(1-n)\,\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}}) + \beta i c'(q_{\mathrm{s}})\left[\widehat{q}_{\mathrm{b}} - (1-n)\,q_{\mathrm{b}}\right]. \tag{44}$$

Define

$$\begin{aligned} g(i,\beta) &= (1-\beta)(1+i)\,c'(q_{\mathrm{s}})\,nq_{\mathrm{b}} \\ f(i,\beta) &= \beta(1-n)\,\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}}) + \beta i c'(q_{\mathrm{s}})\left[\widehat{q}_{\mathrm{b}} - (1-n)\,q_{\mathrm{b}}\right]. \end{aligned}$$

Note that $g(0,\beta) > 0$ and $f(0,\beta) = 0$ for all $0 < \beta < 1$ since $\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}})|_{(0,\beta)} = 0$. So (44) is violated at $i = 0$ and $\beta < 1$. Define $\Delta(i,\beta) \equiv g(i,\beta) - f(i,\beta)$ and consider solutions to $\Delta(i,\beta) = 0$. Note that $\Delta(0,1) = 0$ since $q_{\mathrm{b}}|_{(0,1)} = \widehat{q}_{\mathrm{b}}|_{(0,1)} = q^*$. Let $\Delta_i(i,\beta) \equiv \partial\Delta(i,\beta)/\partial i$ and $\Delta_\beta(i,\beta) \equiv \partial\Delta(i,\beta)/\partial\beta$. We have

$$\frac{\partial g(i,\beta)}{\partial i} = (1-\beta)\,nq_{\mathrm{b}}c'(q_{\mathrm{s}}) + (1-\beta)\left[c'(q_{\mathrm{s}})\frac{\partial q_{\mathrm{b}}}{\partial i} + q_{\mathrm{b}}c''(q_{\mathrm{s}})\frac{\partial q_{\mathrm{s}}}{\partial i}\right](1+i)n$$

and

$$\begin{aligned} \frac{\partial f(i,\beta)}{\partial i} = \beta\Bigg\{&(1-n)\frac{\partial\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}})}{\partial i} + c'(q_{\mathrm{s}})\left[\widehat{q}_{\mathrm{b}} - (1-n)\,q_{\mathrm{b}}\right] \\ &+ ic''(q_{\mathrm{s}})\frac{\partial q_{\mathrm{s}}}{\partial i}\left[\widehat{q}_{\mathrm{b}} - (1-n)\,q_{\mathrm{b}}\right] + ic'(q_{\mathrm{s}})\left[\frac{\partial\widehat{q}_{\mathrm{b}}}{\partial i} - (1-n)\frac{\partial q_{\mathrm{b}}}{\partial i}\right]\Bigg\}. \end{aligned}$$

These partial derivatives are continuous with

$$\left.\frac{\partial g(i,\beta)}{\partial i}\right|_{(0,1)} = 0 \quad \text{and} \quad \left.\frac{\partial f(i,\beta)}{\partial i}\right|_{(0,1)} = c'\left(\frac{1-n}{n}q^*\right)nq^* > 0.$$

Therefore $\Delta_i(i,\beta)$ is continuous and non-zero with

$$\Delta_i(0,1) = -c'\left(\frac{1-n}{n}q^*\right)nq^* < 0.$$

We also see that

$$\frac{\partial g\left(i,\beta\right)}{\partial\beta}=-nq_{\mathrm{b}}c'\left(q_{\mathrm{s}}\right)\left(1+i\right)+\left(1-\beta\right)\left[c'\left(q_{\mathrm{s}}\right)\frac{\partial q_{\mathrm{b}}}{\partial\beta}+q_{\mathrm{b}}c''\left(q_{\mathrm{s}}\right)\frac{\partial q_{\mathrm{s}}}{\partial\beta}\right]\left(1+i\right)n,$$

$$\frac{\partial f\left(i,\beta\right)}{\partial\beta}=\frac{f\left(i,\beta\right)}{\beta}+\beta\left\{\left(1-n\right)\frac{\partial\Psi\left(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}}\right)}{\partial\beta}+ic''\left(q_{\mathrm{s}}\right)\frac{\partial q_{\mathrm{s}}}{\partial\beta}\left[\widehat{q}_{\mathrm{b}}-\left(1-n\right)q_{\mathrm{b}}\right]\right.$$
$$\left.+ic'\left(q_{\mathrm{s}}\right)\left[\frac{\partial\widehat{q}_{\mathrm{b}}}{\partial\beta}-\left(1-n\right)\frac{\partial q_{\mathrm{b}}}{\partial\beta}\right]\right\}$$

with

$$\left.\frac{\partial g\left(i,\beta\right)}{\partial\beta}\right|_{(0,1)}=-c'\left(\frac{1-n}{n}q^*\right)nq^*<0,$$

$$\left.\frac{\partial f\left(i,\beta\right)}{\partial\beta}\right|_{(0,1)}=f\left(0,1\right)+\left(1-n\right)\left.\frac{\partial\Psi\left(i,\beta\right)}{\partial\beta}\right|_{(0,1)}=0$$

since $\left.\frac{\partial\Psi(q_{\mathrm{b}},\widehat{q}_{\mathrm{b}})}{\partial\beta}\right|_{(0,1)}=0$ and $f\left(0,1\right)=0$. Therefore $\Delta_{\beta}\left(0,1\right)$ is continuous and

$$\Delta_{\beta}\left(0,1\right)=-c'\left(\frac{1-n}{n}q^*\right)nq^*<0.$$

By the implicit function theorem, it follows that, for β arbitrarily close to one, the expression $\Delta(i,\beta)=0$ defines i as an implicit function of β, i.e., $i=\hat{\imath}(\beta)$. Furthermore, we have

$$\left.\frac{di}{d\beta}\right|_{(0,1)}=-\frac{\Delta_{\beta}\left(0,1\right)}{\Delta_{i}\left(0,1\right)}=-1,$$

so that as β falls i grows. It follows from the implicit function theorem that $\Delta(\hat{\imath},\beta)=0$ for a unique $i=\hat{\imath}>0$ and β sufficiently close to one.

Above we established that $g\left(0,\beta\right)>f(0,\beta)=0$ for all $0<\beta<1$. Thus, fix $\tilde{\beta}<\beta<1$ where $\tilde{\beta}$ is close to 1. We have established that $g\left(\hat{\imath},\beta\right)-f\left(\hat{\imath},\beta\right)=0$ for some $\hat{\imath}>0$. By continuity, we have that if $i>\hat{\imath}$ then $g\left(i,\beta\right)<f\left(i,\beta\right)$ and so an unconstrained equilibrium exists. For $0\leqslant i<\hat{\imath}$, then $g\left(i,\beta\right)>f\left(i,\beta\right)\geqslant 0$, so if an equilibrium exists it is constrained.

Constrained credit equilibrium: We now consider $0\leqslant i<\hat{\imath}$. In a constrained equilibrium the defection constraint must hold with equality implying

$$\left(1+\bar{\imath}\right)nc'\left(\bar{q}_{\mathrm{s}}\right)\bar{q}_{\mathrm{b}}=\frac{\beta}{1-\beta}\left\{\left(1-n\right)\Psi+c'\left(\bar{q}_{\mathrm{s}}\right)\left(\frac{\gamma-\beta}{\beta}\right)\left[\widehat{q}_{\mathrm{b}}-\left(1-n\right)\bar{q}_{\mathrm{b}}\right]\right\},\tag{45}$$

where $\bar{q}_{\mathrm{b}}$ denotes the quantity consumed and $\bar{\imath}$ is the interest rate in a constrained equilibrium. From the first-order conditions on money holdings we have

$$\frac{\gamma-\beta}{\beta}=\left(1-n\right)\left[\frac{u'\left(\bar{q}_{\mathrm{b}}\right)}{c'\left(\bar{q}_{\mathrm{s}}\right)}-1\right]+n\bar{\imath},\tag{46}$$

$$\frac{\gamma-\beta}{\beta}=\left(1-n\right)\left[\frac{u'\left(\widehat{q}_{\mathrm{b}}\right)}{c'\left(\bar{q}_{\mathrm{s}}\right)}-1\right],\tag{47}$$

where $\bar{q}_{\mathrm{s}}=\frac{1-n}{n}\bar{q}_{\mathrm{b}}$. Thus, a constrained equilibrium is a list $\{\bar{q}_{\mathrm{b}},\widehat{q}_{\mathrm{b}},\bar{\imath}\}$ such that (45)-(47) hold.

We now investigate the properties of (45)–(47). At $\bar{\imath} = 0$, from (46) and (47), $\bar{q}_{\text{b}} = \widehat{q}_{\text{b}}$. Then from (45) we have $\gamma = 1$. This implies there is one and only one monetary policy consistent with a nominal interest rate of zero in a constrained credit equilibrium. Taking the total derivative of (45) and evaluating it at $\gamma = 1$, $\bar{\imath} = 0$, $\widehat{q}_{\text{b}} = \bar{q}_{\text{b}}$ using (47) as well to get

$$\frac{d\bar{\imath}}{d\gamma}\Big|_{\gamma=1} = \frac{1}{1-\beta} > 0. \tag{48}$$

These observations imply that for all $1 < \gamma \leqslant \hat{\gamma}$ where $\hat{\imath} = \left(\hat{\gamma} - \beta\right)/\beta$ we have $\bar{\imath} > 0$. It then follows that a constrained credit equilibrium can exist if and only if $0 \leqslant i < \hat{\imath}$. However, we cannot show existence of a constrained equilibrium in general. □

Proof of Proposition 5. Differentiate $\mathcal{W}$ with respect to γ to get

$$\frac{d\mathcal{W}}{d\gamma}\bigg|_{\gamma=1} = \frac{1-n}{1-\beta}\left[u'\left(\bar{q}_{\text{b}}\right) - c'\left(\frac{1-n}{n}\bar{q}_{\text{b}}\right)\right]\frac{d\bar{q}_{\text{b}}}{d\gamma}\bigg|_{\gamma=1}.$$

Since $u'\left(\bar{q}_{\text{b}}\right) - c'\left(\bar{q}_{\text{s}}\right) > 0$ it is sufficient to show that $\frac{d\bar{q}_{\text{b}}}{d\gamma}\Big|_{\gamma=1} > 0$ for $\frac{d\mathcal{W}}{d\gamma}\Big|_{\gamma=1} > 0$. Totally differentiate (46), evaluate at $\gamma = 1$, $\bar{\imath} = 0$, $\widehat{q}_{\text{b}} = \bar{q}_{\text{b}}$ and use (48) to obtain

$$\frac{d\bar{q}_{\text{b}}}{d\gamma}\bigg|_{\gamma=1} = \left[\frac{1-(1+n)\beta}{\beta(1-\beta)(1-n)}\right]\left[\frac{c'\left(\bar{q}_{\text{s}}\right)^2}{u''\left(\bar{q}_{\text{b}}\right)c'\left(\bar{q}_{\text{s}}\right) - u'\left(\bar{q}_{\text{b}}\right)c''\left(\bar{q}_{\text{s}}\right)\frac{1-n}{n}}\right].$$

The second bracketed term is negative. Thus if $\beta > 1/(1+n)$ this derivative is positive and welfare is increasing. □

References

[1] S. Aiyagari, S. Williamson, Money and dynamic credit arrangements with private information, J. Econ. Theory 91 (2000) 248–279.

[2] F. Alvarez, U. Jermann, Efficiency, equilibrium and asset pricing with risk of default, Econometrica 68 (2000) 775–797.

[3] S.B. Aruoba, C. Waller, R. Wright, Money and capital, Working paper, University of Pennsylvania, 2006.

[4] A. Berentsen, G. Camera, C. Waller, The distribution of money and prices in an equilibrium with lotteries, Econ. Theory 24 (2004) 887–906.

[5] A. Berentsen, G. Camera, C. Waller, The distribution of money balances and the non-neutrality of money, Int. Econ. Rev. 46 (2005) 465–487.

[6] A. Berentsen, G. Rocheteau, S. Shi, Friedman meets Hosios: efficiency in search models of money, Econ. J., forthcoming.

[7] A. Berentsen, C. Waller, Optimal stabilization policy with flexible prices, CESifo Working Paper No. 1638, 2005.

[8] T. Bewley, The optimum quantity of money, in: J.H. Kareken, N. Wallace (Eds.), Models of Monetary Economies, Federal Reserve Bank of Minneapolis, Minneapolis, 1980, pp. 169–210.

[9] G. Camera, D. Corbae, Money and price dispersion, Int. Econ. Rev. 40 (1999) 985–1008.

[10] R. Cavalcanti, A. Erosa, T. Temzelides, Private money and reserve management in a random-matching model, J. Polit. Economy 107 (1999) 929–945.

[11] R. Cavalcanti, N. Wallace, Inside and outside money as alternative media of exchange, J. Money, Credit, Banking 31 (Part 2) (1999) 443–457.

[12] R. Cavalcanti, N. Wallace, A model of private bank-note issue, Rev. Econ. Dynam. 2 (1999) 104–136.

[13] M. Faig, Money and banking in an economy with villages, Working paper, University of Toronto, 2004.

[14] M. Faig, X. Huangfu, Competitive search equilibrium in monetary economies, J. Econ. Theory, forthcoming.

[15] E. Green, R. Zhou, Money as a mechanism in a Bewley economy, Int. Econ. Rev. 46 (2005) 351–371.

[16] P. He, L. Huang, R. Wright, Money and banking in search equilibrium, Int. Econ. Rev. 46 (2005) 631–670.

[17] C. Hellwig, G. Lorenzoni, Bubbles and private liquidity, Working paper, UCLA, 2004.
[18] T. Kehoe, D. Levine, Debt-constrained asset markets, Rev. Econ. Stud. 60 (1993) 865–888.
[19] N. Kocherlakota, Money is memory, J. Econ. Theory 81 (1998) 232–251.
[20] N. Kocherlakota, Societal benefits of illiquid bonds, J. Econ. Theory 108 (2003) 179–193.
[21] R. Lagos, G. Rocheteau, Inflation, output and welfare, Int. Econ. Rev. 46 (2005) 495–522.
[22] R. Lagos, R. Wright, A unified framework for monetary theory and policy analysis, J. Polit. Economy 113 (2005) 463–484.
[23] D. Levine, Asset trading mechanisms and expansionary policy, J. Econ. Theory 54 (1991) 148–164.
[24] R.E. Lucas, Equilibrium in a pure currency economy, in: J.H. Karaken, N. Wallace (Eds.), Models of Monetary Economics, Federal Reserve Bank of Minneapolis, Minneapolis, 1980, pp. 131–145.
[26] M. Molico, The distribution of money and prices in search equilibrium, Int. Econ. Rev. 47 (2006) 701–722.
[27] G. Rocheteau, R. Wright, Money in search equilibrium, in competitive equilibrium and in competitive search equilibrium, Econometrica 73 (2004) 175–202.
[28] T. Sargent, N. Wallace, The real-bills doctrine versus the quantity theory: a reconsideration, J. Polit. Economy 90 (6) (1982) 1212–1236.
[29] S. Shi, Credit and money in a search model with divisible commodities, Rev. Econ. Stud. 63 (1996) 627–652.
[30] S. Shi, A divisible search model of fiat money, Econometrica 65 (1997) 75–102.
[31] J. Stiglitz, A. Weiss, Credit rationing in markets with imperfect information, Amer. Econ. Rev. 71 (1981) 393–410.
[32] N. Wallace, Narrow banking meets the Diamond–Dybvig model, Fed. Reserve Bank Minneapolis Quart. Rev. 20 (1) (1996) Winter, 3–13.
[33] N. Wallace, Whither monetary economics?, Int. Econ. Rev. 42 (2001) 847–869.