

# Cigarette Money<sup>1</sup>

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We study how commodities emerge as money, the way cigarettes did in POW camps. We characterize how specialization, trading frictions, intrinsic properties of goods, and the amount of fiat money determine whether a commodity serves as money and its value. In some equilibria, the exchange value of commodity money is pinned down by its consumption value; in others, it is not. The value of fiat money may or may not be pinned down by commodity money. In some equilibria, the total (fiat plus commodity) money supply is independent of the fiat money supply. We also discuss implications for Gresham's law. *Journal of Economic Literature* Classification Numbers: E40, D83. © 2001 Academic Press

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## 1. INTRODUCTION

We study the circumstances under which certain commodities emerge endogenously as media of exchange, the way cigarettes did in prisoner-of-

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war camps, for example, as described by Radford [10]. As the Radford article makes clear, the POW camp is an ideal situation in which to study the issue, for several reasons. First, the POW economy was relatively uncomplicated and focused primarily on exchange (as opposed to, say, production or investment). Second, exchange was of considerable importance to the prisoners. Third, there is no question that cigarettes served as money:

Between individuals there was active trading in all consumer goods and in some services. Most trading was for food against cigarettes or other food stuffs, but cigarettes rose from the status of a normal commodity to that of currency. ... With this development everyone, including nonsmokers, was willing to sell for cigarettes, using them to buy at another time and place. Cigarettes became the normal currency, though, of course, barter was never extinguished. [10, pp. 190–191]

The use of commodities like cigarettes as money is not exclusive to POW camps.<sup>3</sup> Our objective is to develop a theoretical framework to help us understand when some agents stop smoking and start trading cigarettes, and hopefully also generate insights into exchange institutions more generally. We use a search-based model of the trading process because it allows one to endogenously determine the equilibrium pattern of exchange and, hence, which objects are used as money. The model includes general goods that everyone consumes (cigarettes) as well as specialized goods that only certain individuals consume but that yield a lot of utility for those who do (Brits love tea, French prefer coffee). Specialization generates gains from trade, but also makes trade difficult, and leads to a natural role for money. This can cause some individuals to stop consuming and start trading generally desired goods. The extent to which this happens depends on the nature of the trading process, the degree of specialization, the properties of different goods, and the existence of fiat currency.

The paper differs from previous analyses of commodity money using the search or random matching framework in several ways. For one thing, in our model the key economic decision facing an agent is whether to consume a general good now or, alternatively, store it in an attempt to trade for a preferred special good. By contrast, in the model in Kiyotaki and Wright [5] and its extensions, an agent always consumes any good he

<sup>3</sup> Another example is given by Friedman [3, pp. 12–13]: “After World War II [in Germany] the Allied occupational authorities exercised sufficiently rigid control over monetary matters, in the course of trying to enforce price and wage controls, that it was difficult to use foreign currency. Nonetheless, the pressure for a substitute currency was so great that cigarettes and cognac emerged as substitute currencies and attained an economic value far in excess of their value purely as goods to be consumed. ... Foreigners often expressed surprise that Germans were so addicted to American cigarettes that they would pay a fantastic price for them. The usual reply was “Those aren’t for smoking; they’re for trading.” See Neale [8] for related examples.

desires once he gets it, and the only decision is whether to trade one good he does not desire for another good he does not desire in an attempt to acquire a desired good more efficiently. Agents do not stop consuming any good in those models. Here a central issue is to determine the circumstances under which people stop smoking cigarettes, either partially or completely, and begin to trade them. Moreover, this leads directly to the following feature of our model that is missing from most of the related literature: the money supply—that is, the quantity of cigarette money and/or fiat money in circulation—is endogenous.<sup>4</sup>

The rest of the paper and some results can be summarized as follows. We begin in Section 2 by describing a simple version of the model, where there is no fiat money and all goods are indivisible (so that prices are fixed). We show that there is a unique equilibrium where, depending on parameters, either no agents, some agents, or all agents stop consuming the generally desired consumption good and start using it as money. Relevant parameters include the degree to which goods are specialized, the relative supply of general and special goods, their intrinsic properties including depreciation rates, time preference, and the nature of trading frictions. In Section 3 we incorporate divisible goods and let agents bargain over the terms of trade. We generalize the previous findings and generate the following new result: as long as general goods are used for consumption by at least some agents, their value in exchange is tied down by their value in consumption, but if parameter values are such that the general goods are never consumed they will trade at a premium over their consumption value.

Although the indivisible goods model has a unique equilibrium, the model with bargaining can have multiple equilibria. Multiplicity is perhaps more commonly thought to be associated with fiat money, since the value of fiat money is obviously based on beliefs. For certain parameters in our model, in equilibrium cigarettes are consumed by no one, and so it is perhaps not so surprising that we get results similar to a fiat money model, even though cigarettes are in some sense clearly a commodity money. That is, the exchange value of cigarettes can have a fiat component and this can depend on beliefs. However, it is worth emphasizing that for some parameters there coexists an equilibrium where cigarettes are consumed by

<sup>4</sup> There are also some technical differences from earlier models, motivated by thinking about cigarette money. One is that we have general commodities that are symmetrically enjoyed by everyone as well as specialized goods that are enjoyed by some agents and not others, while in the typical model in the search literature *all* commodities are enjoyed by some agents and not others. Also, for the potential commodity money to generate utility here it must be “used up” in consumption, like cigarettes, while in some other models commodity money generates utility (or disutility) while in storage without depreciating, like gold jewelry, perhaps.

no one and another equilibrium where they are consumed by some agents, and in the latter their exchange value is given by their consumption value while in the former their exchange value exceeds their consumption value. In any event, once we incorporate genuine fiat currency in Section 4, the possibility of multiplicities expands further.

More interestingly, perhaps, in the case with both fiat and commodity money we can prove the following: As long as we do not introduce too much fiat money, it does not affect the exchange process at all—each unit of fiat currency simply crowds out a unit of commodity money and the economy proceeds as before, except for a one-time welfare gain due to the fact that someone can now consume the cigarettes that were previously circulating as money. However, if we introduce too much fiat money, we can drive commodity money from circulation completely and this will have real effects on the exchange process. One way to interpret this finding is that the economy can be quite resilient in monetary matters: an endogenous means of payment will emerge from the private sector if and only if an adequate supply of fiat money is not provided by the public sector. Another interpretation is that policy makers in charge of the fiat money supply may actually have far less impact than one would imagine, once it is recognized that the total money supply is endogenous.

In Section 5 we further extend the model to allow the general commodity to come in heterogeneous qualities in an attempt to capture some of Radford's observations about good and bad cigarettes in the POW camp, and to discuss issues related to Gresham's law more generally. Perhaps surprisingly, we find that whether bad cigarettes end up circulating as money while good cigarettes are consumed, or vice-versa, actually depends on parameter values. Intuitively, while good cigarettes are better for smoking they are also better for trading, given that relative prices are endogenous and reflect the intrinsic properties of the different commodities. We also analyze a version of the model where private information implies that relative prices do not reflect true intrinsic differences in quality, and show that, under these assumptions, good money is necessarily driven out of circulation before bad, in conformance to the predictions of Gresham.<sup>5</sup>

<sup>5</sup> Gresham's law has been previously analyzed in search models by Velde *et al.* [15] and Renero [11]. Sargent and Smith [12] provide an analysis in a very different model that generates some similar things to what we find. Another relevant reference is Azariadis *et al.* [1]; although they are looking at circulating public and private liabilities, rather than fiat and commodity money, their motivation and some of their results (if not the theoretical framework) have several things in common with the current project. It is also worth mentioning Williamson [16], who is interested in private money, and Peterson [9], who is especially interested in endogenizing the amount of money in circulation. Finally, work by Engineer and Shi [2] is also related in the sense that they also emphasize the distinction between some goods which are symmetrically enjoyed by all agents and other goods which are enjoyed by some agents and not others.

## 2. THE BASIC MODEL

Time is continuous. The economy is populated by a  $[0, 1]$  continuum of agents who act as if the horizon is infinite and discount at rate  $r > 0$ . There are many goods, all of which are storable and (for now) indivisible. One of them is called the *general good*, to be thought of as cigarettes, and the others are called *special goods*. All agents derive utility  $u_g$  from consuming a unit of the general good. Every agent derives utility  $u_s$  from consuming a unit of a particular type of special good and cannot consume other special goods, where  $u_s > u_g$ . General goods are subject to depreciation, while for simplicity specialized goods are not. Thus, according to a Poisson process with parameter  $d$ , a general good spoils, or simply disappears. New goods enter the economy in the following way: after an agent consumes a general good, or after a general good depreciates, he always produces a special good other than one he consumes; but after he consumes his special good, he produces a special good with probability  $\sigma$  and a general good with probability  $1 - \sigma$ .<sup>6</sup>

There are gains from trade due to specialized tastes and goods. Rather than a centralized market, we assume agents meet bilaterally according to an anonymous random matching process, and a pair trades if and only if it makes both strictly better off. Let the Poisson arrival rate of potential trading partners be  $\alpha$ . On meeting someone with a special good, let  $x \leq 1$  be the probability (common to everyone) that his special good is the type you desire. Given this event, and given that you also have a special good, let  $y < 1$  be the conditional probability that your special good is also the type he desires. Notice that  $y$  measures the extent of the double coincidence problem. A special case is the specification in Kiyotaki and Wright [6], e.g., where  $x = y$ . While everything we say holds when  $x = y$ , it also holds more generally, and so we see no reason to restrict attention to this special case.<sup>7</sup>

<sup>6</sup> This way of modeling production is meant to capture, more or less, the way the endowments arrived in a POW camp. As Radford [10] reports, "Our supplies consisted of rations provided by the detaining power and (principally) the contents of Red Cross food parcels—tinned milk, jam, biscuits, bully, chocolate, sugar, etc., and cigarettes. ... Private parcels of clothing, toilet requisites and cigarettes were also received." We tried various other assumptions concerning exactly how general and special goods arrive, such as allowing either type to be produced at random after a general good is consumed as well as after a special good is consumed, and the results were the same.

<sup>7</sup> According to Radford [10], in the early stages, or in temporary camps, people literally searched randomly, "wandering though the bungalows calling their offers—'cheese for seven' (cigarettes)." In permanent camps, eventually a Exchange and Mart board went up where notices could be posted listing "name," "room number," "wanted" and "offered," and in some permanent camps public shops were set up where you could leave goods to be sold. Although more organized, these institutions still imply that there is a random time until someone will show up, want your good, and also have what you want. Hence, the random matching assumption would seem to be a reasonable abstraction.

It is always rational for an agent to trade whatever he has for the special good he desires and to consume it immediately. Also, in symmetric equilibria, at least if there is any transaction cost, no matter how small, agents will never accept special goods unless they consume them, exactly as in Kiyotaki and Wright [6]. It is always rational to trade a special good for the general good since you can always consume it and produce another special good costlessly. What needs to be decided is whether to consume the general good when you get it, or to store it to facilitate a future trade for your favorite special good. We will say that a fraction  $\theta$  of the population choose to *always* consume the general good when they get it, while the remaining  $1 - \theta$  choose to *never* consume it. It would be equivalent to say that each agent makes the choice of consuming or storing the general good each time he gets it; this amounts to the same thing because, in equilibrium,  $\theta = 1$  when agents prefer consuming the general good,  $\theta = 0$  when they prefer storing the general good, and  $\theta \in (0, 1)$  only when they are indifferent.

Given that a fraction  $\theta$  of individuals consume the general good while  $1 - \theta$  store it, and given the probability of producing the general good and its rate of depreciation, let  $G$  and  $S = 1 - G$  denote the steady state proportions of the population holding general and special goods. The steady state condition (derived in the Appendix) is

$$(1 - S)[d + \alpha x S \sigma] = [\alpha(1 - S)x + \alpha S x y(1 - \sigma)](S - \theta). \quad (1)$$

For any  $\theta$  this implies a unique  $S$ . Alternatively, we can solve for the value of  $\theta$  that generates a particular  $S$ ,

$$\theta = \frac{Q(S)}{1 - S + y(1 - \sigma)S}, \quad (2)$$

where  $Q(S) = -(1 - \sigma)(1 - y)S^2 + (1 - \sigma + d/\alpha x)S - d/\alpha x$ . For any  $S \in [\hat{S}, 1]$  there is a unique fraction of general good consumers  $\theta \in [0, 1]$  such that  $\theta$  implies  $S$  will be the fraction of special good holders in steady state.<sup>8</sup>

We now describe steady state payoffs. Let  $V_s$  and  $V_g$  be the value functions of agents storing special goods and general goods. Consider first an agent with a special good. In principle, there are two types of such agents: those who always consume general goods and those who always store general goods. When someone who consumes the general good acquires it he consumes and produces a special good for a total payoff of  $u_g + V_s$ . When he acquires the special good he desires, he consumes, and with probability  $\sigma$  a new special good is produced and stored, while with probability

<sup>8</sup> This follows because  $\theta$  is strictly increasing in  $S$ ,  $\theta = 1$  if  $S = 1$ , and  $\theta = 0$  if  $S = \hat{S}$ , where  $\hat{S}$  solves  $Q(\hat{S}) = 0$  and satisfies  $\hat{S} \in [0, 1)$  with  $\hat{S} = 0$  if and only if  $d = 0$ .

$1 - \sigma$  a general good is produced and consumed, followed by a special good; hence, the payoff to acquiring the special good is  $u_s + (1 - \sigma)u_g + V_s$ . Consider next someone with a special good who never consumes the general good. When he gets the general good his payoff is simply  $V_g$ . When he gets the special good he desires, he consumes and stores whatever he produces, for a total payoff of  $u_s + \sigma V_s + (1 - \sigma)V_g$ .

Based on these observations, Bellman's equation in flow terms for a special good holder is

$$rV_s = \alpha Gx[\theta u_g + (1 - \theta)(V_g - V_s)] \\ + \alpha Sxy[u_s + (1 - \sigma)\theta u_g + (1 - \sigma)(1 - \theta)(V_g - V_s)], \quad (3)$$

where one can interpret  $\theta$  here as a dummy variable indicating that the agent chooses to consume or store the general good in order to maximize expected life time utility. Intuitively, (3) sets the flow return  $rV_s$  equal to the sum of two terms. The first term is the rate at which a special good holder meets a general good holder,  $\alpha G$ , times the probability the latter desires the special good he holds,  $x$ , times his gain from trade, which is the net payoff from consuming the general good with probability  $\theta$  and storing it with probability  $1 - \theta$ . The second term is the rate at which he meets a special good holder,  $\alpha S$ , times the probability they desire each other's special goods,  $xy$ , times his gain from trade, which in this case is the utility of consuming the special good, plus the probability he produces the general good,  $1 - \sigma$ , times the net payoff to consuming it with probability  $\theta$  and storing it with probability  $1 - \theta$ .

Now consider an agent with a general good. He is allowed to consume it, of course, but suppose he does not (since we are trying to determine the value of *storing* the general good,  $V_g$ ). Then Bellman's equation is given by

$$rV_g = \alpha Sx[u_s + \sigma(V_s - V_g)] + d(V_s - V_g). \quad (4)$$

This sets the flow return  $rV_g$  equal to the rate at which he meets a special good holder with the desired special good,  $\alpha Sx$ , times the gain from trade, plus the rate at which the general good depreciates,  $d$ , times the capital loss  $V_s - V_g$ .<sup>9</sup>

<sup>9</sup> One can also write the term in square brackets in (4) as  $u_s + \sigma(V_s - V_g) + (1 - \sigma)\theta(u_g + V_s - V_g)$ , to indicate that the agent chooses to consume or store a general good after it is produced, but here we have used the fact that any agent with a general good in inventory has already chosen to be a general good non-consumer and so  $\theta = 0$ . This is merely a notational issue and has no effect on any results. The way we solve the agent's problem is by conjecturing a strategy and then checking that there is no profitable deviation, and since we only look at steady state equilibria, it does not matter whether we check, for example, that a general good non-consumer deviates by consuming a general good once, or deviates by always consuming general goods.

The net gain from consuming the general good and storing the special good that you produce, rather than storing the general good, is given by  $\Delta \equiv u_g + V_s - V_g$ . Individual optimization with respect to  $\theta$  entails the best response condition:

$$\begin{aligned} \Delta > 0 &\Rightarrow \theta = 1; \\ \Delta < 0 &\Rightarrow \theta = 0; \\ \Delta = 0 &\Rightarrow \theta \in [0, 1]. \end{aligned} \tag{5}$$

Using (3) and (4), one can solve explicitly for

$$\Delta = A \left[ 1 - S + \frac{r}{\alpha x} + (1 - \sigma) S y + S \sigma + \frac{d}{\alpha x} \right] u_g - A S (1 - y) u_s, \tag{6}$$

where  $A$  is a positive constant that need not concern us, since only the sign of  $\Delta$  matters.

A *steady state equilibrium* may now be defined as a list  $(S, V_s, V_g, \theta, \Delta)$  satisfying (2)–(6), subject to  $V_g \geq 0$ ,  $V_s \geq 0$ ,  $0 \leq S \leq 1$ ,  $0 \leq \theta \leq 1$ . In practice, we simply look for pairs  $(S, \theta)$  satisfying the steady state condition and the best response condition. There are three types of possible outcomes, depending on whether the general good is always consumed ( $\theta = 1$ ), sometimes consumed ( $0 < \theta < 1$ ), or never consumed ( $\theta = 0$ ). If  $\theta < 1$  the general good circulates as a commodity money (indeed, they are a universally acceptable commodity money, since all agents accept cigarettes, even though some may accept them for smoking while others accept them for retrading). The following Proposition establishes when each type of equilibrium exists. To reduce notation, from now on we let  $\rho = r/\alpha x$  and  $\delta = d/\alpha x$ .

**PROPOSITION 1.** *There exists  $y_1 < 1$  and  $y_2 < y_1$  such that: (i) if  $y \geq y_1$  then  $\theta = 1$  and  $S = 1$ ; (ii) if  $y \in (y_2, y_1)$  then  $\theta \in (0, 1)$  and  $S \in (\hat{S}, 1)$ ; and (iii) if  $y \leq y_2$  then  $\theta = 0$  and  $S = \hat{S}$ . These are all of the (steady state) equilibria.*

*Proof.* First consider an equilibrium with  $\theta = 1$ , which implies  $S = 1$  by the steady state condition. For this to be an equilibrium we require  $\Delta \geq 0$ . Setting  $\theta = 1$  and  $S = 1$  in (6), it is immediate that  $\Delta \geq 0$  if and only if  $y \geq y_1$ , where

$$y_1 = \frac{u_s - (\rho + \sigma + \delta) u_g}{u_s + (1 - \sigma) u_g}. \tag{7}$$



We conclude that an equilibrium with  $\theta = 1$  and  $S = 1$  exists if and only if  $y \geq y_1$ .

Next consider the case where  $0 < \theta < 1$ . We need to find  $(S, \theta) \in (0, 1)^2$  such that the steady state condition is satisfied and  $\Delta = 0$ . The method we pursue is to find a value of  $S$ , call it  $S_A$ , for which  $\Delta = 0$ , and then identify conditions under which  $S_A \in (\hat{S}, 1)$ , as this is equivalent to  $(S, \theta) \in (0, 1)^2$  by virtue of (2). Using (6) we have

$$S_A = \frac{(1 + \rho + \delta) u_g}{(1 - y)[u_s + (1 - \sigma) u_g]}.$$

Notice that  $\partial S_A / \partial y > 0$  and  $S_A = 1$  if  $y = y_1$ , and so  $S_A < 1$  if and only if  $y < y_1$ . Using the function  $Q(S)$  defined after (2), one shows that  $S_A > \hat{S}$  if and only if  $y > y_2$  where

$$y_2 = \frac{[u_s - (\delta + \rho + \sigma) u_g][\delta u_s - (1 + \rho)(1 - \sigma) u_g]}{\delta [u_s + (1 - \sigma) u_g]^2}.$$

We conclude that an equilibrium with  $(S, \theta) \in (0, 1)^2$  exists if and only if  $y_2 < y < y_1$ .

Finally, consider  $\theta = 0$ , which implies  $S = \hat{S}$ , and requires  $\Delta \leq 0$ . We first note that  $\Delta$  is increasing in  $y$  (this is easily verified by showing  $\partial \Delta / \partial y > 0$ ,  $\partial \Delta / \partial S < 0$  and  $\partial \hat{S} / \partial y < 0$ ). Then, since  $\Delta = 0$  when  $y = y_2$  and  $S = \hat{S}$ , it follows that this equilibrium exists if and only if  $y \leq y_2$ . This completes the proof. ■

Since  $\theta < 1$  if and only if  $y < y_1$ , Proposition 1 tells us that general goods are more likely to be used as money when  $y$  is small or  $y_1$  big. Now  $y$  is small when barter is difficult because goods and tastes are highly specialized. And  $y_1$  is big when: general goods are not very desirable relative to special goods ( $u_g/u_s$  is low); people are patient ( $r$  is low); general goods do not depreciate very quickly ( $d$  is low); search frictions are not too severe ( $\alpha x$  is big); or special goods are produced infrequently ( $\sigma$  is low). Figure 1 shows where the different equilibria exist in various regions of parameter space. Figure 2 shows  $\theta$ , as well as the amount of commodity money in circulation,  $G$ , as functions of parameters. The top panel varies  $y$ ; the second panel varies  $\alpha$ ; and the third panel varies  $x$  and  $y$  together, since the case  $y = x$  is one that is often analyzed in the literature.<sup>10</sup>

<sup>10</sup> The bounds  $y_2$  and  $y_1$  defined in the proof of Proposition 1 satisfy  $y_2 < y_1 < 1$ . Notice, however, it is possible for  $y_1$  or  $y_2$  to be negative. For example, if  $d = 0$  and general goods do not depreciate at all, then  $y_2 < 0$  and we cannot have  $y \leq y_2$ . This means that when  $d < 0$  we cannot have an equilibrium with  $\theta = 0$ . Intuitively,  $d = \theta = 0$  implies that in steady state everyone has cigarettes in inventory, which means that it is pointless trying to trade a cigarette for one's special good, and so it cannot be a best response to store rather than smoke cigarettes.

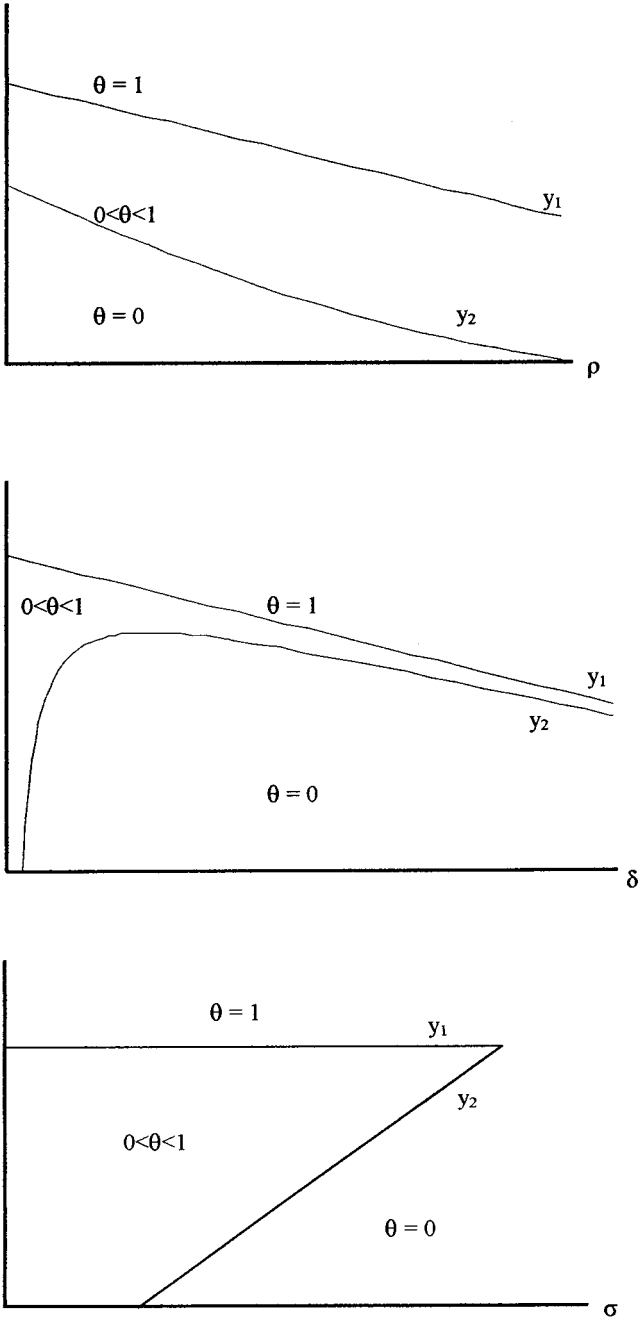
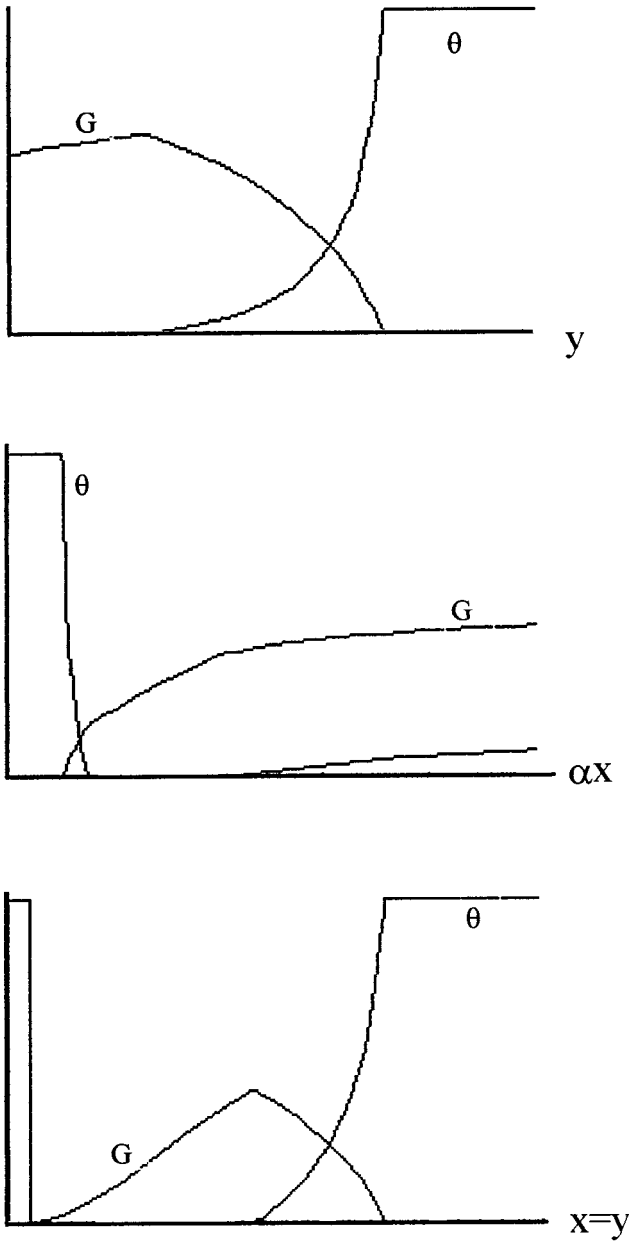


FIG. 1. Regions of parameter space where the different equilibria exist.

FIG. 2. Equilibrium values of  $\theta$  and  $G$ .

## 3. PRICES

Up to this point we have determined when general goods will be used as money, but we have not tried to determine their purchasing power, as all goods were assumed to be indivisible. Following Shi [13] and Trejos and Wright [14], we now endogenize prices by assuming that special goods are perfectly divisible and allowing agents to bargain over the amount of a special good that they trade for an indivisible unit of the general good. It will still be the case in this model that all agents always have either 1 unit or 0 units of the general good. While this is obviously an abstraction, modeling price formation in this way turns out to be an order of magnitude more tractable than proceeding under the assumption that everything is perfectly divisible; see, e.g., Molico [7].

Agents who receive  $q$  units of their special good in trade enjoy utility  $u_s = U_s(q)$ , while agents who produce  $q$  units suffer disutility  $c(q)$ . We assume that  $U_s(0) = 0$ ,  $U'_s(q) > 0$ ,  $U''_s(q) < 0$ , and that there is a  $\hat{q} \in [0, 1]$  such that  $U_s(\hat{q}) = \hat{q}$ . With no loss in generality, we normalize  $c(q) = q$ . The interpretation here is that with probability  $\sigma$  agents get an *opportunity* to produce a special good later, rather than an actual unit of output; i.e., they do not actually produce until they trade, since we do not want the cost to be a sunk cost at the time of bargaining. Denote by  $\Omega$  the expected net utility from bartering one special good for another (to be determined below), and by  $q$  the amount of special good that one gets for a general good. Recalling that  $\rho = r/\alpha x$  and  $\delta = d/\alpha x$ , for any  $q$  and  $\Omega$  Bellman's equations are now given by the generalization of (3) and (4),

$$\begin{aligned} \rho V_s &= (1 - S)[-q + \theta u_g + (1 - \theta)(V_g - V_s)] \\ &\quad + Sy[\Omega + (1 - \sigma)\theta u_g + (1 - \sigma)(1 - \theta)(V_g - V_s)] \end{aligned} \quad (8)$$

$$\rho V_g = S[U_s(q) + \sigma(V_s - V_g)] + \delta(V_s - V_g).$$

For direct barter transactions between two special good holders we adopt the symmetric Nash bargaining solution, which implies both agents produce  $q^*$  units where  $q^*$  satisfies  $U'_s(q^*) = c'(q^*) = 1$ , and therefore  $\Omega = U_s(q^*) - q^*$ . Nothing really depends on this, however; all that happens when we change the bargaining solution in a barter opportunity is that  $\Omega$  changes, and we allow  $\Omega$  to take on any value in what follows. For trades of general against special goods, it simplifies the analysis to assume the

agent with the general good makes a take-it-or-leave-it offer.<sup>11</sup> This implies that

$$q = \theta u_g + (1 - \theta)(V_g - V_s), \quad (9)$$

as this is the greatest  $q$  that a special good holder would produce for a general good. Notice that (9) implies that the first term in the Bellman equation for  $V_s$  vanishes.

An equilibrium is defined as before, except that we make  $q$  endogenous and add the bargaining solution as an equilibrium condition. Thus, we now look for combinations  $(S, \theta, q)$  satisfying (2), (5) and (9). Proposition 2 will show that as long as general goods are sometimes used for consumption purposes (i.e., as long as  $\theta > 0$ ), their value in exchange is pegged to their intrinsic value in consumption,  $q = u_g$ . However, when  $\theta = 0$  general goods trade at a premium over their intrinsic value,  $q > u_g$ , as Friedman [3] observed happening with cigarettes in post-war Germany. Further, we show that there is a range of parameter values such that there are multiple equilibrium values of  $\theta$  (agents may or may not stop smoking cigarettes). Also, there can be more than one equilibrium value of  $q$  for a given equilibrium value of  $\theta$ .

**PROPOSITION 2.** *There exist  $\bar{y}_1$  and  $\bar{y}_2 < \bar{y}_1$  such that: (i) if  $y \geq \bar{y}_1$  then there is an equilibrium with  $\theta = 1$  and  $q = u_g$ ; (ii) if  $y \in (\bar{y}_2, \bar{y}_1)$  then there is an equilibrium with  $\theta \in (0, 1)$  and  $q = u_g$ ; and (iii) if  $y < \bar{y}_2$  then there is an equilibrium with  $\theta = 0$  and  $q > u_g$ . Moreover, there is a  $z > 0$  such that if  $u_g > z$  then these are the only equilibria; but if  $u_g < z$  then  $\bar{y}_2 > 0$  and there exists  $\bar{y}_3 > \bar{y}_2$  such that when  $y \in (\bar{y}_2, \bar{y}_3)$  there are two other equilibria, both having  $\theta = 0$  but different values of  $q > u_g$ . These are all of the (steady state) equilibria.*

*Proof.* First consider equilibrium with  $\theta = 1$ , which implies  $q = u_g$  by (9), and

$$\Delta = \frac{[\rho + \sigma + \delta + (1 - \sigma)y] u_g + y\Omega - U_s(u_g)}{\rho + \sigma + \delta}.$$

<sup>11</sup> Assuming take-it-or-leave-it offers by the agent with the general good means that the other agent gets no surplus in the exchange, which causes some terms in the Bellman's equations to vanish. Other than making the algebra manageable, however, there is nothing special about this bargaining solution, and we expect that the spirit of the model would survive generalization on this dimension.

This equilibrium exists if and only if  $\Delta \geq 0$ , which holds if and only if  $y \geq \bar{y}_1$  where

$$\bar{y}_1 = \frac{U_s(u_g) - (\rho + \sigma + \delta) u_g}{\Omega + (1 - \sigma) u_g}. \quad (10)$$

Next consider  $\theta \in (0, 1)$ , which implies  $u_g = V_g - V_s$ , and therefore again implies  $q = u_g$  by (9). As in Proposition 1, we solve  $\Delta = 0$  for  $S = \bar{S}_\Delta$ , where

$$\bar{S}_\Delta = \frac{(\rho + \delta) u_g}{U_s(u_g) - y\Omega - [\sigma + (1 - \sigma) y] u_g}, \quad (11)$$

and then check when  $\bar{S}_\Delta$  is in  $(\hat{S}, 1)$ . One can show  $\bar{S}_\Delta < 1$  if and only if  $y < \bar{y}_1$ . Also, note that  $\bar{S}_\Delta$  is increasing in  $y$  while  $\hat{S}$  is decreasing in  $y$ . Furthermore, there is a value  $\bar{y}_2$  such that  $\bar{S}_\Delta = \hat{S}$ . Hence,  $\bar{S}_\Delta > \hat{S}$  if and only if  $y > \bar{y}_2$ .

Now consider equilibria where  $\theta = 0$ , which implies  $S = \hat{S}$ , and requires  $\Delta \leq 0$ . We can combine (8) and (9) into the single condition  $T(q) = 0$ , where

$$T(q) = \hat{S}u_s(q) - \hat{S}\Omega y - [\rho + \delta + \sigma\hat{S} + (1 - \sigma) y\hat{S}] q.$$

A solution to  $T(q) = 0$  is an equilibrium if and only if it satisfies  $q \geq u_g$ , since this is equivalent to  $\Delta \leq 0$ . Note that  $T'(0) > 0$ ,  $T''(q) < 0$  for all  $q$ , and  $T(q) < 0$  for large  $q$ , as shown in Fig. 3. Also, if  $y = 0$  then  $T(q)$  has two roots,  $q = 0$  and  $q > 0$ . As  $y$  increases, one can show that  $T(q)$  shifts down, and therefore has two positive roots, until we reach a point  $y = \bar{y}_3$  where  $T(q)$  is tangent to the horizontal axis. Consequently, if  $y > \bar{y}_3$  there are no solutions to  $T(q) = 0$ . Let  $z$  denote the value of  $q$  at which  $T(q)$  is tangent to the axis when  $y = \bar{y}_3$ .

Notice that when  $y = \bar{y}_2$ , where  $\bar{y}_2$  was defined above by  $\bar{S}_\Delta = \hat{S}$ , one solution to  $T(q) = 0$  is always given by  $q = u_g$  (which implies that  $\bar{y}_3 > \bar{y}_2$ ). There are two possible cases. The first case is  $u_g > z$ . This implies that when  $y = \bar{y}_2$ ,  $q = u_g$  is the higher root of  $T(q) = 0$ . Then for all  $y \in (0, \bar{y}_2)$  the higher root of  $T(q) = 0$  is the unique solution such that  $q > u_g$ , and for  $y > \bar{y}_2$  there is no solution to  $T(q) = 0$  such that  $q \geq u_g$ . The second case is  $u_g < z$ . This implies that  $\bar{y}_2 > 0$ , because  $T(u_g) > 0$  when  $y = 0$ . It also implies that when  $y = \bar{y}_2$ ,  $q = u_g$  is the lower root of  $T(q) = 0$ . Then for all  $y \in (0, \bar{y}_2)$  the higher root of  $T(q) = 0$  is the unique solution such that  $q > u_g$ , for  $y \in (\bar{y}_2, \bar{y}_3)$  both roots of  $T(q) = 0$  satisfy  $q > u_g$ , and for  $y > \bar{y}_3$  there is no solution to  $T(q) = 0$ .

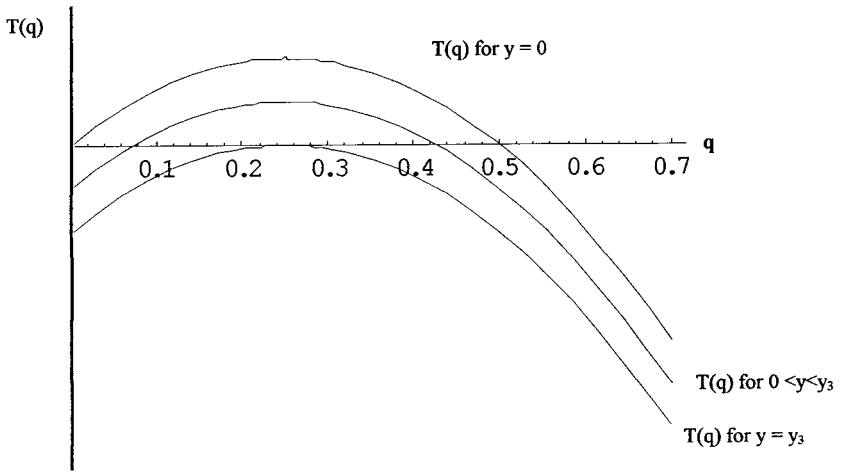


FIG. 3. The function  $T(q)$ .

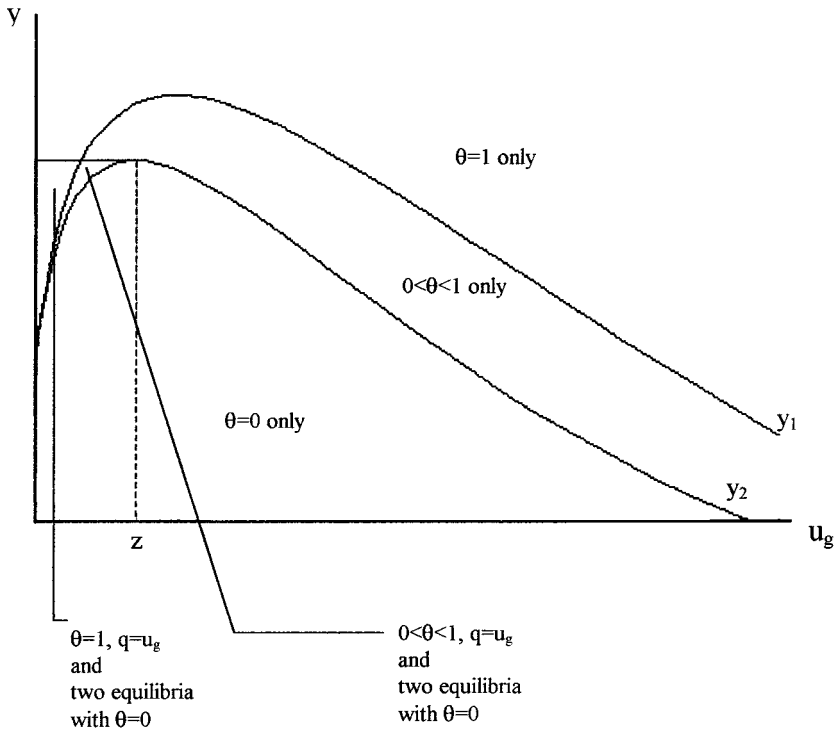


FIG. 4. Equilibria with endogenous prices.

We conclude the following. On the one hand, for  $u_g > z$ , for all  $y \in (0, \bar{y}_2)$  there is a unique equilibrium with  $\theta = 0$ , and for  $y > \bar{y}_2$  there are no equilibria with  $\theta = 0$ . On the other hand, for  $u_g < z$ , for  $y \in (0, \bar{y}_2)$  there is a unique equilibrium  $q$  with  $\theta = 0$ , for  $y \in (\bar{y}_2, \bar{y}_3)$  there are two equilibria with  $\theta = 0$  and different values of  $q$ , and for  $y > \bar{y}_3$  there are no equilibria with  $\theta = 0$ . This completes the proof. ■

Figure 4 shows the regions in  $(y, u_g)$  space where the different equilibria exist. Notice in particular that there are multiple equilibria when  $u_g < z$  and  $y \in (y_2, y_3)$ . First, there are two equilibria with  $\theta = 0$  and different values of  $s$ , both greater than  $u_g$ . Also, in this same region, there is an equilibrium with  $q = u_g$ , and either  $\theta = 1$  or  $\theta \in (0, 1)$  depending on the exact value of  $y$ . We know that  $q$  can never be less than  $u_g$ , since otherwise no one would use the general good as money. What is interesting is that  $q$  can be greater than  $u_g$ . It is perhaps tempting to say that when  $\theta = 0$  the general good is like fiat money, in the sense that its exchange value is not pinned down by its intrinsic value; at least, we could say that cigarettes can have a fiat component that allows them to trade above their consumption value. In any case, in the next section we introduce genuine fiat money.

#### 4. FIAT MONEY

We now introduce a second potential medium of exchange to compete with cigarette money, in the form of fiat currency.<sup>12</sup> We assume fiat currency can be neither produced nor consumed by individuals, and an exogenous fraction  $M$  of the population are simply endowed with it. Following the method in the previous section, we assume that both fiat money and general goods are indivisible, and determine relative prices by making special goods divisible. Also, we restrict attention here to the case

<sup>12</sup> Radford [10] reports that there was some official fiat money (RMk.s) in the camp, but it “had no circulation save for gambling debts.” However, during relatively good economic times the camp introduced its own paper currency. It was initially backed 100 percent by food at the shop and restaurant—hence its name, the “Bully Mark”—and so it was not exactly fiat: “Originally one BMk. was worth one cigarette and for a short time both circulated freely inside and outside the restaurant.” However, “The BMk. was tied to food, but not to cigarettes: as it was issued against food, say 45 for a tin of milk and so on, any reduction in the BMk. prices of food would have meant that there were unbacked BMk.s in circulation.” Hence, even though BMk.s were partially backed, it still seems interesting to consider fiat money in the model. For the record, “In August parcels and cigarettes were halved and the camp was bombed. The Restaurant closed for a short while and sales of food became difficult. ... The BMk. fell to four-fifths of a cigarette and eventually farther still, and it became unacceptable save in the restaurant. There was a flight from the BMk., no longer convertible into cigarettes or popular foods. The cigarette reestablished itself.”



where general goods do not depreciate ( $d=0$ ). This implies that there are no equilibria with  $\theta=0$ , and that without fiat money the equilibrium is unique and satisfies  $q=u_g$ . Hence, any multiplicities that occur in this section are due to the existence of the fiat object. Moreover, it is clear that there is always an equilibrium in which the value of fiat money is  $V_m=0$ . As this case reduces to one analyzed above, from now on we focus on equilibria where  $V_m>0$ .

Let the steady state proportions of the population holding special goods, general goods, and money be given by  $S$ ,  $G$  and  $M$ . A generalized version of the steady state condition from the model without fiat money can be used to show that  $S$  varies monotonically between 0 and  $1-M$  as  $\theta$  varies between 0 and 1. Let  $q_g$  and  $q_m$  be the amount of special good one can get for a unit of general good and a unit of fiat money, respectively. Then Bellman's equations are

$$\begin{aligned} \rho V_s &= Sy[\Omega + (1-\sigma)\theta u_g + (1-\sigma)(1-\theta)(V_g - V_s)] \\ &\quad + G[-q_g + \theta u_g + (1-\theta)(V_g - V_s)] + M[-q_m + V_m - V_s] \\ \rho V_g &= S[U_s(q_g) + \sigma(V_s - V_g)] \\ \rho V_m &= S[U_s(q_m) + \sigma(V_s - V_m) \\ &\quad + (1-\sigma)\theta(u_g + V_s) + (1-\sigma)(1-\theta)(V_g - V_m)]. \end{aligned} \quad (12)$$

Assuming that agents with either general goods or fiat money get to make take-it-or-leave-it offers, bargaining implies

$$q_g = \theta u_g + (1-\theta)(V_g - V_s) \quad (13)$$

$$q_m = V_m - V_s. \quad (14)$$

Notice again that the bargaining solution underlying (13) and (14) implies that some terms in the Bellman equation for  $V_s$  vanish.

An equilibrium is now defined as the obvious generalization of the previous section: we look for combinations  $(S, \theta, q_g, q_m)$  satisfying the relevant conditions. We will show that as long as  $M$  is not too large there is an equilibrium where fiat money simply crowds out commodity money one-for-one and  $q_m = q_g = u_g$ . Hence, the introduction of a small amount of fiat currency can have no effect at all, except for a one-time seigniorage gain that occurs because someone can now consume some general goods that were previously serving as money. Once  $M$  is sufficiently big so as to drive general goods completely from circulation, however, the introduction of more fiat money has real effects on the exchange process. Also, there can be multiple equilibria of the following form: when fiat and commodity

money coexist they may trade at the same value or at different values. Also, there are parameter values for which there cannot be any commodity money in circulation (because everyone always consumes the general good) but there can still be valued fiat money.

**PROPOSITION 3.** *There exists  $\check{y}_1 < 1$  such that: (i) if  $y \geq \check{y}_1$  then  $\theta = 1$ ; and (ii) if  $y < \check{y}_1$  then  $\theta \in (0, 1)$ . In any equilibrium  $q_g = u_g$ , while  $q_m$  depends on parameter values. On the one hand, if  $y < \check{y}_1$ , there is a value  $\check{y}_A < 1$  such that, when  $y < \check{y}_A$  the unique equilibrium has  $q_m = u_g$ , and when  $y > \check{y}_A$  there are two equilibrium values of  $q_m$ , one equal to and one lower than  $u_g$ . On the other hand, if  $y > \check{y}_1$ , there is a value  $\check{y}_B \in (\check{y}_A, 1)$  such that: when  $y < \check{y}_A$  there is a unique equilibrium  $q_m < u_g$ ; when  $y \in (\check{y}_A, \check{y}_B)$  there are two equilibrium values of  $q_m$ , both lower than  $u_g$ ; and when  $y > \check{y}_B$  there are no equilibrium with  $q_m > 0$ . These are all of the (steady state) equilibria with valued fiat money.*

*Proof.* First note that there is no equilibrium with  $\theta = 0$  and  $S = 0$ , since this implies  $\Delta > 0$  and this contradicts  $\theta = 0$ . Now suppose  $\theta = 1$  and  $S = 1 - M$ . Then (13) implies  $q_g = u_g$ , and

$$\Delta = \frac{[\rho/(1-M) + \sigma + (1-\sigma)y]u_g + y\Omega - U_s(u_g)}{\rho/(1-M) + \sigma}.$$

This equilibrium requires  $\Delta \geq 0$ , which holds if and only if  $y \geq \check{y}_1$  where

$$\check{y}_1 = \frac{U_s(u_g) - [\rho/(1-M) + \sigma]u_g}{\Omega + (1-\sigma)u_g}. \quad (15)$$

Now suppose  $\theta \in (0, 1)$ . This requires  $\Delta = 0$ , which again implies  $q_g = u_g$  by (13). We need to solve  $\Delta = 0$  for  $S$  and check when  $S \in (0, 1 - M)$ . The value of  $S$  that solves  $\Delta = 0$  is given by the same  $\bar{S}_\Delta$  that makes  $\Delta = 0$  in the model without fiat money, given in (11) above, which is obviously independent of  $M$ . From this one can easily check that  $\bar{S}_\Delta \in (0, 1 - M)$  if and only if  $y < \check{y}_1$ .

It remains to determine  $q_m$ . We can combine (14) and (12) into the single condition  $q_m = \check{T}(q_m)$ , where

$$\check{T}(q) = S[U_s(q) - y\Omega + (1-y)(1-\sigma)u_g] - (\rho + S)q,$$

and  $S = 1 - M$  when  $\theta = 1$  and  $S = \bar{S}_\Delta$  when  $\theta < 1$ . Note that  $\check{T}'(0) > 0$ ,  $\check{T}''(q) < 0$  for all  $q$ , and  $\check{T}(q) < 0$  for large  $q$ ; hence  $\check{T}$  is qualitatively the same as the function  $T$  shown previously in Fig. 3. The zeros of  $\check{T}$  depend on  $y$ .

First consider the case where  $y > \check{y}_1$ , and let

$$\check{y}_A = \frac{(1 - \sigma) u_g}{\Omega + (1 - \sigma) u_g}.$$

Then  $y < \check{y}_A$  implies  $\check{T}(0) > 0$  and so there is a unique solution to  $\check{T}(q_m) = 0$ . As  $y$  increases,  $\check{T}(q)$  shifts down, until we reach  $\check{y}_B > \check{y}_A$  where  $\check{T}$  is tangent to the horizontal axis, given by

$$\check{y}_B = \frac{U_s(\check{q}) - (\rho + 1 - M)/(1 - M) \check{q} + (1 - \sigma) u_g}{\Omega + (1 - \sigma) u_g}$$

where  $\check{q}$  solves  $(1 - M) u'(\check{q}) = \rho + 1 - M$ . Then  $y \in (\check{y}_A, \check{y}_B)$  implies there are 2 solutions to  $q_m = \check{T}(q_m)$  and  $y > \check{y}_B$  implies there is no solution. Moreover, one checks that  $q_m < u_g$  for all  $y > 0$  by showing  $\check{T}(u_g) \leq 0$  at  $y = 0$ , that  $\check{T}$  is decreasing in  $y$ .

Now consider  $y < \check{y}_1$ . In this case, one can easily check that one solution to  $\check{T}(q) = 0$  is given by  $q = u_g$  for any  $y$ . For  $y < \check{y}_A$ , since  $\check{T}(0) > 0$ ,  $q = u_g$  is the only solution. For  $y > \check{y}_A$ , since  $\check{T}(0) < 0$ , there is also a solution with  $q < u_g$ . This completes the proof. ■

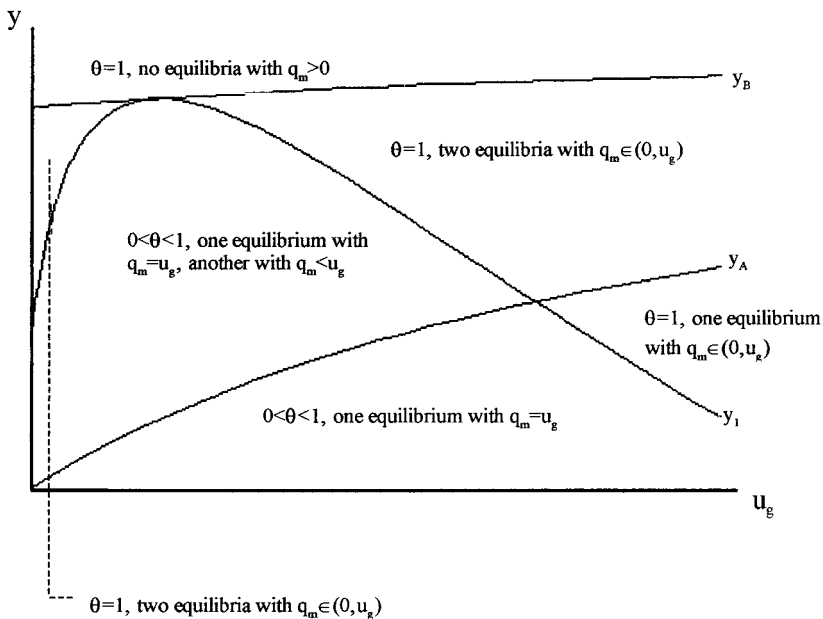


FIG. 5. Equilibria with fiat money.

The set of equilibria in  $(y, u_g)$  space is shown in Fig. 5.<sup>13</sup> Notice that is more likely that  $y < \check{y}_1$ , and hence it is more likely that an equilibrium with  $\theta \in (0, 1)$  exists, if  $M$  is small. However,  $S = \bar{S}_A$  is independent of  $M$ . This implies that as long as we stay in an equilibrium with  $\theta < 1$ , when  $M$  increases  $G$  decreases, and the total amount of money  $G + M$  is independent of  $M$ . However, as  $M$  gets bigger, eventually we must have  $y \geq \check{y}_1$ , which means that  $\theta = 1$  and  $G = 0$ . Once everyone is consuming the general good,  $G$  cannot fall any further. At this point, additional increases in  $M$  must decrease  $S$ . Also notice that for small  $M$  and small  $y$  the value of fiat money must be the same as that of commodity money,  $q_m = q_g = u_g$ . For higher values of  $y$  the value of fiat money may or may not be the same as that of commodity money. For higher values of  $M$  the value of fiat money is necessarily less than  $u_g$ .

## 5. HETEROGENEOUS GOODS

Radford describes how heterogeneous cigarettes traded at different prices in the POW camp. As one might expect from Gresham's Law, certain brands were more regularly used for consumption, while other presumably inferior cigarettes (including hand-rolled ones made from pipe tobacco) were used mainly as money. Moreover, inferior cigarettes tended to circulate at a premium over their intrinsic value. To study this phenomena, suppose now that general commodities come in two qualities: good cigarettes and bad cigarettes. The probability of producing a special good, a good cigarette, and a bad cigarette are given by  $\sigma$ ,  $\gamma$ , and  $\beta$ , and the steady state proportions of agents holding these objects are  $S$ ,  $G$ , and  $B$ . The probability of consuming a good cigarette upon acquiring it is  $\theta_g$  and the probability of consuming a bad cigarette upon acquiring it is  $\theta_b$ . The utility of consuming a good cigarette or bad cigarette is  $u_g$  or  $u_b$ . Cigarette holders make take-it-or-leave offers to special good holders, and the amount of the special good that a good or a bad cigarette commands is  $q_g$  or  $q_b$ . For simplicity, assume no depreciation and no fiat money.

Bellman's equations are

$$\rho V_s = S\gamma[\Omega + \gamma\theta_g u_g + \gamma(1 - \theta_g)(V_g - V_s) + \beta\theta_b u_b + \beta(1 - \theta_b)(V_b - V_s)]$$

$$\rho V_g = S[U_s(u_g) + \sigma(V_s - V_g) + \beta\theta_b(u_b + V_s - V_g) + \beta(1 - \theta_b)(V_b - V_g)]$$

$$\rho V_b = S[U_s(u_b) + \sigma(V_s - V_b) + \gamma\theta_g(u_g + V_s - V_b) + \gamma(1 - \theta_g)(V_g - V_b)].$$

<sup>13</sup> The figure is drawn using the following easily verified results:  $\check{y}_1$ ,  $\check{y}_A$  and  $\check{y}_B$  are 0 when  $u_g = 0$ ;  $\check{y}_1$  increases with  $u_g$  for small  $u_g$ , then decreases and becomes negative as  $u_g$  gets big; and  $\check{y}_A$  and  $\check{y}_B$  increase monotonically to 1 as  $u_g$  gets big.

In writing these in this way we have used the fact that a special good holder gets no surplus from trading with (good or bad) cigarette holders, and that anyone holding a good cigarette must not be a good cigarette consumer although he may be a bad cigarette consumer, and vice versa. We have also inserted  $q_j = u_j$ , which as before follows directly from the bargaining solution.

Consider the case where  $\theta_g = \theta_b = 1$ , which implies  $S = 1$  and  $B = G = 0$ . Bellman's equations reduce in this case to

$$\begin{aligned}\rho V_s &= y(\Omega + \gamma u_g + \beta u_b) \\ \rho V_g &= U_s(u_g) + \sigma(V_s - V_g) + \beta(u_b + V_s - V_g) \\ \rho V_b &= U_s(u_b) + \sigma(V_s - V_b) + \gamma(u_g + V_s - V_b).\end{aligned}$$

The equilibrium condition corresponding to  $\theta_j = 1$  is  $\Delta_j \geq 0$ , where  $\Delta_j$  is proportional to  $u_j + V_s - V_j$ . Solving for the  $\Delta_j$ , we obtain

$$\begin{aligned}\Delta_g &= y\Omega + (\rho + \sigma + \beta + \gamma y) u_g - (1 - y) \beta u_b - U_s(u_g) \\ \Delta_b &= y\Omega + (\rho + \sigma + \gamma + \beta y) u_b - (1 - y) \gamma u_g - U_s(u_b).\end{aligned}$$

It is easy to check that  $\Delta_j \geq 0$  if and only if  $y \geq y_j$ , where

$$\begin{aligned}y_g &= \Sigma [U_s(u_g) + \beta u_b + \gamma u_g - (1 + \rho) u_g] \\ y_b &= \Sigma [U_s(u_b) + \beta u_b + \gamma u_g - (1 + \rho) u_b]\end{aligned}$$

and  $\Sigma = 1/(\Omega + \beta u_b + \gamma u_g)$ .

Hence,  $\theta_g = \theta_b = 1$  is an equilibrium if and only if  $y \geq \max\{y_g, y_b\}$ . One might expect that  $y_g < y_b$ , so that the equilibrium condition for smoking good cigarettes holds automatically as long as the equilibrium condition for smoking bad cigarettes holds; this is not true in general. One can check that  $y_g < y_b$  if and only if

$$\frac{U_s(u_g)}{1 + \rho} - u_g < \frac{U_s(u_b)}{1 + \rho} - u_b. \quad (16)$$

Notice that  $U_s(u_j)/(1 + \rho) - u_j$  can be interpreted as the gain from trading a type  $j$  cigarette next period rather than smoking it now, and  $y_g < y_b$  if and only if this gain is bigger for bad quality cigarettes. The point is that good cigarettes are not only better for smoking, they are also better for trading, and so it is not unambiguous whether the incentive condition to smoke rather than trade a cigarette is more severe for good or bad cigarettes.

Consider now an equilibrium where  $\theta_g = 1$  and  $\theta_b < 1$ . Since good cigarettes are always consumed, only bad cigarettes and special goods circulate, and we can use the conditions from the homogeneous general good model (with  $B$  replacing  $G$ ) to find the steady state. To construct an equilibrium of this sort, one proceeds as follows. First solve for the  $\Delta_j = u_j + V_s - V_j$  as functions of  $S$ ; then solve  $\Delta_b = 0$  for the steady state  $S$  and check that it is in  $(0, 1)$ ; and finally substitute  $S$  into  $\Delta_g$  and check that it is nonnegative (one can recover  $\theta_b$  from  $S$  using the steady state condition, but it is not necessary to check anything else because  $0 < \theta_b < 1$  if and only if  $0 < S < 1$ ). What one finds is that  $S \in (0, 1)$  if and only if  $y < y_b$ , where  $y_b$  was defined above, and  $\Delta_g \geq 0$  if and only if  $y \leq \hat{y}$ , where

$$\hat{y} = \Sigma \left[ \frac{u_g U_s(u_b) - u_b U_s(u_g)}{u_g - u_b} + \beta u_b + \gamma u_g \right].$$

Next, consider an equilibrium where  $\theta_g < 1$  and  $\theta_b = 1$ . Similar to the previous case, one shows that  $S \in (0, 1)$  if and only if  $y < y_g$ , where  $y_g$  was defined above, and  $\Delta_b \geq 0$  if and only if  $y \geq \hat{y}$ . Finally, consider the case where  $\theta_g < 1$  and  $\theta_b < 1$ . This cannot be an equilibrium, except possibly for parameter values in a set of measure zero, because we would have to find a value of  $S$  that satisfies both  $\Delta_g = 0$  and  $\Delta_b = 0$ . This completes the analysis of the different possible equilibria.

In terms of describing the results, we break things into two cases:  $y_g < y_b$  or  $y_g > y_b$ , corresponding to whether condition (16) does or does not hold. In the first case where  $y_g < y_b$ , one can easily show  $\hat{y} > y_b$ , and so based on the above analysis we conclude the following: for  $y > y_b$  the unique equilibrium is  $\theta_g = 1$  and  $\theta_b = 1$ ; and for  $y < y_b$  the unique equilibrium is  $\theta_g = 1$  and  $\theta_b < 1$ . In the second case where  $y_g > y_b$ , one can show  $\hat{y} < y_b$ , and so we conclude the following: for  $y > y_g$  the unique equilibrium is  $\theta_g = 1$  and  $\theta_b = 1$ ; for  $y \in (\hat{y}, y_g)$  the unique equilibrium is  $\theta_g < 1$  and  $\theta_b = 1$ ; and for  $y < \hat{y}$  the unique equilibrium is  $\theta_g = 1$  and  $\theta_b < 1$ . These results are all depicted in Fig. 6.

In particular, observe that when  $y$  is sufficiently big both good and bad cigarettes are smoked; when  $y$  is sufficiently small, good cigarettes are smoked and bad cigarettes are traded; and, as long as  $u_g$  is not too big relative to  $u_b$ , there is an intermediate range of  $y$  such that good cigarettes are traded while bad cigarettes are smoked. This last possibility—that good commodities circulate as money while bad commodities are consumed—seems to fly in the face of at least a naive version of Gresham's law that says "bad money drives out good." The explanation here is that the relative price of good cigarettes makes them more desirable for trading, even

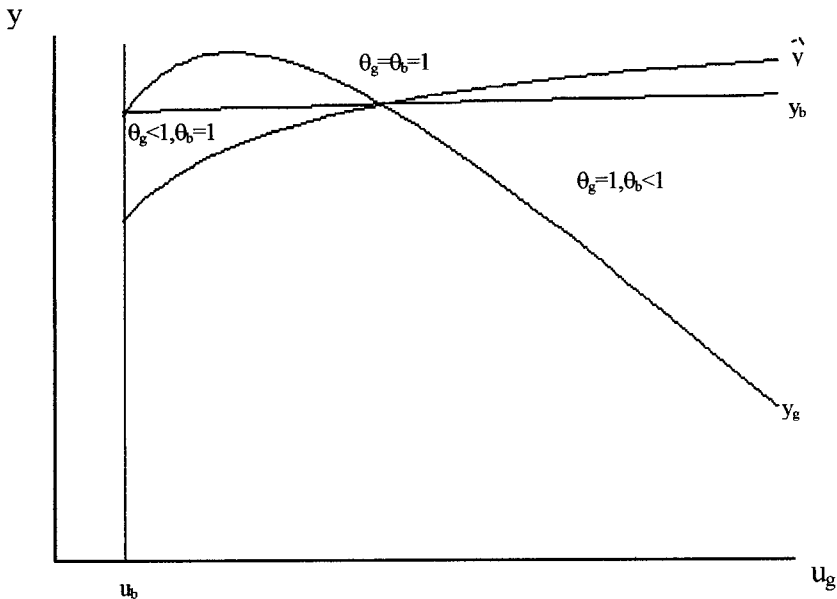


FIG. 6. Equilibria with heterogeneous quality.

though their intrinsic properties also make them more desirable for consumption, and the net outcome depends on the difficulty of trade and the relative intrinsic values of the commodities ( $y$  and  $u_g/u_b$ ).

As discussed by Friedman and Schwartz [4, footnote 27], a more sophisticated version of Gresham's law says that it applies "only when there is a fixed rate of exchange between the two" candidate monies. Following Williamson and Wright [17], suppose we introduce private information by assuming agents being offered a cigarette cannot tell whether it is good or bad until after a trade has been completed. Then obviously we must have  $q_g = q_b$ . This implies that trading good and bad cigarettes has the same payoff:  $V_g = V_b$ . Hence, given that  $u_g > u_b$ , we have  $\Delta_g = u_g + V_s - V_g > u_b + V_s - V_b = \Delta_b$ , and therefore it is not possible to have  $\theta_g < 1$  and  $\theta_b = 1$  (since that requires  $\Delta_g = 0 \leq \Delta_b$ ). In other words, anyone who is indifferent between smoking and trading a bad cigarette now must strictly prefer smoking to trading a good cigarette. This implies that we cannot have good money circulating while bad money is being consumed.

Given this, the only possible equilibria are  $\theta_g = \theta_b = 1$ , or  $\theta_g = 1$  and  $\theta_b \in (0, 1)$ , and in either case we have  $q = u_b$ . One can easily verify that  $\theta_b = 1$  if  $y \geq y_b$  and  $\theta_b \in (0, 1)$  if  $y < y_b$ . Hence, in Fig. 6, all that matters now is whether  $y$  is above or below  $y_b$ . In the region where we formerly

had  $\theta_g \in (0, 1)$  and  $\theta_b = 1$  with no private information—i.e., the region where the equilibrium flew in the face of Gresham's law—good cigarettes are now driven from circulation because they are forced to trade at the same price as bad cigarettes. There are two different cases in this region: if  $y \geq y_b$  then good cigarettes are driven out of circulation and replaced by direct barter; and if  $y < y_b$  then good cigarettes are driven from circulation and replaced by bad cigarettes as money (although not one-for-one: it is easy to show there are more bad cigarettes in circulation with private information than there were good cigarettes in circulation with full information).

We summarize the main results that have been derived in this section with the following proposition.

**PROPOSITION 4.** *Assume no depreciation, no fiat money, and two types of general commodities—good and bad. With full information,  $q_b = u_b$  and  $q_g = u_g$ , and there exist  $y_b, y_g$ , and  $\hat{y}$  defined in the text such that the following is true: if  $y > \max\{y_b, y_g\}$  then  $\theta_b = \theta_g = 1$ ; if  $y < \min\{y_b, \hat{y}\}$  then  $0 < \theta_b < 1$  and  $\theta_g = 1$ ; and for  $y \in (\hat{y}, y_g)$ , which is nonempty if and only if  $u_g$  is not too big relative to  $u_b$ , then  $\theta_b = 1$  and  $0 < \theta_g < 1$ . With private information,  $q_g = q_b = u_b$  and  $\theta_g = 1$  for all parameters, while  $\theta_b = 1$  if and only if  $y \geq y_b$ . These are all of the (steady state) equilibria.*

## 6. SUMMARY

We have analyzed when generally desired commodities will be used as money, depending on parameters that measure the extent of specialization, trading frictions, depreciation rates, and so on. We solved for the amount of money in circulation and its value. If the general good is ever consumed its value in trade is pinned down by its utility in consumption, but when it is used exclusively as money it trades at a premium over its consumption value. When commodity and fiat money both circulate, for some parameter values (e.g.,  $M$  small), the only equilibria is one where the two monies are treated as perfect substitutes. However, in other circumstances there is a multiplicity of equilibria, and fiat money may be less valuable than commodity money. When a small amount of fiat money is introduced it crowds out commodity money one-for-one, and has no other effect on the trading process. When too much fiat money is introduced, it drives commodity money from circulation and has real effects. We also analyzed the circumstances in which Gresham's law does or does not hold.

It seems that the model does fairly well at capturing some of the phenomena described by Radford [10] concerning the exchange process in a primitive economy like a POW camp. There are several other interesting



phenomena discussed by Radford that the model could in principle be used to address, such as the effects of time-varying cigarette endowments, for example. At a more general level, the model generates predictions about the exchange process that we think may also apply to more complicated economies. For example, it predicts that the private sector can create institutions like money without public intervention in terms of providing fiat currency. Of course, other models (e.g., Kiyotaki and Wright [5]) demonstrate this as well. However, something quite new here is that the model predicts that trying to control the amount of money in circulation is not easy, because changes in  $M$  imply changes in  $G$ , and over some range the net money supply is actually independent of  $M$ . We leave a more detailed exploration of policy and welfare along these lines to future research.

## APPENDIX

Here we derive (1). Begin by considering the probability that a general good holder becomes a special good holder. First, his general good can depreciate, which occurs with probability  $d$  per unit time. Second, he can trade for his desired special good, which happens with probability  $\alpha Sx$  per unit time, after which he produces a special good with probability  $\sigma$  (note that if he produces a general good he stores it, given he was holding a general good in the first place). Hence, the probability per unit time that a general good holder becomes a special good holder is  $P_{gs} = d + \alpha Sx\sigma$ .

Next consider the probability that a special good holder becomes a general good holder. First he must get his hands on the general good, which can happen in two ways: by trading for the general good, which occurs with probability  $\alpha(1 - S)x$  per unit time; or by trading for his special good, which occurs with probability  $\alpha Sxy$  per unit time, and then producing the general good, which occurs with probability  $1 - \sigma$ . In either case, we claim that *conditional* on having held the special good, he will store rather than consume the general good once he gets his hands on it with probability  $1 - \theta/S$  (which generally differs from the *unconditional* probability  $1 - \theta$ ). This can be seen as follows. First note that general good consumers never store the general good. Now let  $\omega$  denote the probability that an agent consumes the general good given he currently has the special good. The total number of special good holders includes all the general good consumers,  $\theta$ , plus the general good non-consumers who happen to hold the special good,  $(1 - \omega)S$ . Hence,  $S = \theta + (1 - \omega)S$ , which means  $\omega = \theta/S$ , which was our claim.

Therefore, the probability per unit time that a special good holder becomes a general good holder is  $P_{sg} = [\alpha(1 - S)x + \alpha Sxy(1 - \sigma)](S - \theta)/S$ . Equating  $SP_{sg} = (1 - S)P_{gs}$ , we get the steady state condition (1).

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